Computer algebra independent integration tests

`1_Algebraic_functions/1.2_Trinomial_products/1.2.2Quartic/1.2.2.6P(x)(dx)^m(a+bx^2+c`

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

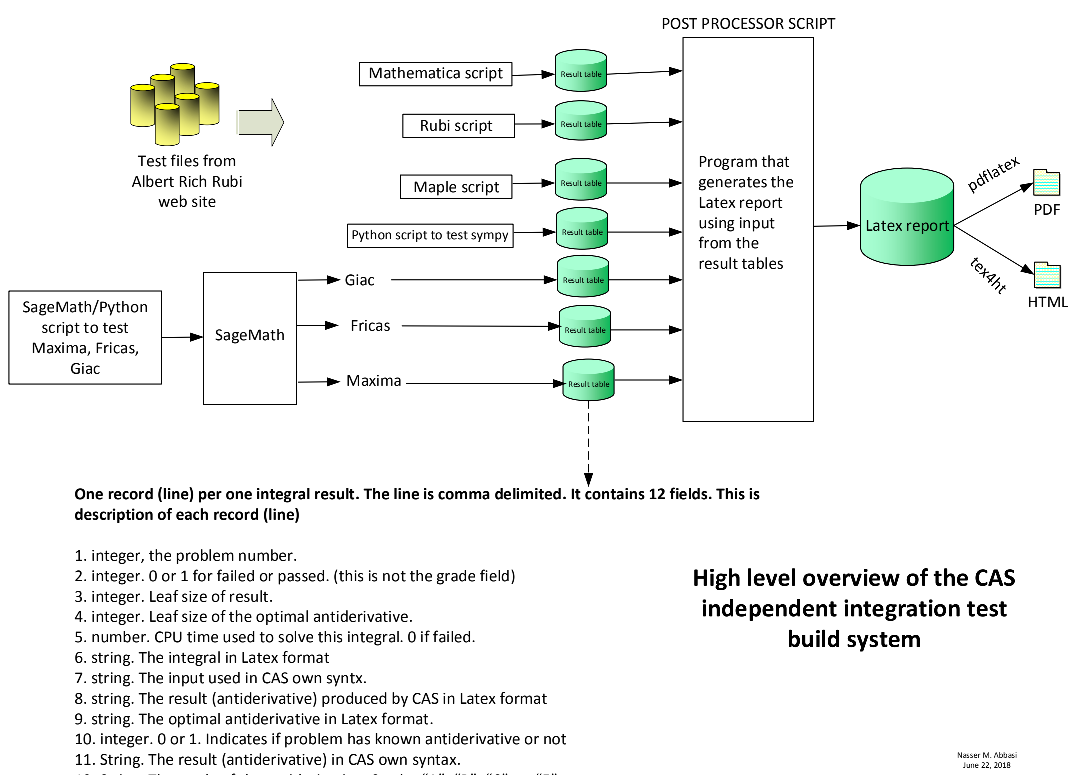
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (145)	% 0. (0)
Rubi in Sympy	% 66.21 (96)	% 33.79 (49)
Mathematica	% 100. (145)	% 0. (0)
Maple	% 98.62 (143)	% 1.38 (2)
Maxima	% 42.07 (61)	% 57.93 (84)
Fricas	% 79.31 (115)	% 20.69 (30)
Sympy	% 57.24 (83)	% 42.76 (62)
Giac	% 64.83 (94)	% 35.17 (51)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

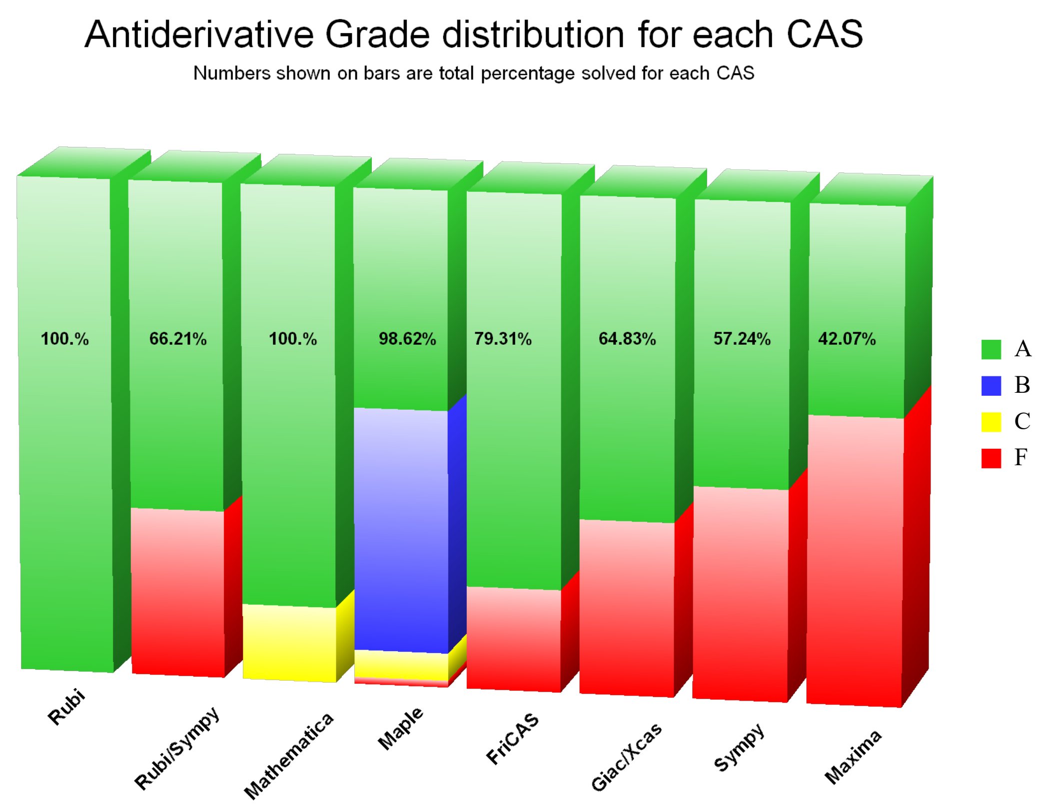
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

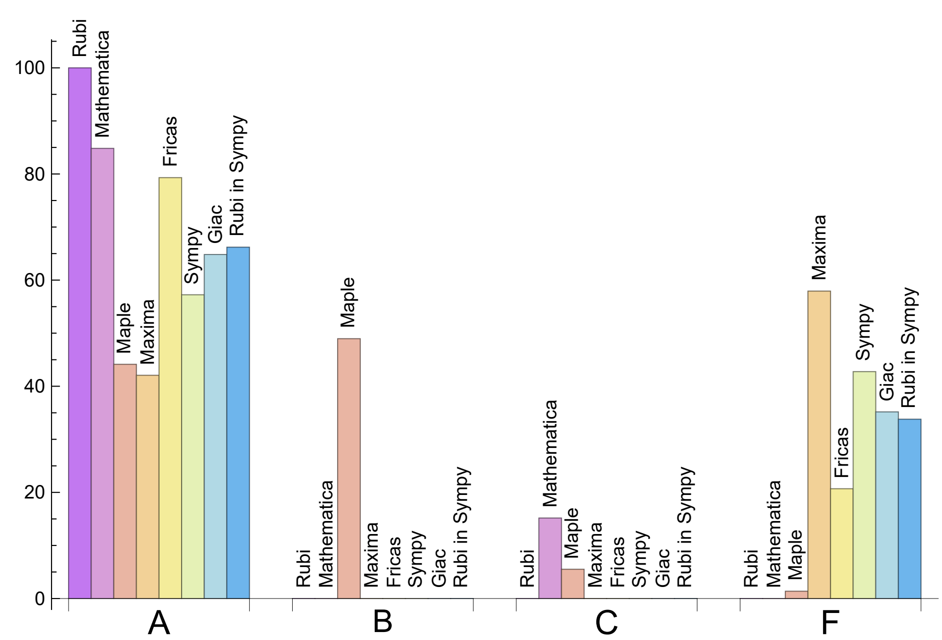
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	66.21	0.	0.	33.79
Mathematica	84.83	0.	15.17	0.
Maple	44.14	48.97	5.52	1.38
Maxima	42.07	0.	0.	57.93
Fricas	79.31	0.	0.	20.69
Sympy	57.24	0.	0.	42.76
Giac	64.83	0.	0.	35.17

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.86	207.48	1.03	179.	1.
Rubi in Sympy	52.89	201.95	1.06	194.	1.02
Mathematica	0.91	209.72	0.99	139.	1.
Maple	0.03	1220.35	3.6	227.	1.68
Maxima	0.75	111.54	1.23	81.	1.18
Fricas	1.07	1593.22	5.89	154.	1.97
Sympy	19.28	187.4	1.52	68.	0.94
Giac	0.48	95.7	0.96	77.5	1.19

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 31, 33, 35, 36, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 60, 61, 62, 68, 72, 74, 75, 76, 83, 100, 101, 125, 126, 129, 130, 132}

Not solved by Mathematica {}

Not solved by Maple {40, 41}

Not solved by Maxima {21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 141, 142, 143, 144, 145}

Not solved by Fricas {21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 68, 73, 126, 127, 128, 129, 130}

Not solved by Sympy {21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 143, 144, 145}

Not solved by Giac {29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 141, 142}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {40, 41}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	81	1	68	86	68
normalized size	1	1.	1.	0.82	1.09	0.01	0.92	1.16	0.92
time (sec)	N/A	0.155	0.034	0.002	0.698	0.228	0.103	0.281	23.235

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	81	1	68	86	0
normalized size	1	1.	1.	0.82	1.09	0.01	0.92	1.16	0.
time (sec)	N/A	0.118	0.022	0.001	0.704	0.24	0.101	0.279	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	1	65	82	0
normalized size	1	1.	1.	0.84	1.12	0.01	0.94	1.19	0.
time (sec)	N/A	0.081	0.027	0.001	0.707	0.232	0.101	0.281	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	60	74	74	63	81	0
normalized size	1	1.	1.	0.92	1.14	1.14	0.97	1.25	0.
time (sec)	N/A	0.085	0.03	0.004	0.701	0.255	1.162	0.278	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	74	84	58	77	0
normalized size	1	1.	1.	0.9	1.17	1.33	0.92	1.22	0.
time (sec)	N/A	0.106	0.041	0.008	0.7	0.249	1.226	0.281	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	74	84	60	78	0
normalized size	1	1.	0.92	0.92	1.17	1.33	0.95	1.24	0.
time (sec)	N/A	0.101	0.076	0.008	0.712	0.249	1.7	0.281	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	57	76	84	61	76	0
normalized size	1	1.	0.95	0.9	1.21	1.33	0.97	1.21	0.
time (sec)	N/A	0.106	0.088	0.009	0.704	0.246	2.534	0.28	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	58	76	84	61	77	0
normalized size	1	1.	0.98	0.92	1.21	1.33	0.97	1.22	0.
time (sec)	N/A	0.105	0.052	0.01	0.701	0.247	7.428	0.282	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	60	76	84	63	77	0
normalized size	1	1.	1.	0.95	1.21	1.33	1.	1.22	0.
time (sec)	N/A	0.102	0.136	0.009	0.699	0.251	21.738	0.282	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	63	80	84	66	81	63
normalized size	1	1.	1.	0.93	1.18	1.24	0.97	1.19	0.93
time (sec)	N/A	0.095	0.094	0.009	0.711	0.248	59.196	0.281	18.88

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	193	1	168	208	160
normalized size	1	1.	1.	0.89	1.21	0.01	1.06	1.31	1.01
time (sec)	N/A	0.422	0.088	0.001	0.701	0.23	0.172	0.28	48.55

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	193	1	163	208	0
normalized size	1	1.	1.	0.89	1.21	0.01	1.03	1.31	0.
time (sec)	N/A	0.358	0.086	0.001	0.698	0.242	0.17	0.282	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	189	1	165	204	0
normalized size	1	1.	1.	0.9	1.23	0.01	1.07	1.32	0.
time (sec)	N/A	0.283	0.057	0.001	0.698	0.231	0.168	0.278	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	150	149	186	186	156	201	0
normalized size	1	1.	1.	0.99	1.24	1.24	1.04	1.34	0.
time (sec)	N/A	0.225	0.076	0.004	0.702	0.257	1.713	0.281	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	145	147	185	196	156	198	0
normalized size	1	1.	1.	1.01	1.28	1.35	1.08	1.37	0.
time (sec)	N/A	0.289	0.31	0.01	0.702	0.246	1.847	0.286	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	139	148	188	196	151	200	0
normalized size	1	1.	0.93	0.99	1.26	1.32	1.01	1.34	0.
time (sec)	N/A	0.308	0.218	0.01	0.695	0.252	2.301	0.28	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	151	146	189	196	158	197	0
normalized size	1	1.	1.01	0.98	1.27	1.32	1.06	1.32	0.
time (sec)	N/A	0.305	0.174	0.01	0.69	0.248	3.336	0.286	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	130	144	188	196	151	192	0
normalized size	1	1.	0.88	0.97	1.27	1.32	1.02	1.3	0.
time (sec)	N/A	0.327	0.19	0.011	0.693	0.247	9.81	0.283	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	142	144	186	196	151	189	0
normalized size	1	1.	0.99	1.01	1.3	1.37	1.06	1.32	0.
time (sec)	N/A	0.34	0.172	0.012	0.694	0.25	31.229	0.28	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	144	148	189	196	153	190	0
normalized size	1	1.	0.97	0.99	1.27	1.32	1.03	1.28	0.
time (sec)	N/A	0.317	0.208	0.012	0.704	0.251	98.901	0.282	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	460	1622	0	0	0	1	347
normalized size	1	1.	1.36	4.78	0.	0.	0.	0.	1.02
time (sec)	N/A	4.187	1.234	0.068	0.	0.	0.	1.585	163.925

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	377	1171	0	0	0	1	0
normalized size	1	1.	1.36	4.21	0.	0.	0.	0.	0.
time (sec)	N/A	1.092	0.843	0.048	0.	0.	0.	1.305	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	360	1327	0	0	0	1	274
normalized size	1	1.	1.33	4.91	0.	0.	0.	0.	1.01
time (sec)	N/A	1.898	0.764	0.05	0.	0.	0.	1.439	104.165

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	240	728	0	0	0	1	209
normalized size	1	1.	1.08	3.26	0.	0.	0.	0.	0.94
time (sec)	N/A	0.521	0.79	0.035	0.	0.	0.	1.215	64.451

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	234	616	0	0	0	1	221
normalized size	1	1.	1.11	2.92	0.	0.	0.	0.	1.05
time (sec)	N/A	0.597	0.403	0.025	0.	0.	0.	1.069	53.705

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	285	488	0	0	0	1	216
normalized size	1	1.	1.24	2.13	0.	0.	0.	0.	0.94
time (sec)	N/A	0.628	0.932	0.038	0.	0.	0.	0.867	76.694

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	315	811	0	0	0	1	264
normalized size	1	1.	1.21	3.12	0.	0.	0.	0.	1.02
time (sec)	N/A	1.059	2.891	0.042	0.	0.	0.	1.095	100.472

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	377	1054	0	0	0	1	287
normalized size	1	1.	1.31	3.66	0.	0.	0.	0.	1.
time (sec)	N/A	1.102	1.954	0.057	0.	0.	0.	1.293	117.703

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	444	5283	0	0	0	0	376
normalized size	1	1.	1.08	12.82	0.	0.	0.	0.	0.91
time (sec)	N/A	2.982	2.805	0.13	0.	0.	0.	0.	179.049

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	358	3041	0	0	0	0	289
normalized size	1	1.	1.03	8.76	0.	0.	0.	0.	0.83
time (sec)	N/A	1.366	1.842	0.101	0.	0.	0.	0.	136.213

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	4063	0	0	0	0	0
normalized size	1	1.	1.06	11.41	0.	0.	0.	0.	0.
time (sec)	N/A	1.985	2.056	0.088	0.	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	335	1344	0	0	0	0	291
normalized size	1	1.	1.06	4.24	0.	0.	0.	0.	0.92
time (sec)	N/A	0.992	3.315	0.162	0.	0.	0.	0.	98.381

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	393	2851	0	0	0	0	0
normalized size	1	1.	1.07	7.75	0.	0.	0.	0.	0.
time (sec)	N/A	1.954	2.88	0.141	0.	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	458	4871	0	0	0	0	382
normalized size	1	1.	1.14	12.09	0.	0.	0.	0.	0.95
time (sec)	N/A	2.008	3.003	0.097	0.	0.	0.	0.	167.269

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	559	6960	0	0	0	0	0
normalized size	1	1.	1.09	13.54	0.	0.	0.	0.	0.
time (sec)	N/A	3.792	4.337	0.119	0.	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	655	6930	0	0	0	0	0
normalized size	1	1.	1.23	12.98	0.	0.	0.	0.	0.
time (sec)	N/A	4.173	5.209	0.122	0.	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	296	5520	0	5262	0	1	396
normalized size	1	1.	0.74	13.83	0.	13.19	0.	0.	0.99
time (sec)	N/A	0.945	4.458	0.017	0.	0.353	0.	0.347	120.922

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	187	2187	0	2164	0	1	248
normalized size	1	1.	0.72	8.41	0.	8.32	0.	0.	0.95
time (sec)	N/A	0.488	1.565	0.013	0.	0.346	0.	0.302	75.846

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	92	585	0	599	3735	1	122
normalized size	1	1.	0.67	4.27	0.	4.37	27.26	0.01	0.89
time (sec)	N/A	0.198	0.248	0.007	0.	0.312	7.635	0.297	39.764

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	438	0	0	0	0	0	335
normalized size	1	1.	1.19	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	1.481	0.429	0.041	0.	0.	0.	0.	117.687

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	685	670	1136	0	0	0	0	0	0
normalized size	1	0.98	1.66	0.	0.	0.	0.	0.	0.
time (sec)	N/A	5.212	9.019	0.041	0.	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	4063	0	0	0	0	0
normalized size	1	1.	1.06	11.41	0.	0.	0.	0.	0.
time (sec)	N/A	2.024	2.101	0.	0.	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	4063	0	0	0	0	0
normalized size	1	1.	1.06	11.41	0.	0.	0.	0.	0.
time (sec)	N/A	1.198	0.281	0.007	0.	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	4063	0	0	0	0	0
normalized size	1	1.	1.06	11.41	0.	0.	0.	0.	0.
time (sec)	N/A	1.11	0.273	0.007	0.	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	4063	0	0	0	0	0
normalized size	1	1.	1.06	11.41	0.	0.	0.	0.	0.
time (sec)	N/A	1.119	0.269	0.007	0.	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	4063	0	0	0	0	0
normalized size	1	1.	1.06	11.41	0.	0.	0.	0.	0.
time (sec)	N/A	1.233	0.268	0.007	0.	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	260	622	0	1	1392	413	0
normalized size	1	1.	0.95	2.28	0.	0.	5.1	1.51	0.
time (sec)	N/A	1.636	0.394	0.009	0.	0.612	149.096	0.318	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	193	474	0	1	1044	289	0
normalized size	1	1.	0.95	2.33	0.	0.	5.14	1.42	0.
time (sec)	N/A	0.85	0.262	0.008	0.	0.458	110.105	0.304	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	136	321	0	1	721	190	0
normalized size	1	1.	0.94	2.23	0.	0.01	5.01	1.32	0.
time (sec)	N/A	0.536	0.192	0.006	0.	0.324	63.468	0.295	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	100	211	0	1	498	134	0
normalized size	1	1.	0.97	2.05	0.	0.01	4.83	1.3	0.
time (sec)	N/A	0.347	0.119	0.005	0.	0.29	36.95	0.318	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	178	165	0	1	0	131	90
normalized size	1	1.	1.84	1.7	0.	0.01	0.	1.35	0.93
time (sec)	N/A	0.398	0.265	0.01	0.	0.447	0.	0.322	62.573

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	203	227	0	1	0	182	112
normalized size	1	1.	1.72	1.92	0.	0.01	0.	1.54	0.95
time (sec)	N/A	0.556	0.274	0.013	0.	0.527	0.	0.323	85.631

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	314	356	0	1	0	286	167
normalized size	1	1.	1.8	2.05	0.	0.01	0.	1.64	0.96
time (sec)	N/A	0.782	0.681	0.016	0.	0.927	0.	0.295	136.097

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	416	523	0	1	0	423	0
normalized size	1	1.	1.7	2.14	0.	0.	0.	1.73	0.
time (sec)	N/A	1.177	0.789	0.019	0.	2.047	0.	0.295	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	456	1450	0	20880	0	1	405
normalized size	1	1.	1.24	3.93	0.	56.59	0.	0.	1.1
time (sec)	N/A	10.939	1.169	0.05	0.	14.722	0.	1.822	164.076

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	365	1035	0	12641	0	1	294
normalized size	1	1.	1.29	3.67	0.	44.83	0.	0.	1.04
time (sec)	N/A	7.573	1.025	0.04	0.	3.656	0.	1.69	69.889

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	258	676	0	7814	0	1	221
normalized size	1	1.	1.18	3.09	0.	35.68	0.	0.	1.01
time (sec)	N/A	1.34	0.658	0.031	0.	1.895	0.	1.257	51.899

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	253	563	0	8006	0	1	235
normalized size	1	1.	1.19	2.64	0.	37.59	0.	0.	1.1
time (sec)	N/A	1.754	0.607	0.031	0.	0.931	0.	1.268	74.169

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	284	727	0	13298	0	1	258
normalized size	1	1.	1.06	2.72	0.	49.81	0.	0.	0.97
time (sec)	N/A	2.248	0.688	0.036	0.	5.123	0.	1.309	72.48

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	394	1121	0	21371	0	1	0
normalized size	1	1.	1.2	3.41	0.	64.96	0.	0.	0.
time (sec)	N/A	4.143	1.111	0.044	0.	18.487	0.	1.761	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	309	1764	0	1	0	0	0
normalized size	1	1.	0.97	5.51	0.	0.	0.	0.	0.
time (sec)	N/A	2.463	1.041	0.026	0.	0.658	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	236	1266	0	1	0	0	0
normalized size	1	1.	1.	5.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.928	0.683	0.021	0.	0.429	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	175	671	0	1	0	0	231
normalized size	1	1.	1.06	4.07	0.	0.01	0.	0.	1.4
time (sec)	N/A	0.562	0.488	0.019	0.	0.329	0.	0.	59.545

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	130	205	0	1	474	0	114
normalized size	1	1.	1.06	1.67	0.	0.01	3.85	0.	0.93
time (sec)	N/A	0.352	0.197	0.014	0.	0.295	131.162	0.	19.585

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	268	744	0	1	0	0	194
normalized size	1	1.	1.61	4.48	0.	0.01	0.	0.	1.17
time (sec)	N/A	0.826	0.928	0.024	0.	1.349	0.	0.	37.104

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	403	1156	0	1	0	0	265
normalized size	1	1.	1.72	4.94	0.	0.	0.	0.	1.13
time (sec)	N/A	1.444	1.432	0.03	0.	3.096	0.	0.	59.252

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	592	1675	0	1	0	0	384
normalized size	1	1.	1.8	5.09	0.	0.	0.	0.	1.17
time (sec)	N/A	2.363	3.257	0.038	0.	6.938	0.	0.	119.625

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	648	6868	0	0	0	0	0
normalized size	1	1.	1.18	12.49	0.	0.	0.	0.	0.
time (sec)	N/A	25.03	5.352	0.101	0.	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	511	5928	0	17006	0	0	478
normalized size	1	1.	1.17	13.6	0.	39.	0.	0.	1.1
time (sec)	N/A	12.662	3.499	0.086	0.	9.911	0.	0.	127.085

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	414	5528	0	12084	0	0	374
normalized size	1	1.	1.14	15.27	0.	33.38	0.	0.	1.03
time (sec)	N/A	5.507	2.363	0.079	0.	5.475	0.	0.	75.033

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	382	5350	0	12138	0	0	364
normalized size	1	1.	1.1	15.46	0.	35.08	0.	0.	1.05
time (sec)	N/A	3.999	2.183	0.12	0.	5.474	0.	0.	72.8

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	444	5142	0	17700	0	0	0
normalized size	1	1.	1.11	12.89	0.	44.36	0.	0.	0.
time (sec)	N/A	6.044	2.645	0.086	0.	11.933	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	575	548	6122	0	0	0	0	216
normalized size	1	1.	0.95	10.65	0.	0.	0.	0.	0.38
time (sec)	N/A	18.204	3.605	0.095	0.	0.	0.	0.	76.151

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	56	78	111	61	85	0
normalized size	1	1.	0.91	0.82	1.15	1.63	0.9	1.25	0.
time (sec)	N/A	0.199	0.048	0.024	0.731	0.26	0.412	0.286	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	51	72	104	54	78	0
normalized size	1	1.	1.	0.84	1.18	1.7	0.89	1.28	0.
time (sec)	N/A	0.18	0.05	0.022	0.737	0.274	0.416	0.275	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	46	65	97	48	72	0
normalized size	1	1.	1.	0.85	1.2	1.8	0.89	1.33	0.
time (sec)	N/A	0.173	0.043	0.022	0.723	0.256	0.407	0.271	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	41	58	90	42	61	48
normalized size	1	1.	1.	0.84	1.18	1.84	0.86	1.24	0.98
time (sec)	N/A	0.144	0.04	0.025	0.739	0.283	0.413	0.274	22.557

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	51	77	36	54	48
normalized size	1	1.	1.	0.86	1.21	1.83	0.86	1.29	1.14
time (sec)	N/A	0.084	0.031	0.022	0.725	0.252	0.395	0.274	17.386

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	59	96	39	63	56
normalized size	1	1.	1.	0.86	1.34	2.18	0.89	1.43	1.27
time (sec)	N/A	0.141	0.035	0.027	0.722	0.26	0.453	0.272	22.162

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	45	72	124	51	72	61
normalized size	1	1.	0.91	0.82	1.31	2.25	0.93	1.31	1.11
time (sec)	N/A	0.168	0.043	0.028	0.72	0.279	0.545	0.272	22.649

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	50	76	131	56	89	75
normalized size	1	1.	0.88	0.78	1.19	2.05	0.88	1.39	1.17
time (sec)	N/A	0.172	0.051	0.029	0.739	0.265	0.614	0.277	22.832

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	56	78	107	66	78	66
normalized size	1	1.	1.01	0.8	1.11	1.53	0.94	1.11	0.94
time (sec)	N/A	0.137	0.092	0.029	0.801	0.276	0.604	0.272	25.135

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	58	49	69	100	54	69	0
normalized size	1	1.	1.02	0.86	1.21	1.75	0.95	1.21	0.
time (sec)	N/A	0.13	0.087	0.016	0.793	0.266	0.586	0.275	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	46	65	93	53	65	53
normalized size	1	1.	1.02	0.82	1.16	1.66	0.95	1.16	0.95
time (sec)	N/A	0.122	0.083	0.019	0.797	0.265	0.579	0.272	21.632

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	50	41	58	99	48	58	48
normalized size	1	1.	1.02	0.84	1.18	2.02	0.98	1.18	0.98
time (sec)	N/A	0.114	0.07	0.016	0.793	0.271	0.581	0.276	19.503

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	46	38	54	92	44	54	44
normalized size	1	1.	0.96	0.79	1.12	1.92	0.92	1.12	0.92
time (sec)	N/A	0.058	0.072	0.015	0.792	0.27	0.58	0.27	10.463

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	43	61	104	49	61	48
normalized size	1	1.	0.96	0.81	1.15	1.96	0.92	1.15	0.91
time (sec)	N/A	0.119	0.098	0.018	0.792	0.277	0.656	0.27	19.025

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	48	70	117	56	70	32
normalized size	1	1.	0.9	0.77	1.13	1.89	0.9	1.13	0.52
time (sec)	N/A	0.132	0.112	0.021	0.794	0.271	0.731	0.271	18.923

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	53	77	124	61	77	41
normalized size	1	1.	0.88	0.77	1.12	1.8	0.88	1.12	0.59
time (sec)	N/A	0.144	0.112	0.023	0.797	0.282	0.806	0.27	22.456

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	77	58	84	131	66	84	48
normalized size	1	1.	1.01	0.76	1.11	1.72	0.87	1.11	0.63
time (sec)	N/A	0.154	0.127	0.023	0.794	0.291	0.905	0.27	25.909

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	71	64	96	154	75	82	76
normalized size	1	1.	0.88	0.79	1.19	1.9	0.93	1.01	0.94
time (sec)	N/A	0.177	0.12	0.018	0.796	0.265	0.8	0.272	37.397

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	62	92	159	75	78	76
normalized size	1	1.	0.82	0.78	1.15	1.99	0.94	0.98	0.95
time (sec)	N/A	0.165	0.101	0.018	0.797	0.284	0.775	0.272	32.119

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	56	85	151	68	72	70
normalized size	1	1.	0.8	0.75	1.13	2.01	0.91	0.96	0.93
time (sec)	N/A	0.156	0.112	0.018	0.786	0.269	0.791	0.27	27.439

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	54	81	146	65	68	66
normalized size	1	1.	0.76	0.75	1.12	2.03	0.9	0.94	0.92
time (sec)	N/A	0.122	0.109	0.016	0.798	0.28	0.764	0.274	20.768

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	53	81	146	65	68	66
normalized size	1	1.	0.78	0.74	1.12	2.03	0.9	0.94	0.92
time (sec)	N/A	0.119	0.105	0.017	0.791	0.272	0.796	0.271	19.413

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	53	81	146	65	68	66
normalized size	1	1.	0.78	0.74	1.12	2.03	0.9	0.94	0.92
time (sec)	N/A	0.084	0.104	0.016	0.797	0.266	0.806	0.272	13.393

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	63	58	88	158	70	74	71
normalized size	1	1.	0.8	0.73	1.11	2.	0.89	0.94	0.9
time (sec)	N/A	0.17	0.113	0.021	0.785	0.268	0.905	0.272	26.93

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	64	97	173	76	84	58
normalized size	1	1.	0.91	0.74	1.13	2.01	0.88	0.98	0.67
time (sec)	N/A	0.191	0.116	0.024	0.789	0.273	0.952	0.271	22.941

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	73	68	104	180	82	90	65
normalized size	1	1.	0.78	0.73	1.12	1.94	0.88	0.97	0.7
time (sec)	N/A	0.209	0.147	0.024	0.776	0.278	1.052	0.271	26.496

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	74	96	140	85	103	0
normalized size	1	1.	0.91	0.86	1.12	1.63	0.99	1.2	0.
time (sec)	N/A	0.239	0.079	0.014	0.79	0.271	0.453	0.271	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	69	89	132	80	96	0
normalized size	1	1.	0.9	0.85	1.1	1.63	0.99	1.19	0.
time (sec)	N/A	0.219	0.054	0.013	0.792	0.276	0.455	0.27	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	64	80	126	73	89	78
normalized size	1	1.	0.89	0.86	1.08	1.7	0.99	1.2	1.05
time (sec)	N/A	0.215	0.051	0.013	0.798	0.262	0.449	0.271	29.305

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	59	73	120	66	73	73
normalized size	1	1.	0.94	0.91	1.12	1.85	1.02	1.12	1.12
time (sec)	N/A	0.18	0.047	0.012	0.793	0.271	0.455	0.274	26.339

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	54	66	104	60	66	53
normalized size	1	1.	1.	0.93	1.14	1.79	1.03	1.14	0.91
time (sec)	N/A	0.119	0.038	0.012	0.784	0.274	0.442	0.273	17.505

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	93	58	74	127	65	84	80
normalized size	1	1.	1.41	0.88	1.12	1.92	0.98	1.27	1.21
time (sec)	N/A	0.199	0.103	0.016	0.784	0.268	0.487	0.272	25.851

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	97	63	89	158	75	89	85
normalized size	1	1.	1.37	0.89	1.25	2.23	1.06	1.25	1.2
time (sec)	N/A	0.219	0.086	0.02	0.799	0.271	0.583	0.272	26.494

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	105	68	96	165	80	107	95
normalized size	1	1.	1.31	0.85	1.2	2.06	1.	1.34	1.19
time (sec)	N/A	0.231	0.105	0.02	0.805	0.288	0.642	0.274	27.022

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	110	73	103	171	85	113	102
normalized size	1	1.	1.26	0.84	1.18	1.97	0.98	1.3	1.17
time (sec)	N/A	0.244	0.112	0.02	0.79	0.258	0.725	0.27	27.674

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	145	427	0	1040	71	0	352
normalized size	1	1.	0.58	1.72	0.	4.19	0.29	0.	1.42
time (sec)	N/A	0.731	0.337	0.106	0.	0.313	1.954	0.	62.769

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	132	419	0	1031	63	0	342
normalized size	1	1.	0.56	1.77	0.	4.35	0.27	0.	1.44
time (sec)	N/A	0.725	0.299	0.043	0.	0.303	1.969	0.	50.098

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	129	416	0	1013	58	0	338
normalized size	1	1.	0.56	1.79	0.	4.37	0.25	0.	1.46
time (sec)	N/A	0.701	0.319	0.037	0.	0.29	1.973	0.	44.317

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	121	412	0	1057	51	0	332
normalized size	1	1.	0.54	1.83	0.	4.7	0.23	0.	1.48
time (sec)	N/A	0.741	0.315	0.038	0.	0.301	1.932	0.	36.78

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	115	408	0	1038	48	0	328
normalized size	1	1.	0.51	1.82	0.	4.63	0.21	0.	1.46
time (sec)	N/A	0.618	0.525	0.036	0.	0.315	1.928	0.	27.55

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	126	414	0	1077	53	0	313
normalized size	1	1.	0.55	1.81	0.	4.7	0.23	0.	1.37
time (sec)	N/A	0.728	0.35	0.035	0.	0.336	2.003	0.	33.395

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	131	419	0	1112	60	0	320
normalized size	1	1.	0.55	1.76	0.	4.67	0.25	0.	1.34
time (sec)	N/A	0.745	0.555	0.038	0.	0.3	2.179	0.	37.298

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	140	424	0	1126	65	0	326
normalized size	1	1.	0.57	1.73	0.	4.6	0.27	0.	1.33
time (sec)	N/A	0.734	0.609	0.037	0.	0.291	2.18	0.	41.361

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	156	429	0	1202	82	0	362
normalized size	1	1.	0.64	1.77	0.	4.95	0.34	0.	1.49
time (sec)	N/A	0.854	0.423	0.033	0.	0.298	2.152	0.	65.015

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	155	426	0	1188	80	0	360
normalized size	1	1.	0.64	1.76	0.	4.91	0.33	0.	1.49
time (sec)	N/A	0.757	0.392	0.033	0.	0.316	2.192	0.	53.118

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	138	422	0	1173	71	0	354
normalized size	1	1.	0.59	1.8	0.	4.99	0.3	0.	1.51
time (sec)	N/A	0.758	0.699	0.033	0.	0.307	2.218	0.	43.103

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	129	418	0	1165	68	0	350
normalized size	1	1.	0.54	1.76	0.	4.89	0.29	0.	1.47
time (sec)	N/A	0.725	0.665	0.039	0.	0.302	2.128	0.	38.595

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	133	418	0	1079	68	0	350
normalized size	1	1.	0.54	1.7	0.	4.39	0.28	0.	1.42
time (sec)	N/A	0.763	0.607	0.038	0.	0.317	2.163	0.	37.209

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	129	418	0	1177	68	0	350
normalized size	1	1.	0.52	1.69	0.	4.75	0.27	0.	1.41
time (sec)	N/A	0.705	0.616	0.037	0.	0.316	2.092	0.	31.126

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	140	424	0	1189	73	0	333
normalized size	1	1.	0.55	1.68	0.	4.7	0.29	0.	1.32
time (sec)	N/A	0.833	0.785	0.037	0.	0.297	2.267	0.	38.16

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	139	429	0	1241	80	0	342
normalized size	1	1.	0.53	1.64	0.	4.74	0.31	0.	1.31
time (sec)	N/A	0.872	0.676	0.04	0.	0.292	2.338	0.	42.001

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	142	357	0	1	789	197	0
normalized size	1	1.	0.95	2.4	0.	0.01	5.3	1.32	0.
time (sec)	N/A	0.594	0.231	0.006	0.	0.357	164.926	0.298	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	721	8533	0	0	0	0	0
normalized size	1	1.	1.21	14.37	0.	0.	0.	0.	0.
time (sec)	N/A	28.539	6.641	0.115	0.	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	575	7318	0	0	0	0	547
normalized size	1	1.	1.22	15.54	0.	0.	0.	0.	1.16
time (sec)	N/A	15.823	4.393	0.093	0.	0.	0.	0.	168.249

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	512	8358	0	0	0	0	508
normalized size	1	1.	1.14	18.61	0.	0.	0.	0.	1.13
time (sec)	N/A	7.177	3.529	0.126	0.	0.	0.	0.	99.317

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	529	6807	0	0	0	0	0
normalized size	1	1.	1.15	14.8	0.	0.	0.	0.	0.
time (sec)	N/A	8.157	5.957	0.092	0.	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	612	7512	0	0	0	0	0
normalized size	1	1.	1.13	13.86	0.	0.	0.	0.	0.
time (sec)	N/A	19.333	4.885	0.095	0.	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	42	42	0	86	17
normalized size	1	1.	1.	1.05	2.1	2.1	0.	4.3	0.85
time (sec)	N/A	0.021	0.053	0.011	0.806	0.28	0.	0.303	13.607

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	278	149	145	398	554	0	328	0
normalized size	1	1.32	0.71	0.69	1.9	2.64	0.	1.56	0.
time (sec)	N/A	0.818	0.164	0.011	0.789	0.284	0.	0.3	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	213	116	109	293	435	0	239	177
normalized size	1	1.34	0.73	0.69	1.84	2.74	0.	1.5	1.11
time (sec)	N/A	0.556	0.128	0.01	0.789	0.274	0.	0.295	29.005

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	149	80	73	188	313	0	153	117
normalized size	1	1.37	0.73	0.67	1.72	2.87	0.	1.4	1.07
time (sec)	N/A	0.357	0.093	0.007	0.798	0.272	0.	0.286	19.777

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	151	84	143	0	270	304	4	114
normalized size	1	1.62	0.9	1.54	0.	2.9	3.27	0.04	1.23
time (sec)	N/A	0.482	0.21	0.048	0.	0.279	101.102	0.67	33.853

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	155	102	163	0	378	270	4	114
normalized size	1	1.57	1.03	1.65	0.	3.82	2.73	0.04	1.15
time (sec)	N/A	0.555	0.272	0.032	0.	0.266	144.224	0.684	36.235

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	182	135	222	0	567	0	4	143
normalized size	1	1.44	1.07	1.76	0.	4.5	0.	0.03	1.13
time (sec)	N/A	0.625	0.248	0.033	0.	0.304	0.	0.706	38.391

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	248	177	306	0	923	0	4	196
normalized size	1	1.17	0.83	1.44	0.	4.35	0.	0.02	0.92
time (sec)	N/A	0.759	0.41	0.052	0.	0.484	0.	0.743	42.982

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	245	136	273	309	921	0	257	194
normalized size	1	1.13	0.63	1.26	1.43	4.26	0.	1.19	0.9
time (sec)	N/A	0.601	0.229	0.046	0.787	0.387	0.	0.316	27.855

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	179	99	191	201	571	325	170	141
normalized size	1	1.4	0.77	1.49	1.57	4.46	2.54	1.33	1.1
time (sec)	N/A	0.26	0.149	0.029	0.796	0.283	104.666	0.309	19.001

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	155	82	148	0	424	287	0	114
normalized size	1	1.52	0.8	1.45	0.	4.16	2.81	0.	1.12
time (sec)	N/A	0.392	0.167	0.031	0.	0.277	118.302	0.	20.133

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	84	146	0	396	257	0	133
normalized size	1	1.	0.54	0.93	0.	2.52	1.64	0.	0.85
time (sec)	N/A	0.418	0.181	0.031	0.	0.289	167.912	0.	19.851

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	87	82	0	505	0	1	153
normalized size	1	1.	0.54	0.51	0.	3.16	0.	0.01	0.96
time (sec)	N/A	0.445	0.119	0.009	0.	0.334	0.	0.536	21.905

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	122	118	0	713	0	1	207
normalized size	1	1.	0.54	0.52	0.	3.15	0.	0.	0.92
time (sec)	N/A	0.545	0.15	0.009	0.	0.557	0.	0.864	26.383

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	157	154	0	922	0	1	262
normalized size	1	1.	0.54	0.53	0.	3.16	0.	0.	0.9
time (sec)	N/A	0.684	0.187	0.014	0.	1.143	0.	1.167	31.379

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [0.4643]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	26	0.038
2	A	2	1	1.	24	0.042
3	A	2	1	1.	23	0.043
4	A	2	1	1.	26	0.038
5	A	2	1	1.	26	0.038
6	A	2	1	1.	26	0.038
7	A	2	1	1.	26	0.038
8	A	2	1	1.	26	0.038
9	A	2	1	1.	26	0.038
10	A	2	1	1.	26	0.038
11	A	2	1	1.	28	0.036
12	A	2	1	1.	26	0.038
13	A	2	1	1.	25	0.04
14	A	2	1	1.	28	0.036
15	A	2	1	1.	28	0.036
16	A	2	1	1.	28	0.036
17	A	2	1	1.	28	0.036
18	A	2	1	1.	28	0.036
19	A	2	1	1.	28	0.036
20	A	2	1	1.	28	0.036
21	A	13	11	1.	28	0.393
22	A	12	11	1.	28	0.393
23	A	11	10	1.	28	0.357
24	A	10	9	1.	26	0.346
25	A	8	7	1.	25	0.28
26	A	12	10	1.	28	0.357
27	A	13	12	1.	28	0.429
28	A	13	11	1.	28	0.393
29	A	11	10	1.	28	0.357
30	A	10	9	1.	28	0.321
31	A	10	9	1.	28	0.321
32	A	10	9	1.	26	0.346
33	A	10	9	1.	25	0.36
34	A	14	12	1.	28	0.429
35	A	15	13	1.	28	0.464
36	A	15	13	1.	28	0.464
37	A	2	1	1.	30	0.033
38	A	2	1	1.	30	0.033
39	A	2	1	1.	28	0.036

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	8	5	1.	30	0.167
41	A	10	6	0.98	30	0.2
42	A	10	9	1.	28	0.321
43	A	11	10	1.	30	0.333
44	A	11	10	1.	31	0.323
45	A	11	10	1.	34	0.294
46	A	11	10	1.	34	0.294
47	A	7	6	1.	30	0.2
48	A	7	6	1.	30	0.2
49	A	7	6	1.	30	0.2
50	A	7	6	1.	28	0.214
51	A	7	6	1.	30	0.2
52	A	7	6	1.	30	0.2
53	A	7	6	1.	30	0.2
54	A	7	6	1.	30	0.2
55	A	5	3	1.	30	0.1
56	A	5	3	1.	30	0.1
57	A	5	3	1.	27	0.111
58	A	5	3	1.	30	0.1
59	A	5	3	1.	30	0.1
60	A	5	3	1.	30	0.1
61	A	8	7	1.	30	0.233
62	A	7	7	1.	30	0.233
63	A	6	6	1.	30	0.2
64	A	5	5	1.	28	0.179
65	A	8	7	1.	30	0.233
66	A	8	7	1.	30	0.233
67	A	8	7	1.	30	0.233
68	A	6	4	1.	30	0.133
69	A	6	4	1.	30	0.133
70	A	4	3	1.	30	0.1
71	A	4	3	1.	27	0.111
72	A	6	4	1.	30	0.133
73	A	6	4	1.	30	0.133
74	A	7	5	1.	31	0.161
75	A	7	5	1.	31	0.161
76	A	7	5	1.	31	0.161
77	A	7	5	1.	31	0.161
78	A	5	4	1.	29	0.138
79	A	4	3	1.	31	0.097
80	A	4	3	1.	31	0.097
81	A	4	3	1.	31	0.097
82	A	6	4	1.	31	0.129
83	A	6	4	1.	31	0.129
84	A	6	4	1.	31	0.129
85	A	6	4	1.	31	0.129
86	A	4	3	1.	28	0.107

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
87	A	5	3	1.	31	0.097
88	A	5	3	1.	31	0.097
89	A	5	3	1.	31	0.097
90	A	5	3	1.	31	0.097
91	A	7	5	1.	31	0.161
92	A	7	5	1.	31	0.161
93	A	7	5	1.	31	0.161
94	A	5	4	1.	31	0.129
95	A	5	4	1.	31	0.129
96	A	5	4	1.	28	0.143
97	A	6	3	1.	31	0.097
98	A	6	3	1.	31	0.097
99	A	6	3	1.	31	0.097
100	A	8	7	1.	31	0.226
101	A	8	7	1.	31	0.226
102	A	8	7	1.	31	0.226
103	A	8	7	1.	31	0.226
104	A	6	6	1.	29	0.207
105	A	8	7	1.	31	0.226
106	A	8	7	1.	31	0.226
107	A	8	7	1.	31	0.226
108	A	8	7	1.	31	0.226
109	A	12	7	1.	31	0.226
110	A	12	7	1.	31	0.226
111	A	12	7	1.	31	0.226
112	A	12	7	1.	31	0.226
113	A	10	6	1.	28	0.214
114	A	12	7	1.	31	0.226
115	A	12	7	1.	31	0.226
116	A	12	7	1.	31	0.226
117	A	13	8	1.	31	0.258
118	A	13	8	1.	31	0.258
119	A	13	8	1.	31	0.258
120	A	11	7	1.	31	0.226
121	A	11	7	1.	31	0.226
122	A	11	7	1.	28	0.25
123	A	13	7	1.	31	0.226
124	A	13	7	1.	31	0.226
125	A	7	6	1.	33	0.182
126	A	6	4	1.	35	0.114
127	A	6	4	1.	35	0.114
128	A	4	3	1.	32	0.094
129	A	6	4	1.	35	0.114
130	A	6	4	1.	35	0.114
131	A	1	1	1.	42	0.024
132	A	5	4	1.32	35	0.114
133	A	4	3	1.34	35	0.086

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
134	A	4	3	1.37	33	0.091
135	A	6	5	1.62	35	0.143
136	A	6	6	1.57	35	0.171
137	A	6	6	1.44	35	0.171
138	A	7	7	1.17	35	0.2
139	A	6	6	1.13	35	0.171
140	A	5	5	1.4	32	0.156
141	A	6	6	1.52	35	0.171
142	A	6	6	1.	35	0.171
143	A	5	4	1.	35	0.114
144	A	6	5	1.	35	0.143
145	A	7	5	1.	35	0.143

3 Listing of integrals

3.1 $\int x^2 \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=74

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + (
(A*c + b*C)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Rubi [A] time = 0.15507, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + (
(A*c + b*C)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Rubi in Sympy [A] time = 23.2355, size = 68, normalized size = 0.92

$$\frac{Aax^3}{3}+\frac{Bax^4}{4}+\frac{Bbx^6}{6}+\frac{Bcx^8}{8}+\frac{Ccx^9}{9}+x^7\left(\frac{Ac}{7}+\frac{Cb}{7}\right)+x^5\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + B*b*x**6/6 + B*c*x**8/8 + C*c*x**9/9 +
x**7*(A*c/7 + C*b/7) + x**5*(A*b/5 + C*a/5)
```

Mathematica [A] time = 0.0339102, size = 74, normalized size = 1.

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + (
(A*c + b*C)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Maple [A] time = 0.002, size = 61, normalized size = 0.8

$$\frac{aAx^3}{3}+\frac{aBx^4}{4}+\frac{(Ab+aC)\,x^5}{5}+\frac{bBx^6}{6}+\frac{(Ac+bC)\,x^7}{7}+\frac{Bcx^8}{8}+\frac{cCx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*
b)*x^7+1/8*B*c*x^8+1/9*c*C*x^9
```

Maxima [A] time = 0.698025, size = 81, normalized size = 1.09

$$\frac{1}{9}\,Ccx^{9} + \frac{1}{8}\,Bcx^{8} + \frac{1}{6}\,Bbx^{6} + \frac{1}{7}\,(Cb + Ac)x^{7} + \frac{1}{4}\,Bax^{4} + \frac{1}{5}\,(Ca + Ab)x^{5} + \frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)*x^2,x, algorithm="maxima")
```

```
[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1
/4*B*a*x^4 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3
```

Fricas [A] time = 0.228166, size = 1, normalized size = 0.01

$$\frac{1}{9}x^{9}cC + \frac{1}{8}x^{8}cB + \frac{1}{7}x^{7}bC + \frac{1}{7}x^{7}cA + \frac{1}{6}x^{6}bB + \frac{1}{5}x^{5}aC + \frac{1}{5}x^{5}bA + \frac{1}{4}x^{4}aB + \frac{1}{3}x^{3}aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)*x^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c*C + 1/8*x^8*c*B + 1/7*x^7*b*C + 1/7*x^7*c*A + 1/6*x^6*b
*B + 1/5*x^5*a*C + 1/5*x^5*b*A + 1/4*x^4*a*B + 1/3*x^3*a*A
```

Sympy [A] time = 0.103473, size = 68, normalized size = 0.92

$$\frac{Aax^{3}}{3} + \frac{Bax^{4}}{4} + \frac{Bbx^{6}}{6} + \frac{Bcx^{8}}{8} + \frac{Ccx^{9}}{9} + x^{7}\left(\frac{Ac}{7} + \frac{Cb}{7}\right) + x^{5}\left(\frac{Ab}{5} + \frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + B*b*x**6/6 + B*c*x**8/8 + C*c*x**9/9 +
x**7*(A*c/7 + C*b/7) + x**5*(A*b/5 + C*a/5)
```

GIAC/XCAS [A] time = 0.280558, size = 86, normalized size = 1.16

$$\frac{1}{9}\,Ccx^{9} + \frac{1}{8}\,Bcx^{8} + \frac{1}{7}\,Cbx^{7} + \frac{1}{7}\,Acx^{7} + \frac{1}{6}\,Bbx^{6} + \frac{1}{5}\,Cax^{5} + \frac{1}{5}\,Abx^{5} + \frac{1}{4}\,Bax^{4} + \frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)*x^2,x, algorithm="giac")
```

```
[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/7*C*b*x^7 + 1/7*A*c*x^7 + 1/6*B*b*x
^6 + 1/5*C*a*x^5 + 1/5*A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

3.2 $\int x \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=74

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + (
(A*c + b*C)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Rubi [A] time = 0.117787, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

Antiderivative was successfully verified.

```
[In]  Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + (
(A*c + b*C)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Aa\int x\,dx+\frac{Bax^3}{3}+\frac{Bbx^5}{5}+\frac{Bcx^7}{7}+\frac{Ccx^8}{8}+x^6\left(\frac{Ac}{6}+\frac{Cb}{6}\right)+x^4\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*Integral(x, x) + B*a*x**3/3 + B*b*x**5/5 + B*c*x**7/7 + C*c*x
**8/8 + x**6*(A*c/6 + C*b/6) + x**4*(A*b/4 + C*a/4)
```

Mathematica [A] time = 0.0221838, size = 74, normalized size = 1.

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + (
(A*c + b*C)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Maple [A] time = 0.001, size = 61, normalized size = 0.8

$$\frac{aAx^2}{2}+\frac{aBx^3}{3}+\frac{(Ab+aC)\,x^4}{4}+\frac{bBx^5}{5}+\frac{(Ac+bC)\,x^6}{6}+\frac{Bcx^7}{7}+\frac{cCx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*
b)*x^6+1/7*B*c*x^7+1/8*c*C*x^8
```

Maxima [A] time = 0.703562, size = 81, normalized size = 1.09

$$\frac{1}{8}\,Ccx^{8} + \frac{1}{7}\,Bcx^{7} + \frac{1}{5}\,Bbx^{5} + \frac{1}{6}\,(Cb + Ac)x^{6} + \frac{1}{3}\,Bax^{3} + \frac{1}{4}\,(Ca + Ab)x^{4} + \frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)*x,x, algorithm="maxima")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1
/3*B*a*x^3 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

Fricas [A] time = 0.240343, size = 1, normalized size = 0.01

$$\frac{1}{8}x^{8}cC + \frac{1}{7}x^{7}cB + \frac{1}{6}x^{6}bC + \frac{1}{6}x^{6}cA + \frac{1}{5}x^{5}bB + \frac{1}{4}x^{4}aC + \frac{1}{4}x^{4}bA + \frac{1}{3}x^{3}aB + \frac{1}{2}x^{2}aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)*x,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c*C + 1/7*x^7*c*B + 1/6*x^6*b*C + 1/6*x^6*c*A + 1/5*x^5*b
*B + 1/4*x^4*a*C + 1/4*x^4*b*A + 1/3*x^3*a*B + 1/2*x^2*a*A
```

Sympy [A] time = 0.101254, size = 68, normalized size = 0.92

$$\frac{Aax^{2}}{2} + \frac{Bax^{3}}{3} + \frac{Bbx^{5}}{5} + \frac{Bcx^{7}}{7} + \frac{Ccx^{8}}{8} + x^{6}\left(\frac{Ac}{6} + \frac{Cb}{6}\right) + x^{4}\left(\frac{Ab}{4} + \frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x**2/2 + B*a*x**3/3 + B*b*x**5/5 + B*c*x**7/7 + C*c*x**8/8 +
x**6*(A*c/6 + C*b/6) + x**4*(A*b/4 + C*a/4)
```

GIAC/XCAS [A] time = 0.278687, size = 86, normalized size = 1.16

$$\frac{1}{8}\,Ccx^{8} + \frac{1}{7}\,Bcx^{7} + \frac{1}{6}\,Cbx^{6} + \frac{1}{6}\,Acx^{6} + \frac{1}{5}\,Bbx^{5} + \frac{1}{4}\,Cax^{4} + \frac{1}{4}\,Abx^{4} + \frac{1}{3}\,Bax^{3} + \frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)*x,x, algorithm="giac")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/6*C*b*x^6 + 1/6*A*c*x^6 + 1/5*B*b*x
^5 + 1/4*C*a*x^4 + 1/4*A*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

3.3 $\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=69

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c +
 b*C)*x^5)/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Rubi [A] time = 0.0811486, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c +
 b*C)*x^5)/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Ba\int x\,dx+\frac{Bbx^4}{4}+\frac{Bcx^6}{6}+\frac{Ccx^7}{7}+a\int A\,dx+x^5\left(\frac{Ac}{5}+\frac{Cb}{5}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] B*a*Integral(x, x) + B*b*x**4/4 + B*c*x**6/6 + C*c*x**7/7 + a*Int
egral(A, x) + x**5*(A*c/5 + C*b/5) + x**3*(A*b/3 + C*a/3)
```

Mathematica [A] time = 0.02666, size = 69, normalized size = 1.

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c +
 b*C)*x^5)/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$aAx+\frac{aBx^2}{2}+\frac{(Ab+aC)\,x^3}{3}+\frac{bBx^4}{4}+\frac{(Ac+bC)\,x^5}{5}+\frac{Bcx^6}{6}+\frac{cCx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5
+1/6*B*c*x^6+1/7*c*C*x^7
```

Maxima [A] time = 0.706545, size = 77, normalized size = 1.12

$$\frac{1}{7}Ccx^7+\frac{1}{6}Bcx^6+\frac{1}{4}Bbx^4+\frac{1}{5}(Cb+Ac)x^5+\frac{1}{2}Bax^2+\frac{1}{3}(Ca+Ab)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1
/2*B*a*x^2 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

Fricas [A] time = 0.231648, size = 1, normalized size = 0.01

$$\frac{1}{7}x^7cC+\frac{1}{6}x^6cB+\frac{1}{5}x^5bC+\frac{1}{5}x^5cA+\frac{1}{4}x^4bB+\frac{1}{3}x^3aC+\frac{1}{3}x^3bA+\frac{1}{2}x^2aB+xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c*C + 1/6*x^6*c*B + 1/5*x^5*b*C + 1/5*x^5*c*A + 1/4*x^4*b
*B + 1/3*x^3*a*C + 1/3*x^3*b*A + 1/2*x^2*a*B + x*a*A
```

Sympy [A] time = 0.100609, size = 65, normalized size = 0.94

$$Aax+\frac{Bax^2}{2}+\frac{Bbx^4}{4}+\frac{Bcx^6}{6}+\frac{Ccx^7}{7}+x^5\left(\frac{Ac}{5}+\frac{Cb}{5}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x + B*a*x**2/2 + B*b*x**4/4 + B*c*x**6/6 + C*c*x**7/7 + x**5*
(A*c/5 + C*b/5) + x**3*(A*b/3 + C*a/3)
```

GIAC/XCAS [A] time = 0.2813, size = 82, normalized size = 1.19

$$\frac{1}{7}Ccx^7+\frac{1}{6}Bcx^6+\frac{1}{5}Cbx^5+\frac{1}{5}Acx^5+\frac{1}{4}Bbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Abx^3+\frac{1}{2}Bax^2+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A),x, algorithm="giac")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/5*C*b*x^5 + 1/5*A*c*x^5 + 1/4*B*b*x
^4 + 1/3*C*a*x^3 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

3.4 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} dx$

Optimal. Leaf size=65

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 +
 (B*c*x^5)/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Rubi [A] time = 0.0849859, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 +
 (B*c*x^5)/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Aa\log(x)+\frac{Bbx^3}{3}+\frac{Bcx^5}{5}+\frac{Ccx^6}{6}+a\int B\,dx+x^4\left(\frac{Ac}{4}+\frac{Cb}{4}\right)+(Ab+Ca)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] A*a*log(x) + B*b*x**3/3 + B*c*x**5/5 + C*c*x**6/6 + a*Integral(B,
 x) + x**4*(A*c/4 + C*b/4) + (A*b + C*a)*Integral(x, x)
```

Mathematica [A] time = 0.030041, size = 65, normalized size = 1.

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 +
 (B*c*x^5)/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Maple [A] time = 0.004, size = 60, normalized size = 0.9

$$\frac{cCx^6}{6}+\frac{Bcx^5}{5}+\frac{Ax^4c}{4}+\frac{Cx^4b}{4}+\frac{bBx^3}{3}+\frac{Ax^2b}{2}+\frac{Cx^2a}{2}+aBx+aA\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x)
```

```
[Out] 1/6*c*C*x^6+1/5*B*c*x^5+1/4*A*x^4*c+1/4*C*x^4*b+1/3*b*B*x^3+1/2*A
*x^2*b+1/2*C*x^2*a+a*B*x+a*A*ln(x)
```

Maxima [A] time = 0.700626, size = 74, normalized size = 1.14

$$\frac{1}{6}Ccx^6 + \frac{1}{5}Bcx^5 + \frac{1}{3}Bbx^3 + \frac{1}{4}(Cb + Ac)x^4 + Bax + \frac{1}{2}(Ca + Ab)x^2 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x,x, algorithm="maxima")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B
*a*x + 1/2*(C*a + A*b)*x^2 + A*a*log(x)
```

Fricas [A] time = 0.255305, size = 74, normalized size = 1.14

$$\frac{1}{6}Ccx^6 + \frac{1}{5}Bcx^5 + \frac{1}{3}Bbx^3 + \frac{1}{4}(Cb + Ac)x^4 + Bax + \frac{1}{2}(Ca + Ab)x^2 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x,x, algorithm="fricas")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B
*a*x + 1/2*(C*a + A*b)*x^2 + A*a*log(x)
```

Sympy [A] time = 1.16242, size = 63, normalized size = 0.97

$$Aa\log(x) + Bax + \frac{Bbx^3}{3} + \frac{Bcx^5}{5} + \frac{Ccx^6}{6} + x^4\left(\frac{Ac}{4} + \frac{Cb}{4}\right) + x^2\left(\frac{Ab}{2} + \frac{Ca}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*c*x**6/6 + x**4*
(A*c/4 + C*b/4) + x**2*(A*b/2 + C*a/2)
```

GIAC/XCAS [A] time = 0.278075, size = 81, normalized size = 1.25

$$\frac{1}{6}Ccx^6 + \frac{1}{5}Bcx^5 + \frac{1}{4}Cbx^4 + \frac{1}{4}Acx^4 + \frac{1}{3}Bbx^3 + \frac{1}{2}Cax^2 + \frac{1}{2}Abx^2 + Bax + Aa\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x,x, algorithm="giac")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/4*C*b*x^4 + 1/4*A*c*x^4 + 1/3*B*b*x
^3 + 1/2*C*a*x^2 + 1/2*A*b*x^2 + B*a*x + A*a*ln(abs(x))
```

3.5 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx$

Optimal. Leaf size=63

$$x(aC+Ab)-\frac{aA}{x}+aB\log(x)+\frac{1}{3}x^3(Ac+bC)+\frac{1}{2}bBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 +
(B*c*x^4)/4 + (c*C*x^5)/5 + a*B*Log[x]
```

Rubi [A] time = 0.105777, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$x(aC+Ab)-\frac{aA}{x}+aB\log(x)+\frac{1}{3}x^3(Ac+bC)+\frac{1}{2}bBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 +
(B*c*x^4)/4 + (c*C*x^5)/5 + a*B*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{x}+Ba\log(x)+Bb\int x\,dx+\frac{Bcx^4}{4}+\frac{Ccx^5}{5}+x^3\left(\frac{Ac}{3}+\frac{Cb}{3}\right)+\frac{(Ab+Ca)\int A\,dx}{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + B*b*Integral(x, x) + B*c*x**4/4 + C*c*x**5/
5 + x**3*(A*c/3 + C*b/3) + (A*b + C*a)*Integral(A, x)/A
```

Mathematica [A] time = 0.0406286, size = 63, normalized size = 1.

$$x(aC+Ab)-\frac{aA}{x}+aB\log(x)+\frac{1}{3}x^3(Ac+bC)+\frac{1}{2}bBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 +
(B*c*x^4)/4 + (c*C*x^5)/5 + a*B*Log[x]
```

Maple [A] time = 0.008, size = 57, normalized size = 0.9

$$\frac{cCx^5}{5}+\frac{Bcx^4}{4}+\frac{Ax^3c}{3}+\frac{Cx^3b}{3}+\frac{bBx^2}{2}+Axb+Cxa+aB\ln(x)-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x)
```

```
[Out] 1/5*c*C*x^5+1/4*B*c*x^4+1/3*A*x^3*c+1/3*C*x^3*b+1/2*b*B*x^2+A*x*b
+C*x*a+a*B*ln(x)-a*A/x
```

Maxima [A] time = 0.69975, size = 74, normalized size = 1.17

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Bcx^4 + \frac{1}{2}Bbx^2 + \frac{1}{3}(Cb + Ac)x^3 + Ba\log(x) + (Ca + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/3*(C*b + A*c)*x^3 + B
*a*log(x) + (C*a + A*b)*x - A*a/x
```

Fricas [A] time = 0.249013, size = 84, normalized size = 1.33

$$\frac{12\,Ccx^6 + 15\,Bcx^5 + 30\,Bbx^3 + 20\,(Cb + Ac)x^4 + 60\,Bax\log(x) + 60\,(Ca + Ab)x^2 - 60\,Aa}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c*x^6 + 15*B*c*x^5 + 30*B*b*x^3 + 20*(C*b + A*c)*x^4 +
 60*B*a*x*log(x) + 60*(C*a + A*b)*x^2 - 60*A*a)/x
```

Sympy [A] time = 1.22588, size = 58, normalized size = 0.92

$$-\frac{Aa}{x} + Ba\log(x) + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} + \frac{Ccx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Cb}{3}\right) + x\,(Ab + Ca)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3
*(A*c/3 + C*b/3) + x*(A*b + C*a)
```

GIAC/XCAS [A] time = 0.280626, size = 77, normalized size = 1.22

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Bcx^4 + \frac{1}{3}Cbx^3 + \frac{1}{3}Acx^3 + \frac{1}{2}Bbx^2 + Cax + Abx + Ba\ln(|x|) - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*b*x^3 + 1/3*A*c*x^3 + 1/2*B*b*x
^2 + C*a*x + A*b*x + B*a*ln(abs(x)) - A*a/x
```

3.6 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx$

Optimal. Leaf size=63

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}-\frac{aB}{x}+\frac{1}{2}x^2(Ac+bC)+bBx+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4$$

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + b*B*x + ((A*c + b*C)*x^2)/2 + (B*c*x^3
)/3 + (c*C*x^4)/4 + (A*b + a*C)*Log[x]
```

Rubi [A] time = 0.100985, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}-\frac{aB}{x}+\frac{1}{2}x^2(Ac+bC)+bBx+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + b*B*x + ((A*c + b*C)*x^2)/2 + (B*c*x^3
)/3 + (c*C*x^4)/4 + (A*b + a*C)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{2x^2}-\frac{Ba}{x}+\frac{Bcx^3}{3}+\frac{Ccx^4}{4}+b\int B\,dx+(Ab+Ca)\log(x)+(Ac+Cb)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -A*a/(2*x**2) - B*a/x + B*c*x**3/3 + C*c*x**4/4 + b*Integral(B, x
) + (A*b + C*a)*log(x) + (A*c + C*b)*Integral(x, x)
```

Mathematica [A] time = 0.075531, size = 58, normalized size = 0.92

$$\log(x)(aC+Ab)-\frac{a(A+2Bx)}{2x^2}+\frac{1}{12}x\left(cx\left(6A+4Bx+3Cx^2\right)+6b(2B+Cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -(a*(A + 2*B*x))/(2*x^2) + (x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x
 + 3*C*x^2)))/12 + (A*b + a*C)*Log[x]
```

Maple [A] time = 0.008, size = 58, normalized size = 0.9

$$\frac{cCx^4}{4}+\frac{Bcx^3}{3}+\frac{Acx^2}{2}+\frac{Cx^2b}{2}+bBx+A\ln(x)\,b+C\ln(x)\,a-\frac{Ba}{x}-\frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x)
```

```
[Out] 1/4*c*C*x^4+1/3*B*c*x^3+1/2*A*c*x^2+1/2*C*x^2*b+b*B*x+A*ln(x)*b+C
*ln(x)*a-a*B/x-1/2*a*A/x^2
```

Maxima [A] time = 0.712012, size = 74, normalized size = 1.17

$$\frac{1}{4}Ccx^4 + \frac{1}{3}Bcx^3 + Bbx + \frac{1}{2}(Cb + Ac)x^2 + (Ca + Ab)\log(x) - \frac{2Bax + Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + B*b*x + 1/2*(C*b + A*c)*x^2 + (C*a +
A*b)*log(x) - 1/2*(2*B*a*x + A*a)/x^2
```

Fricas [A] time = 0.248793, size = 84, normalized size = 1.33

$$\frac{3Ccx^6 + 4Bcx^5 + 12Bbx^3 + 6(Cb + Ac)x^4 + 12(Ca + Ab)x^2\log(x) - 12Bax - 6Aa}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c*x^6 + 4*B*c*x^5 + 12*B*b*x^3 + 6*(C*b + A*c)*x^4 + 12
*(C*a + A*b)*x^2*log(x) - 12*B*a*x - 6*A*a)/x^2
```

Sympy [A] time = 1.70048, size = 60, normalized size = 0.95

$$Bbx + \frac{Bcx^3}{3} + \frac{Ccx^4}{4} + x^2\left(\frac{Ac}{2} + \frac{Cb}{2}\right) + (Ab + Ca)\log(x) - \frac{Aa + 2Bax}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] B*b*x + B*c*x**3/3 + C*c*x**4/4 + x**2*(A*c/2 + C*b/2) + (A*b + C
*a)*log(x) - (A*a + 2*B*a*x)/(2*x**2)
```

GIAC/XCAS [A] time = 0.280595, size = 78, normalized size = 1.24

$$\frac{1}{4}Ccx^4 + \frac{1}{3}Bcx^3 + \frac{1}{2}Cbx^2 + \frac{1}{2}Acx^2 + Bbx + (Ca + Ab)\ln(|x|) - \frac{2Bax + Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + 1/2*C*b*x^2 + 1/2*A*c*x^2 + B*b*x + (
C*a + A*b)*ln(abs(x)) - 1/2*(2*B*a*x + A*a)/x^2
```

3.7 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx$

Optimal. Leaf size=63

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}-\frac{aB}{2x^2}+x(Ac+bC)+bB\log(x)+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + (A*c + b*C)*x +
(B*c*x^2)/2 + (c*C*x^3)/3 + b*B*Log[x]
```

Rubi [A] time = 0.10621, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}-\frac{aB}{2x^2}+x(Ac+bC)+bB\log(x)+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + (A*c + b*C)*x +
(B*c*x^2)/2 + (c*C*x^3)/3 + b*B*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{3x^3}-\frac{Ba}{2x^2}+Bb\log(x)+Bc\int x\,dx+\frac{Ccx^3}{3}-\frac{Ab+Ca}{x}+\frac{(Ac+Cb)\int A\,dx}{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] -A*a/(3*x**3) - B*a/(2*x**2) + B*b*log(x) + B*c*Integral(x, x) +
C*c*x**3/3 - (A*b + C*a)/x + (A*c + C*b)*Integral(A, x)/A
```

Mathematica [A] time = 0.0877233, size = 60, normalized size = 0.95

$$-\frac{a(2A+3x(B+2Cx))}{6x^3}-\frac{Ab}{x}+Acx+bB\log(x)+bCx+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + A*c*x + b*C*x + (B*c*x^2)/2 + (c*C*x^3)/3 - (a*(2*A
+ 3*x*(B + 2*C*x)))/(6*x^3) + b*B*Log[x]
```

Maple [A] time = 0.009, size = 57, normalized size = 0.9

$$\frac{cCx^3}{3}+\frac{Bcx^2}{2}+Acx+Cxb-\frac{Aa}{3x^3}+bB\ln(x)-\frac{Ab}{x}-\frac{aC}{x}-\frac{Ba}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x)
```

```
[Out] 1/3*c*C*x^3+1/2*B*c*x^2+A*c*x+C*x*b-1/3*a*A/x^3+b*B*ln(x)-1/x*A*b
-1/x*a*C-1/2*a*B/x^2
```

Maxima [A] time = 0.703787, size = 76, normalized size = 1.21

$$\frac{1}{3}Ccx^3 + \frac{1}{2}Bcx^2 + Bb\log(x) + (Cb + Ac)x - \frac{3Bax + 6(Ca + Ab)x^2 + 2Aa}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + B*b*log(x) + (C*b + A*c)*x - 1/6*(3*B
*a*x + 6*(C*a + A*b)*x^2 + 2*A*a)/x^3
```

Fricas [A] time = 0.246375, size = 84, normalized size = 1.33

$$\frac{2Ccx^6 + 3Bcx^5 + 6Bbx^3\log(x) + 6(Cb + Ac)x^4 - 3Bax - 6(Ca + Ab)x^2 - 2Aa}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*c*x^6 + 3*B*c*x^5 + 6*B*b*x^3*log(x) + 6*(C*b + A*c)*x^4
 - 3*B*a*x - 6*(C*a + A*b)*x^2 - 2*A*a)/x^3
```

Sympy [A] time = 2.5344, size = 61, normalized size = 0.97

$$Bb\log(x) + \frac{Bcx^2}{2} + \frac{Ccx^3}{3} + x(Ac + Cb) - \frac{2Aa + 3Bax + x^2(6Ab + 6Ca)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] B*b*log(x) + B*c*x**2/2 + C*c*x**3/3 + x*(A*c + C*b) - (2*A*a + 3
*B*a*x + x**2*(6*A*b + 6*C*a))/(6*x**3)
```

GIAC/XCAS [A] time = 0.27994, size = 76, normalized size = 1.21

$$\frac{1}{3}Ccx^3 + \frac{1}{2}Bcx^2 + Cbx + Acx + Bb\ln(|x|) - \frac{3Bax + 6(Ca + Ab)x^2 + 2Aa}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + C*b*x + A*c*x + B*b*ln(abs(x)) - 1/6*
(3*B*a*x + 6*(C*a + A*b)*x^2 + 2*A*a)/x^3
```

3.8 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx$

Optimal. Leaf size=63

$$-\frac{aC+Ab}{2x^2}-\frac{aA}{4x^4}-\frac{aB}{3x^3}+\log(x)(Ac+bC)-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2$$

```
[Out] -(a*A)/(4*x^4) - (a*B)/(3*x^3) - (A*b + a*C)/(2*x^2) - (b*B)/x +
B*c*x + (c*C*x^2)/2 + (A*c + b*C)*Log[x]
```

Rubi [A] time = 0.104733, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$-\frac{aC+Ab}{2x^2}-\frac{aA}{4x^4}-\frac{aB}{3x^3}+\log(x)(Ac+bC)-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -(a*A)/(4*x^4) - (a*B)/(3*x^3) - (A*b + a*C)/(2*x^2) - (b*B)/x +
B*c*x + (c*C*x^2)/2 + (A*c + b*C)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{4x^4}-\frac{Ba}{3x^3}-\frac{Bb}{x}+Cc\int x\,dx+c\int B\,dx+(Ac+Cb)\log(x)-\frac{\frac{Ab}{2}+\frac{Ca}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] -A*a/(4*x**4) - B*a/(3*x**3) - B*b/x + C*c*Integral(x, x) + c*Int
egral(B, x) + (A*c + C*b)*log(x) - (A*b/2 + C*a/2)/x**2
```

Mathematica [A] time = 0.0515569, size = 62, normalized size = 0.98

$$-\frac{a\left(3A+4Bx+6Cx^2\right)}{12x^4}+\frac{-Ab-2bBx+cx^3(2B+Cx)}{2x^2}+\log(x)(Ac+bC)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -(a*(3*A + 4*B*x + 6*C*x^2))/(12*x^4) + (-(A*b) - 2*b*B*x + c*x^3
*(2*B + C*x))/(2*x^2) + (A*c + b*C)*Log[x]
```

Maple [A] time = 0.01, size = 58, normalized size = 0.9

$$\frac{cCx^2}{2}+Bcx-\frac{Ba}{3\,x^3}+A\ln(x)\,c+C\ln(x)\,b-\frac{bB}{x}-\frac{Ab}{2\,x^2}-\frac{aC}{2\,x^2}-\frac{Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x)
```

```
[Out] 1/2*c*C*x^2+B*c*x-1/3*a*B/x^3+A*ln(x)*c+C*ln(x)*b-b*B/x-1/2/x^2*A
*b-1/2/x^2*a*C-1/4*a*A/x^4
```

Maxima [A] time = 0.701371, size = 76, normalized size = 1.21

$$\frac{1}{2}\,Ccx^2 + Bcx + (Cb + Ac)\log(x) - \frac{12\,Bbx^3 + 4\,Bax + 6\,(Ca + Ab)x^2 + 3\,Aa}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(x) - 1/12*(12*B*b*x^3 + 4*B
*a*x + 6*(C*a + A*b)*x^2 + 3*A*a)/x^4
```

Fricas [A] time = 0.247114, size = 84, normalized size = 1.33

$$\frac{6\,Ccx^6 + 12\,Bcx^5 + 12\,(Cb + Ac)x^4\log(x) - 12\,Bbx^3 - 4\,Bax - 6\,(Ca + Ab)x^2 - 3\,Aa}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(6*C*c*x^6 + 12*B*c*x^5 + 12*(C*b + A*c)*x^4*log(x) - 12*B*b
*x^3 - 4*B*a*x - 6*(C*a + A*b)*x^2 - 3*A*a)/x^4
```

Sympy [A] time = 7.4285, size = 61, normalized size = 0.97

$$Bcx + \frac{Ccx^2}{2} + (Ac + Cb)\log(x) - \frac{3Aa + 4Bax + 12Bbx^3 + x^2\,(6Ab + 6Ca)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] B*c*x + C*c*x**2/2 + (A*c + C*b)*log(x) - (3*A*a + 4*B*a*x + 12*B
*b*x**3 + x**2*(6*A*b + 6*C*a))/(12*x**4)
```

GIAC/XCAS [A] time = 0.282393, size = 77, normalized size = 1.22

$$\frac{1}{2}\,Ccx^2 + Bcx + (Cb + Ac)\ln(|x|) - \frac{12\,Bbx^3 + 4\,Bax + 6\,(Ca + Ab)x^2 + 3\,Aa}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*ln(abs(x)) - 1/12*(12*B*b*x^3 +
 4*B*a*x + 6*(C*a + A*b)*x^2 + 3*A*a)/x^4
```

3.9 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6} dx$

Optimal. Leaf size=63

$$-\frac{aC+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ac+bC}{x}-\frac{bB}{2x^2}+Bc\log(x)+cCx$$

```
[Out] -(a*A)/(5*x^5) - (a*B)/(4*x^4) - (A*b + a*C)/(3*x^3) - (b*B)/(2*x
^2) - (A*c + b*C)/x + c*C*x + B*c*Log[x]
```

Rubi [A] time = 0.102326, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$-\frac{aC+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ac+bC}{x}-\frac{bB}{2x^2}+Bc\log(x)+cCx$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) - (a*B)/(4*x^4) - (A*b + a*C)/(3*x^3) - (b*B)/(2*x
^2) - (A*c + b*C)/x + c*C*x + B*c*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa}{5x^5}-\frac{Ba}{4x^4}-\frac{Bb}{2x^2}+Bc\log(x)+c\int C\,dx-\frac{Ac+Cb}{x}-\frac{\frac{Ab}{3}+\frac{Ca}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] -A*a/(5*x**5) - B*a/(4*x**4) - B*b/(2*x**2) + B*c*log(x) + c*Inte
gral(C, x) - (A*c + C*b)/x - (A*b/3 + C*a/3)/x**3
```

Mathematica [A] time = 0.135777, size = 63, normalized size = 1.

$$Bc\log(x)-\frac{12aA+5ax(3B+4Cx)+20Ax^2\left(b+3cx^2\right)+30bx^3(B+2Cx)-60cCx^6}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -(12*a*A - 60*c*C*x^6 + 30*b*x^3*(B + 2*C*x) + 5*a*x*(3*B + 4*C*x
) + 20*A*x^2*(b + 3*c*x^2))/(60*x^5) + B*c*Log[x]
```

Maple [A] time = 0.009, size = 60, normalized size = 1.

$$cCx-\frac{Ab}{3x^3}-\frac{aC}{3x^3}+Bc\ln(x)-\frac{Ac}{x}-\frac{bC}{x}-\frac{bB}{2x^2}-\frac{Aa}{5x^5}-\frac{Ba}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x)
```

```
[Out] c*C*x-1/3/x^3*A*b-1/3/x^3*a*C+B*c*ln(x)-1/x*A*c-1/x*b*C-1/2*b*B/x
^2-1/5*a*A/x^5-1/4*a*B/x^4
```

Maxima [A] time = 0.698668, size = 76, normalized size = 1.21

$$Ccx + Bc\log(x) - \frac{30\,Bbx^3 + 60\,(Cb + Ac)x^4 + 15\,Bax + 20\,(Ca + Ab)x^2 + 12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^6,x, algorithm="maxima")
```

```
[Out] C*c*x + B*c*log(x) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B
*a*x + 20*(C*a + A*b)*x^2 + 12*A*a)/x^5
```

Fricas [A] time = 0.251467, size = 84, normalized size = 1.33

$$\frac{60\,Ccx^6 + 60\,Bcx^5\log(x) - 30\,Bbx^3 - 60\,(Cb + Ac)x^4 - 15\,Bax - 20\,(Ca + Ab)x^2 - 12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*C*c*x^6 + 60*B*c*x^5*log(x) - 30*B*b*x^3 - 60*(C*b + A*c
)*x^4 - 15*B*a*x - 20*(C*a + A*b)*x^2 - 12*A*a)/x^5
```

Sympy [A] time = 21.7377, size = 63, normalized size = 1.

$$Bc\log(x) + Ccx - \frac{12Aa + 15Bax + 30Bbx^3 + x^4\,(60Ac + 60Cb) + x^2\,(20Ab + 20Ca)}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] B*c*log(x) + C*c*x - (12*A*a + 15*B*a*x + 30*B*b*x**3 + x**4*(60*
A*c + 60*C*b) + x**2*(20*A*b + 20*C*a))/(60*x**5)
```

GIAC/XCAS [A] time = 0.281511, size = 77, normalized size = 1.22

$$Ccx + Bc\ln(|x|) - \frac{30\,Bbx^3 + 60\,(Cb + Ac)x^4 + 15\,Bax + 20\,(Ca + Ab)x^2 + 12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^6,x, algorithm="giac")
```

```
[Out] C*c*x + B*c*ln(abs(x)) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 +
15*B*a*x + 20*(C*a + A*b)*x^2 + 12*A*a)/x^5
```

3.10 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$

Optimal. Leaf size=68

$$-\frac{aC+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ac+bC}{2x^2}-\frac{bB}{3x^3}-\frac{Bc}{x}+cC\log(x)$$

```
[Out] -(a*A)/(6*x^6) - (a*B)/(5*x^5) - (A*b + a*C)/(4*x^4) - (b*B)/(3*x
^3) - (A*c + b*C)/(2*x^2) - (B*c)/x + c*C*Log[x]
```

Rubi [A] time = 0.0953815, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$-\frac{aC+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ac+bC}{2x^2}-\frac{bB}{3x^3}-\frac{Bc}{x}+cC\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -(a*A)/(6*x^6) - (a*B)/(5*x^5) - (A*b + a*C)/(4*x^4) - (b*B)/(3*x
^3) - (A*c + b*C)/(2*x^2) - (B*c)/x + c*C*Log[x]
```

Rubi in Sympy [A] time = 18.8801, size = 63, normalized size = 0.93

$$-\frac{Aa}{6x^6}-\frac{Ba}{5x^5}-\frac{Bb}{3x^3}-\frac{Bc}{x}+Cc\log(x)-\frac{\frac{Ac}{2}+\frac{Cb}{2}}{x^2}-\frac{\frac{Ab}{4}+\frac{Ca}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] -A*a/(6*x**6) - B*a/(5*x**5) - B*b/(3*x**3) - B*c/x + C*c*log(x)
- (A*c/2 + C*b/2)/x**2 - (A*b/4 + C*a/4)/x**4
```

Mathematica [A] time = 0.0944231, size = 68, normalized size = 1.

$$cC\log(x)-\frac{a(10A+3x(4B+5Cx))+5x^2\left(3A\left(b+2cx^2\right)+2x\left(2bB+3bCx+6Bcx^2\right)\right)}{60x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -(a*(10*A + 3*x*(4*B + 5*C*x)) + 5*x^2*(3*A*(b + 2*c*x^2) + 2*x*(
2*b*B + 3*b*C*x + 6*B*c*x^2)))/(60*x^6) + c*C*Log[x]
```

Maple [A] time = 0.009, size = 63, normalized size = 0.9

$$-\frac{bB}{3x^3}-\frac{Aa}{6x^6}+cC\ln(x)-\frac{Bc}{x}-\frac{Ac}{2x^2}-\frac{bC}{2x^2}-\frac{Ba}{5x^5}-\frac{Ab}{4x^4}-\frac{aC}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x)
```

```
[Out] -1/3*b*B/x^3-1/6*a*A/x^6+c*C*ln(x)-B*c/x-1/2/x^2*A*c-1/2/x^2*b*C-
1/5*a*B/x^5-1/4/x^4*A*b-1/4/x^4*a*C
```

Maxima [A] time = 0.711381, size = 80, normalized size = 1.18

$$Cc\log(x) - \frac{60\,Bcx^5 + 20\,Bbx^3 + 30\,(Cb + Ac)x^4 + 12\,Bax + 15\,(Ca + Ab)x^2 + 10\,Aa}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^7,x, algorithm="maxima")
```

```
[Out] C*c*log(x) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 +
 12*B*a*x + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

Fricas [A] time = 0.247652, size = 84, normalized size = 1.24

$$\frac{60\,Ccx^6\log(x) - 60\,Bcx^5 - 20\,Bbx^3 - 30\,(Cb + Ac)x^4 - 12\,Bax - 15\,(Ca + Ab)x^2 - 10\,Aa}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*C*c*x^6*log(x) - 60*B*c*x^5 - 20*B*b*x^3 - 30*(C*b + A*c
)*x^4 - 12*B*a*x - 15*(C*a + A*b)*x^2 - 10*A*a)/x^6
```

Sympy [A] time = 59.1961, size = 66, normalized size = 0.97

$$Cc\log(x) - \frac{10Aa + 12Bax + 20Bbx^3 + 60Bcx^5 + x^4(30Ac + 30Cb) + x^2(15Ab + 15Ca)}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] C*c*log(x) - (10*A*a + 12*B*a*x + 20*B*b*x**3 + 60*B*c*x**5 + x**
4*(30*A*c + 30*C*b) + x**2*(15*A*b + 15*C*a))/(60*x**6)
```

GIAC/XCAS [A] time = 0.280958, size = 81, normalized size = 1.19

$$Cc\ln(|x|) - \frac{60\,Bcx^5 + 20\,Bbx^3 + 30\,(Cb + Ac)x^4 + 12\,Bax + 15\,(Ca + Ab)x^2 + 10\,Aa}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)/x^7,x, algorithm="giac")
```

```
[Out] C*c*ln(abs(x)) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x
^4 + 12*B*a*x + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

3.11 $\int x^2 \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=159

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{7}x^7\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+\frac{1}{3}abBx^6+\frac{1}{11}cx^{11}(Ac+2bC)+\frac{1}{5}bBcx^{10}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*
x^6)/3 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x
^8)/8 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^9)/9 + (b*B*c*x^10)/5 + (c
*(A*c + 2*b*C)*x^11)/11 + (B*c^2*x^12)/12 + (c^2*C*x^13)/13
```

Rubi [A] time = 0.422046, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{7}x^7\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+\frac{1}{3}abBx^6+\frac{1}{11}cx^{11}(Ac+2bC)+\frac{1}{5}bBcx^{10}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*
x^6)/3 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x
^8)/8 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^9)/9 + (b*B*c*x^10)/5 + (c
*(A*c + 2*b*C)*x^11)/11 + (B*c^2*x^12)/12 + (c^2*C*x^13)/13
```

Rubi in Sympy [A] time = 48.55, size = 160, normalized size = 1.01

$$\frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Babx^6}{3}+\frac{Bbcx^{10}}{5}+\frac{Bc^2x^{12}}{12}+\frac{Bx^8\left(2ac+b^2\right)}{8}+\frac{Cc^2x^{13}}{13}+\frac{ax^5\left(2Ab+Ca\right)}{5}+\frac{cx^{11}\left(Ac+2Cb\right)}{11}+x^9\left(\frac{2Abc}{9}+\frac{2Cac}{9}+\frac{Cb^2}{9}\right)+x^7\left(\frac{2Aac}{7}+\frac{Ab^2}{7}+\frac{2Cab}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + B*a*b*x**6/3 + B*b*c*x**10/5 + B*
c**2*x**12/12 + B*x**8*(2*a*c + b**2)/8 + C*c**2*x**13/13 + a*x**
5*(2*A*b + C*a)/5 + c*x**11*(A*c + 2*C*b)/11 + x**9*(2*A*b*c/9 +
2*C*a*c/9 + C*b**2/9) + x**7*(2*A*a*c/7 + A*b**2/7 + 2*C*a*b/7)
```

Mathematica [A] time = 0.0879835, size = 159, normalized size = 1.

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(2acC+2Abc+b^2C\right)+\frac{1}{7}x^7\left(2aAc+2abC+Ab^2\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+\frac{1}{3}abBx^6+\frac{1}{11}cx^{11}(Ac+2bC)+\frac{1}{5}bBcx^{10}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*
x^6)/3 + ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x
^8)/8 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^9)/9 + (b*B*c*x^10)/5 + (c
*(A*c + 2*b*C)*x^11)/11 + (B*c^2*x^12)/12 + (c^2*C*x^13)/13
```

Maple [A] time = 0.001, size = 142, normalized size = 0.9

$$\frac{c^2Cx^{13}}{13}+\frac{Bc^2x^{12}}{12}+\frac{\left(Ac^2+2\,Cbc\right)x^{11}}{11}+\frac{bBcx^{10}}{5}+\frac{\left(2\,Abc+\left(2\,ac+b^2\right)C\right)x^9}{9}+\frac{B\left(2\,ac+b^2\right)x^8}{8}+\frac{\left(A\left(2\,ac+b^2\right)+2\,abC\right)x^7}{7}+\frac{abBx^6}{3}+\frac{\left(2\,abA+a^2C\right)x^5}{5}+\frac{a^2Bx^4}{4}+\frac{a^2Ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/13*c^2*C*x^13+1/12*B*c^2*x^12+1/11*(A*c^2+2*C*b*c)*x^11+1/5*b*B
*c*x^10+1/9*(2*A*b*c+(2*a*c+b^2)*C)*x^9+1/8*B*(2*a*c+b^2)*x^8+1/7
*(A*(2*a*c+b^2)+2*a*b*C)*x^7+1/3*a*b*B*x^6+1/5*(2*A*a*b+C*a^2)*x^
5+1/4*a^2*B*x^4+1/3*a^2*A*x^3
```

Maxima [A] time = 0.700648, size = 193, normalized size = 1.21

$$\frac{1}{13}\,Cc^2x^{13}+\frac{1}{12}\,Bc^2x^{12}+\frac{1}{5}\,Bbcx^{10}+\frac{1}{11}\left(2\,Cbc+Ac^2\right)x^{11}+\frac{1}{9}\left(Cb^2+2\,(Ca+Ab)c\right)x^9+\frac{1}{3}\,Babx^6+\frac{1}{8}\left(Bb^2+2\,Bac\right)x^8+\frac{1}{7}\left(2\,Cab+Ab^2+2\,Aac\right)x^7+\frac{1}{4}\,Ba^2x^4+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\left(Ca^2+2\,Aab\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)*x^2,x, algorithm="maxima")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*
c + A*c^2)*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x
^6 + 1/8*(B*b^2 + 2*B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*
x^7 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

Fricas [A] time = 0.230039, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}c^2C+\frac{1}{12}x^{12}c^2B+\frac{2}{11}x^{11}cbC+\frac{1}{11}x^{11}c^2A+\frac{1}{5}x^{10}cbB+\frac{1}{9}x^9b^2C+\frac{2}{9}x^9caC+\frac{2}{9}x^9cbA+\frac{1}{8}x^8b^2B+\frac{1}{4}x^8caB+\frac{2}{7}x^7baC+\frac{1}{7}x^7b^2A+\frac{2}{7}x^7caA+\frac{1}{3}x^6baB+\frac{1}{5}x^5a^2C+\frac{2}{5}x^5baA+\frac{1}{4}x^4a^2B+\frac{1}{3}x^3a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)*x^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*c^2*C + 1/12*x^12*c^2*B + 2/11*x^11*c*b*C + 1/11*x^11*c
^2*A + 1/5*x^10*c*b*B + 1/9*x^9*b^2*C + 2/9*x^9*c*a*C + 2/9*x^9*c
*b*A + 1/8*x^8*b^2*B + 1/4*x^8*c*a*B + 2/7*x^7*b*a*C + 1/7*x^7*b^
2*A + 2/7*x^7*c*a*A + 1/3*x^6*b*a*B + 1/5*x^5*a^2*C + 2/5*x^5*b*a
*A + 1/4*x^4*a^2*B + 1/3*x^3*a^2*A
```

Sympy [A] time = 0.171828, size = 168, normalized size = 1.06

$$\frac{Aa^2x^3}{3} + \frac{Ba^2x^4}{4} + \frac{Babx^6}{3} + \frac{Bbcx^{10}}{5} + \frac{Bc^2x^{12}}{12} + \frac{Cc^2x^{13}}{13} + x^{11}\left(\frac{Ac^2}{11} + \frac{2Cbc}{11}\right) + x^9\left(\frac{2Abc}{9} + \frac{2Cac}{9} + \frac{Cb^2}{9}\right) + x^8\left(\frac{Bac}{4} + \frac{Bb^2}{8}\right) + x^7\left(\frac{2Aac}{7} + \frac{Ab^2}{7} + \frac{2Cab}{7}\right) + x^5\left(\frac{2Aab}{5} + \frac{Ca^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + B*a*b*x**6/3 + B*b*c*x**10/5 + B*
c**2*x**12/12 + C*c**2*x**13/13 + x**11*(A*c**2/11 + 2*C*b*c/11)
+ x**9*(2*A*b*c/9 + 2*C*a*c/9 + C*b**2/9) + x**8*(B*a*c/4 + B*b**
2/8) + x**7*(2*A*a*c/7 + A*b**2/7 + 2*C*a*b/7) + x**5*(2*A*a*b/5
+ C*a**2/5)
```

GIAC/XCAS [A] time = 0.280326, size = 208, normalized size = 1.31

$$\frac{1}{13}Cc^2x^{13} + \frac{1}{12}Bc^2x^{12} + \frac{2}{11}Cbcx^{11} + \frac{1}{11}Ac^2x^{11} + \frac{1}{5}Bbcx^{10} + \frac{1}{9}Cb^2x^9 + \frac{2}{9}Cacx^9 + \frac{2}{9}Abcx^9 + \frac{1}{8}Bb^2x^8 + \frac{1}{4}Bacx^8 + \frac{2}{7}Cabx^7 + \frac{1}{7}Ab^2x^7 + \frac{2}{7}Aacx^7 + \frac{1}{3}Babx^6 + \frac{1}{5}Ca^2x^5 + \frac{2}{5}Aabx^5 + \frac{1}{4}Ba^2x^4 + \frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)*x^2,x, algorithm="giac")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 2/11*C*b*c*x^11 + 1/11*A*c^2*
x^11 + 1/5*B*b*c*x^10 + 1/9*C*b^2*x^9 + 2/9*C*a*c*x^9 + 2/9*A*b*c
*x^9 + 1/8*B*b^2*x^8 + 1/4*B*a*c*x^8 + 2/7*C*a*b*x^7 + 1/7*A*b^2*
x^7 + 2/7*A*a*c*x^7 + 1/3*B*a*b*x^6 + 1/5*C*a^2*x^5 + 2/5*A*a*b*x
^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

3.12 $\int x \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=159

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{6}x^6\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)+\frac{2}{5}abBx^5+\frac{1}{10}cx^{10}(Ac+2bC)+\frac{2}{9}bBcx^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*
B*x^5)/5 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)
*x^7)/7 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^8)/8 + (2*b*B*c*x^9)/9 +
 (c*(A*c + 2*b*C)*x^10)/10 + (B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Rubi [A] time = 0.358422, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{6}x^6\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)+\frac{2}{5}abBx^5+\frac{1}{10}cx^{10}(Ac+2bC)+\frac{2}{9}bBcx^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*
B*x^5)/5 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)
*x^7)/7 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^8)/8 + (2*b*B*c*x^9)/9 +
 (c*(A*c + 2*b*C)*x^10)/10 + (B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Aa^2\int x\,dx+\frac{Ba^2x^3}{3}+\frac{2Babx^5}{5}+\frac{2Bbcx^9}{9}+\frac{Bc^2x^{11}}{11}+\frac{Bx^7\left(2ac+b^2\right)}{7}+\frac{Cc^2x^{12}}{12}+\frac{ax^4\left(2Ab+Ca\right)}{4}+\frac{cx^{10}\left(Ac+2Cb\right)}{10}+x^8\left(\frac{Abc}{4}+\frac{Cac}{4}+\frac{Cb^2}{8}\right)+x^6\left(\frac{Aac}{3}+\frac{Ab^2}{6}+\frac{Cab}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*Integral(x, x) + B*a**2*x**3/3 + 2*B*a*b*x**5/5 + 2*B*b*c*
x**9/9 + B*c**2*x**11/11 + B*x**7*(2*a*c + b**2)/7 + C*c**2*x**12
/12 + a*x**4*(2*A*b + C*a)/4 + c*x**10*(A*c + 2*C*b)/10 + x**8*(A
*b*c/4 + C*a*c/4 + C*b**2/8) + x**6*(A*a*c/3 + A*b**2/6 + C*a*b/3
)
```

Mathematica [A] time = 0.0859919, size = 159, normalized size = 1.

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(2acC+2Abc+b^2C\right)+\frac{1}{6}x^6\left(2aAc+2abC+Ab^2\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)+\frac{2}{5}abBx^5+\frac{1}{10}cx^{10}(Ac+2bC)+\frac{2}{9}bBcx^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*
B*x^5)/5 + ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)
*x^7)/7 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^8)/8 + (2*b*B*c*x^9)/9 +
 (c*(A*c + 2*b*C)*x^10)/10 + (B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Maple [A] time = 0.001, size = 142, normalized size = 0.9

$$\frac{c^2Cx^{12}}{12}+\frac{Bc^2x^{11}}{11}+\frac{\left(Ac^2+2\,Cbc\right)x^{10}}{10}+\frac{2\,bBcx^9}{9}+\frac{\left(2\,Abc+\left(2\,ac+b^2\right)C\right)x^8}{8}+\frac{B\left(2\,ac+b^2\right)x^7}{7}+\frac{\left(A\left(2\,ac+b^2\right)+2\,abC\right)x^6}{6}+\frac{2\,abBx^5}{5}+\frac{\left(2\,abA+a^2C\right)x^4}{4}+\frac{a^2Bx^3}{3}+\frac{a^2Ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/12*c^2*C*x^12+1/11*B*c^2*x^11+1/10*(A*c^2+2*C*b*c)*x^10+2/9*b*B
*c*x^9+1/8*(2*A*b*c+(2*a*c+b^2)*C)*x^8+1/7*B*(2*a*c+b^2)*x^7+1/6*
(A*(2*a*c+b^2)+2*a*b*C)*x^6+2/5*a*b*B*x^5+1/4*(2*A*a*b+C*a^2)*x^4
+1/3*a^2*B*x^3+1/2*a^2*A*x^2
```

Maxima [A] time = 0.698476, size = 193, normalized size = 1.21

$$\frac{1}{12}\,Cc^2x^{12}+\frac{1}{11}\,Bc^2x^{11}+\frac{2}{9}\,Bbcx^9+\frac{1}{10}\left(2\,Cbc+Ac^2\right)x^{10}+\frac{1}{8}\left(Cb^2+2\left(Ca+Ab\right)c\right)x^8+\frac{2}{5}\,Babx^5+\frac{1}{7}\left(Bb^2+2\,Bac\right)x^7+\frac{1}{6}\left(2\,Cab+Ab^2+2\,Aac\right)x^6+\frac{1}{3}\,Ba^2x^3+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\left(Ca^2+2\,Aab\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)*x,x, algorithm="maxima")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c
 + A*c^2)*x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^
5 + 1/7*(B*b^2 + 2*B*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x
^6 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

Fricas [A] time = 0.241563, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}c^2C+\frac{1}{11}x^{11}c^2B+\frac{1}{5}x^{10}cbC+\frac{1}{10}x^{10}c^2A+\frac{2}{9}x^9cbB+\frac{1}{8}x^8b^2C+\frac{1}{4}x^8caC+\frac{1}{4}x^8cbA+\frac{1}{7}x^7b^2B+\frac{2}{7}x^7caB+\frac{1}{3}x^6baC+\frac{1}{6}x^6b^2A+\frac{1}{3}x^6caA+\frac{2}{5}x^5baB+\frac{1}{4}x^4a^2C+\frac{1}{2}x^4baA+\frac{1}{3}x^3a^2B+\frac{1}{2}x^2a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)*x,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*c^2*C + 1/11*x^11*c^2*B + 1/5*x^10*c*b*C + 1/10*x^10*c^
2*A + 2/9*x^9*c*b*B + 1/8*x^8*b^2*C + 1/4*x^8*c*a*C + 1/4*x^8*c*b
*A + 1/7*x^7*b^2*B + 2/7*x^7*c*a*B + 1/3*x^6*b*a*C + 1/6*x^6*b^2*
A + 1/3*x^6*c*a*A + 2/5*x^5*b*a*B + 1/4*x^4*a^2*C + 1/2*x^4*b*a*A
 + 1/3*x^3*a^2*B + 1/2*x^2*a^2*A
```

Sympy [A] time = 0.170042, size = 163, normalized size = 1.03

$$\frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{2Babx^5}{5}+\frac{2Bbcx^9}{9}+\frac{Bc^2x^{11}}{11}+\frac{Cc^2x^{12}}{12}+x^{10}\left(\frac{Ac^2}{10}+\frac{Cbc}{5}\right)+x^8\left(\frac{Abc}{4}+\frac{Cac}{4}+\frac{Cb^2}{8}\right)+x^7\left(\frac{2Bac}{7}+\frac{Bb^2}{7}\right)+x^6\left(\frac{Aac}{3}+\frac{Ab^2}{6}+\frac{Cab}{3}\right)+x^4\left(\frac{Aab}{2}+\frac{Ca^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + 2*B*a*b*x**5/5 + 2*B*b*c*x**9/9 +
 B*c**2*x**11/11 + C*c**2*x**12/12 + x**10*(A*c**2/10 + C*b*c/5)
+ x**8*(A*b*c/4 + C*a*c/4 + C*b**2/8) + x**7*(2*B*a*c/7 + B*b**2/
7) + x**6*(A*a*c/3 + A*b**2/6 + C*a*b/3) + x**4*(A*a*b/2 + C*a**2
/4)
```

GIAC/XCAS [A] time = 0.281645, size = 208, normalized size = 1.31

$$\frac{1}{12}Cc^2x^{12}+\frac{1}{11}Bc^2x^{11}+\frac{1}{5}Cbcx^{10}+\frac{1}{10}Ac^2x^{10}+\frac{2}{9}Bbcx^9+\frac{1}{8}Cb^2x^8+\frac{1}{4}Cacx^8+\frac{1}{4}Abcx^8+\frac{1}{7}Bb^2x^7+\frac{2}{7}Bacx^7+\frac{1}{3}Cabx^6+\frac{1}{6}Ab^2x^6+\frac{1}{3}Aacx^6+\frac{2}{5}Babx^5+\frac{1}{4}Ca^2x^4+\frac{1}{2}Aabx^4+\frac{1}{3}Ba^2x^3+\frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)*x,x, algorithm="giac")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 1/5*C*b*c*x^10 + 1/10*A*c^2*x
^10 + 2/9*B*b*c*x^9 + 1/8*C*b^2*x^8 + 1/4*C*a*c*x^8 + 1/4*A*b*c*x
^8 + 1/7*B*b^2*x^7 + 2/7*B*a*c*x^7 + 1/3*C*a*b*x^6 + 1/6*A*b^2*x^
6 + 1/3*A*a*c*x^6 + 2/5*B*a*b*x^5 + 1/4*C*a^2*x^4 + 1/2*A*a*b*x^4
 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

3.13 $\int \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=154

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}abBx^4+\frac{1}{9}cx^9(Ac+2bC)+\frac{1}{4}bBcx^8+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2
 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6
+ ((2*A*b*c + (b^2 + 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c +
 2*b*C)*x^9)/9 + (B*c^2*x^10)/10 + (c^2*C*x^11)/11
```

Rubi [A] time = 0.283172, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}abBx^4+\frac{1}{9}cx^9(Ac+2bC)+\frac{1}{4}bBcx^8+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2
 + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6
+ ((2*A*b*c + (b^2 + 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c +
 2*b*C)*x^9)/9 + (B*c^2*x^10)/10 + (c^2*C*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Ba^2\int x\,dx+\frac{Babx^4}{2}+\frac{Bbcx^8}{4}+\frac{Bc^2x^{10}}{10}+\frac{Bx^6\left(2ac+b^2\right)}{6}+\frac{Cc^2x^{11}}{11}+a^2\int A\,dx+\frac{ax^3\left(2Ab+Ca\right)}{3}+\frac{cx^9\left(Ac+2Cb\right)}{9}+x^7\left(\frac{2Abc}{7}+\frac{2Cac}{7}+\frac{Cb^2}{7}\right)+x^5\left(\frac{2Aac}{5}+\frac{Ab^2}{5}+\frac{2Cab}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] B*a**2*Integral(x, x) + B*a*b*x**4/2 + B*b*c*x**8/4 + B*c**2*x**1
0/10 + B*x**6*(2*a*c + b**2)/6 + C*c**2*x**11/11 + a**2*Integral(
A, x) + a*x**3*(2*A*b + C*a)/3 + c*x**9*(A*c + 2*C*b)/9 + x**7*(2
*A*b*c/7 + 2*C*a*c/7 + C*b**2/7) + x**5*(2*A*a*c/5 + A*b**2/5 + 2
*C*a*b/5)
```

Mathematica [A] time = 0.0567707, size = 154, normalized size = 1.

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(2acC+2Abc+b^2C\right)+\frac{1}{5}x^5\left(2aAc+2abC+Ab^2\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}abBx^4+\frac{1}{9}cx^9(Ac+2bC)+\frac{1}{4}bBcx^8+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2
 + ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6
+ ((2*A*b*c + b^2*C + 2*a*c*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c +
 2*b*C)*x^9)/9 + (B*c^2*x^10)/10 + (c^2*C*x^11)/11
```

Maple [A] time = 0.001, size = 139, normalized size = 0.9

$$\frac{c^2Cx^{11}}{11}+\frac{Bc^2x^{10}}{10}+\frac{\left(Ac^2+2\,Cbc\right)x^9}{9}+\frac{bBcx^8}{4}+\frac{\left(2\,Abc+\left(2\,ac+b^2\right)C\right)x^7}{7}+\frac{B\left(2\,ac+b^2\right)x^6}{6}+\frac{\left(A\left(2\,ac+b^2\right)+2\,abC\right)x^5}{5}+\frac{abBx^4}{2}+\frac{\left(2\,abA+a^2C\right)x^3}{3}+\frac{a^2Bx^2}{2}+a^2Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/11*c^2*C*x^11+1/10*B*c^2*x^10+1/9*(A*c^2+2*C*b*c)*x^9+1/4*b*B*c
*x^8+1/7*(2*A*b*c+(2*a*c+b^2)*C)*x^7+1/6*B*(2*a*c+b^2)*x^6+1/5*(A
*(2*a*c+b^2)+2*a*b*C)*x^5+1/2*a*b*B*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1
/2*a^2*B*x^2+a^2*A*x
```

Maxima [A] time = 0.697516, size = 189, normalized size = 1.23

$$\frac{1}{11}\,Cc^2x^{11}+\frac{1}{10}\,Bc^2x^{10}+\frac{1}{4}\,Bbcx^8+\frac{1}{9}\left(2\,Cbc+Ac^2\right)x^9+\frac{1}{7}\left(Cb^2+2\,(Ca+Ab)c\right)x^7+\frac{1}{2}\,Babx^4+\frac{1}{6}\left(Bb^2+2\,Bac\right)x^6+\frac{1}{5}\left(2\,Cab+Ab^2+2\,Aac\right)x^5+\frac{1}{2}\,Ba^2x^2+Aa^2x+\frac{1}{3}\left(Ca^2+2\,Aab\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c
+ A*c^2)*x^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4
+ 1/6*(B*b^2 + 2*B*a*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5
 + 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*b)*x^3
```

Fricas [A] time = 0.231253, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}c^2C+\frac{1}{10}x^{10}c^2B+\frac{2}{9}x^9cbC+\frac{1}{9}x^9c^2A+\frac{1}{4}x^8cbB+\frac{1}{7}x^7b^2C+\frac{2}{7}x^7caC+\frac{2}{7}x^7cbA+\frac{1}{6}x^6b^2B+\frac{1}{3}x^6caB+\frac{2}{5}x^5baC+\frac{1}{5}x^5b^2A+\frac{2}{5}x^5caA+\frac{1}{2}x^4baB+\frac{1}{3}x^3a^2C+\frac{2}{3}x^3baA+\frac{1}{2}x^2a^2B+xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2*C + 1/10*x^10*c^2*B + 2/9*x^9*c*b*C + 1/9*x^9*c^2*A
 + 1/4*x^8*c*b*B + 1/7*x^7*b^2*C + 2/7*x^7*c*a*C + 2/7*x^7*c*b*A
+ 1/6*x^6*b^2*B + 1/3*x^6*c*a*B + 2/5*x^5*b*a*C + 1/5*x^5*b^2*A +
 2/5*x^5*c*a*A + 1/2*x^4*b*a*B + 1/3*x^3*a^2*C + 2/3*x^3*b*a*A +
1/2*x^2*a^2*B + x*a^2*A
```

Sympy [A] time = 0.168474, size = 165, normalized size = 1.07

$$Aa^2x + \frac{Ba^2x^2}{2} + \frac{Babx^4}{2} + \frac{Bbcx^8}{4} + \frac{Bc^2x^{10}}{10} + \frac{Cc^2x^{11}}{11} + x^9\left(\frac{Ac^2}{9} + \frac{2Cbc}{9}\right) + x^7\left(\frac{2Abc}{7} + \frac{2Cac}{7} + \frac{Cb^2}{7}\right) + x^6\left(\frac{Bac}{3} + \frac{Bb^2}{6}\right) + x^5\left(\frac{2Aac}{5} + \frac{Ab^2}{5} + \frac{2Cab}{5}\right) + x^3\left(\frac{2Aab}{3} + \frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b*c*x**8/4 + B*c**2*x
**10/10 + C*c**2*x**11/11 + x**9*(A*c**2/9 + 2*C*b*c/9) + x**7*(2
*A*b*c/7 + 2*C*a*c/7 + C*b**2/7) + x**6*(B*a*c/3 + B*b**2/6) + x*
*5*(2*A*a*c/5 + A*b**2/5 + 2*C*a*b/5) + x**3*(2*A*a*b/3 + C*a**2/
3)
```

GIAC/XCAS [A] time = 0.27819, size = 204, normalized size = 1.32

$$\frac{1}{11}Cc^2x^{11} + \frac{1}{10}Bc^2x^{10} + \frac{2}{9}Cbcx^9 + \frac{1}{9}Ac^2x^9 + \frac{1}{4}Bbcx^8 + \frac{1}{7}Cb^2x^7 + \frac{2}{7}Cacx^7 + \frac{2}{7}Abcx^7 + \frac{1}{6}Bb^2x^6 + \frac{1}{3}Bacx^6 + \frac{2}{5}Cabx^5 + \frac{1}{5}Ab^2x^5 + \frac{2}{5}Aacx^5 + \frac{1}{2}Babx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aabx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 2/9*C*b*c*x^9 + 1/9*A*c^2*x^9
 + 1/4*B*b*c*x^8 + 1/7*C*b^2*x^7 + 2/7*C*a*c*x^7 + 2/7*A*b*c*x^7
+ 1/6*B*b^2*x^6 + 1/3*B*a*c*x^6 + 2/5*C*a*b*x^5 + 1/5*A*b^2*x^5 +
 2/5*A*a*c*x^5 + 1/2*B*a*b*x^4 + 1/3*C*a^2*x^3 + 2/3*A*a*b*x^3 +
1/2*B*a^2*x^2 + A*a^2*x
```

3.14 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx$

Optimal. Leaf size=150

$$a^2A\log(x)+a^2Bx+\frac{1}{6}x^6\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)+\frac{2}{3}abBx^3+\frac{1}{8}cx^8(Ac+2bC)+\frac{2}{7}bBcx^7+\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}$$

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*(b^2 +
2*a*c) + 2*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c +
(b^2 + 2*a*c)*C)*x^6)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)
/8 + (B*c^2*x^9)/9 + (c^2*C*x^10)/10 + a^2*A*Log[x]
```

Rubi [A] time = 0.224873, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$a^2A\log(x)+a^2Bx+\frac{1}{6}x^6\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)+\frac{2}{3}abBx^3+\frac{1}{8}cx^8(Ac+2bC)+\frac{2}{7}bBcx^7+\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*(b^2 +
2*a*c) + 2*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c +
(b^2 + 2*a*c)*C)*x^6)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)
/8 + (B*c^2*x^9)/9 + (c^2*C*x^10)/10 + a^2*A*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$Aa^2\log(x)+\frac{2Babx^3}{3}+\frac{2Bbcx^7}{7}+\frac{Bc^2x^9}{9}+\frac{Bx^5\left(2ac+b^2\right)}{5}+\frac{Cc^2x^{10}}{10}+a^2\int B\,dx+a(2Ab+Ca)\int x\,dx+\frac{cx^8(Ac+2Cb)}{8}+x^6\left(\frac{Abc}{3}+\frac{Cac}{3}+\frac{Cb^2}{6}\right)+x^4\left(\frac{Aac}{2}+\frac{Ab^2}{4}+\frac{Cab}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] A*a**2*log(x) + 2*B*a*b*x**3/3 + 2*B*b*c*x**7/7 + B*c**2*x**9/9 +
 B*x**5*(2*a*c + b**2)/5 + C*c**2*x**10/10 + a**2*Integral(B, x)
+ a*(2*A*b + C*a)*Integral(x, x) + c*x**8*(A*c + 2*C*b)/8 + x**6*
(A*b*c/3 + C*a*c/3 + C*b**2/6) + x**4*(A*a*c/2 + A*b**2/4 + C*a*b
/2)
```

Mathematica [A] time = 0.0764305, size = 150, normalized size = 1.

$$a^2A\log(x)+a^2Bx+\frac{1}{6}x^6\left(2acC+2Abc+b^2C\right)+\frac{1}{4}x^4\left(2aAc+2abC+Ab^2\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)+\frac{2}{3}abBx^3+\frac{1}{8}cx^8(Ac+2bC)+\frac{2}{7}bBcx^7+\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*b^2 + 2
*a*A*c + 2*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c +
b^2*C + 2*a*c*C)*x^6)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)
/8 + (B*c^2*x^9)/9 + (c^2*C*x^10)/10 + a^2*A*Log[x]
```

Maple [A] time = 0.004, size = 149, normalized size = 1.

$$\frac{c^2Cx^{10}}{10}+\frac{Bc^2x^9}{9}+\frac{Ax^8c^2}{8}+\frac{Cx^8bc}{4}+\frac{2\,bBcx^7}{7}+\frac{Ax^6bc}{3}+\frac{Cx^6ac}{3}+\frac{Cx^6b^2}{6}+\frac{2\,Bx^5ac}{5}+\frac{Bx^5b^2}{5}+\frac{Ax^4ac}{2}+\frac{Ax^4b^2}{4}+\frac{Cx^4ab}{2}+\frac{2\,abBx^3}{3}+aAbx^2+\frac{Cx^2a^2}{2}+a^2Bx+a^2A\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x)
```

```
[Out] 1/10*c^2*C*x^10+1/9*B*c^2*x^9+1/8*A*x^8*c^2+1/4*C*x^8*b*c+2/7*b*B
*c*x^7+1/3*A*x^6*b*c+1/3*C*x^6*a*c+1/6*C*x^6*b^2+2/5*B*x^5*a*c+1/
5*B*x^5*b^2+1/2*A*x^4*a*c+1/4*A*x^4*b^2+1/2*C*x^4*a*b+2/3*a*b*B*x
^3+a*A*b*x^2+1/2*C*x^2*a^2+a^2*B*x+a^2*A*ln(x)
```

Maxima [A] time = 0.7022, size = 186, normalized size = 1.24

$$\frac{1}{10}\,Cc^2x^{10}+\frac{1}{9}\,Bc^2x^9+\frac{2}{7}\,Bbcx^7+\frac{1}{8}\left(2\,Cbc+Ac^2\right)x^8+\frac{1}{6}\left(Cb^2+2\,(Ca+Ab)c\right)x^6+\frac{2}{3}\,Babx^3+\frac{1}{5}\left(Bb^2+2\,Bac\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2+2\,Aac\right)x^4+Ba^2x+Aa^2\log(x)+\frac{1}{2}\left(Ca^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x,x, algorithm="maxima")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c +
A*c^2)*x^8 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 +
1/5*(B*b^2 + 2*B*a*c)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 +
 B*a^2*x + A*a^2*log(x) + 1/2*(C*a^2 + 2*A*a*b)*x^2
```

Fricas [A] time = 0.256648, size = 186, normalized size = 1.24

$$\frac{1}{10}\,Cc^2x^{10}+\frac{1}{9}\,Bc^2x^9+\frac{2}{7}\,Bbcx^7+\frac{1}{8}\left(2\,Cbc+Ac^2\right)x^8+\frac{1}{6}\left(Cb^2+2\,(Ca+Ab)c\right)x^6+\frac{2}{3}\,Babx^3+\frac{1}{5}\left(Bb^2+2\,Bac\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2+2\,Aac\right)x^4+Ba^2x+Aa^2\log(x)+\frac{1}{2}\left(Ca^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x,x, algorithm="fricas")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c +
A*c^2)*x^8 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 +
1/5*(B*b^2 + 2*B*a*c)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 +
 B*a^2*x + A*a^2*log(x) + 1/2*(C*a^2 + 2*A*a*b)*x^2
```

Sympy [A] time = 1.71315, size = 156, normalized size = 1.04

$$Aa^2\log(x) + Ba^2x + \frac{2Babx^3}{3} + \frac{2Bbcx^7}{7} + \frac{Bc^2x^9}{9} + \frac{Cc^2x^{10}}{10} + x^8\left(\frac{Ac^2}{8} + \frac{Cbc}{4}\right) + x^6\left(\frac{Abc}{3} + \frac{Cac}{3} + \frac{Cb^2}{6}\right) + x^5\left(\frac{2Bac}{5} + \frac{Bb^2}{5}\right) + x^4\left(\frac{Aac}{2} + \frac{Ab^2}{4} + \frac{Cab}{2}\right) + x^2\left(Aab + \frac{Ca^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + 2*B*a*b*x**3/3 + 2*B*b*c*x**7/7 + B*c*
*2*x**9/9 + C*c**2*x**10/10 + x**8*(A*c**2/8 + C*b*c/4) + x**6*(A
*b*c/3 + C*a*c/3 + C*b**2/6) + x**5*(2*B*a*c/5 + B*b**2/5) + x**4
*(A*a*c/2 + A*b**2/4 + C*a*b/2) + x**2*(A*a*b + C*a**2/2)
```

GIAC/XCAS [A] time = 0.280989, size = 201, normalized size = 1.34

$$\frac{1}{10}Cc^2x^{10} + \frac{1}{9}Bc^2x^9 + \frac{1}{4}Cbcx^8 + \frac{1}{8}Ac^2x^8 + \frac{2}{7}Bbcx^7 + \frac{1}{6}Cb^2x^6 + \frac{1}{3}Cacx^6 + \frac{1}{3}Abcx^6 + \frac{1}{5}Bb^2x^5 + \frac{2}{5}Bacx^5 + \frac{1}{2}Cabx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{2}Aacx^4 + \frac{2}{3}Babx^3 + \frac{1}{2}Ca^2x^2 + Aabx^2 + Ba^2x + Aa^2\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x,x, algorithm="giac")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 1/4*C*b*c*x^8 + 1/8*A*c^2*x^8 +
 2/7*B*b*c*x^7 + 1/6*C*b^2*x^6 + 1/3*C*a*c*x^6 + 1/3*A*b*c*x^6 +
1/5*B*b^2*x^5 + 2/5*B*a*c*x^5 + 1/2*C*a*b*x^4 + 1/4*A*b^2*x^4 + 1
/2*A*a*c*x^4 + 2/3*B*a*b*x^3 + 1/2*C*a^2*x^2 + A*a*b*x^2 + B*a^2*
x + A*a^2*ln(abs(x))
```

3.15 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx$

Optimal. Leaf size=145

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+abBx^2+\frac{1}{7}cx^7(Ac+2bC)+\frac{1}{3}bBcx^6+\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9$$

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*(b^2 + 2*a*c)
+ 2*a*b*C)*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + (b^2 +
2*a*c)*C)*x^5)/5 + (b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c
^2*x^8)/8 + (c^2*C*x^9)/9 + a^2*B*Log[x]
```

Rubi [A] time = 0.289457, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+abBx^2+\frac{1}{7}cx^7(Ac+2bC)+\frac{1}{3}bBcx^6+\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*(b^2 + 2*a*c)
+ 2*a*b*C)*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + (b^2 +
2*a*c)*C)*x^5)/5 + (b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c
^2*x^8)/8 + (c^2*C*x^9)/9 + a^2*B*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{x}+Ba^2\log(x)+2Bab\int x\,dx+\frac{Bbcx^6}{3}+\frac{Bc^2x^8}{8}+\frac{Bx^4\left(2ac+b^2\right)}{4}+\frac{Cc^2x^9}{9}+\frac{cx^7\left(Ac+2Cb\right)}{7}+x^5\left(\frac{2Abc}{5}+\frac{2Cac}{5}+\frac{Cb^2}{5}\right)+x^3\left(\frac{2Aac}{3}+\frac{Ab^2}{3}+\frac{2Cab}{3}\right)+\frac{a\left(2Ab+Ca\right)\int C\,dx}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + 2*B*a*b*Integral(x, x) + B*b*c*x**6/3
 + B*c**2*x**8/8 + B*x**4*(2*a*c + b**2)/4 + C*c**2*x**9/9 + c*x*
*7*(A*c + 2*C*b)/7 + x**5*(2*A*b*c/5 + 2*C*a*c/5 + C*b**2/5) + x*
*3*(2*A*a*c/3 + A*b**2/3 + 2*C*a*b/3) + a*(2*A*b + C*a)*Integral(
C, x)/C
```

Mathematica [A] time = 0.309841, size = 145, normalized size = 1.

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(2acC+2Abc+b^2C\right)+\frac{1}{3}x^3\left(2aAc+2abC+Ab^2\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+abBx^2+\frac{1}{7}cx^7(Ac+2bC)+\frac{1}{3}bBcx^6+\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*b^2 + 2*a*A*c
+ 2*a*b*C)*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + b^2*C +
 2*a*c*C)*x^5)/5 + (b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c
^2*x^8)/8 + (c^2*C*x^9)/9 + a^2*B*Log[x]
```

Maple [A] time = 0.01, size = 147, normalized size = 1.

$$\frac{c^2Cx^9}{9}+\frac{Bc^2x^8}{8}+\frac{Ax^7c^2}{7}+\frac{2\,Cx^7bc}{7}+\frac{bBcx^6}{3}+\frac{2\,Ax^5bc}{5}+\frac{2\,Cx^5ac}{5}+\frac{Cx^5b^2}{5}+\frac{Bx^4ac}{2}+\frac{Bx^4b^2}{4}+\frac{2\,Ax^3ac}{3}+\frac{Ax^3b^2}{3}+\frac{2\,Cx^3ab}{3}+abBx^2+2\,Axab+Cxa^2+a^2B\ln(x)-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x)
```

```
[Out] 1/9*c^2*C*x^9+1/8*B*c^2*x^8+1/7*A*x^7*c^2+2/7*C*x^7*b*c+1/3*b*B*c
*x^6+2/5*A*x^5*b*c+2/5*C*x^5*a*c+1/5*C*x^5*b^2+1/2*B*x^4*a*c+1/4*
B*x^4*b^2+2/3*A*x^3*a*c+1/3*A*x^3*b^2+2/3*C*x^3*a*b+a*b*B*x^2+2*A
*x*a*b+C*x*a^2+a^2*B*ln(x)-a^2*A/x
```

Maxima [A] time = 0.702254, size = 185, normalized size = 1.28

$$\frac{1}{9}\,Cc^2x^9+\frac{1}{8}\,Bc^2x^8+\frac{1}{3}\,Bbcx^6+\frac{1}{7}\left(2\,Cbc+Ac^2\right)x^7+\frac{1}{5}\left(Cb^2+2\,(Ca+Ab)c\right)x^5+Babx^2+\frac{1}{4}\left(Bb^2+2\,Bac\right)x^4+\frac{1}{3}\left(2\,Cab+Ab^2+2\,Aac\right)x^3+Ba^2\log(x)-\frac{Aa^2}{x}+\left(Ca^2+2\,Aab\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/7*(2*C*b*c + A*
c^2)*x^7 + 1/5*(C*b^2 + 2*(C*a + A*b)*c)*x^5 + B*a*b*x^2 + 1/4*(B
*b^2 + 2*B*a*c)*x^4 + 1/3*(2*C*a*b + A*b^2 + 2*A*a*c)*x^3 + B*a^2
*log(x) - A*a^2/x + (C*a^2 + 2*A*a*b)*x
```

Fricas [A] time = 0.246103, size = 196, normalized size = 1.35

$$\frac{280\,Cc^2x^{10}+315\,Bc^2x^9+840\,Bbcx^7+360\left(2\,Cbc+Ac^2\right)x^8+504\left(Cb^2+2\,(Ca+Ab)c\right)x^6+2520\,Babx^3+630\left(Bb^2+2\,Bac\right)\cdots}{2520\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2520*(280*C*c^2*x^10 + 315*B*c^2*x^9 + 840*B*b*c*x^7 + 360*(2*C
*b*c + A*c^2)*x^8 + 504*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2520*B*a*
b*x^3 + 630*(B*b^2 + 2*B*a*c)*x^5 + 840*(2*C*a*b + A*b^2 + 2*A*a*
c)*x^4 + 2520*B*a^2*x*log(x) - 2520*A*a^2 + 2520*(C*a^2 + 2*A*a*b
)*x^2)/x
```

Sympy [A] time = 1.84658, size = 156, normalized size = 1.08

$$-\frac{Aa^2}{x} + Ba^2\log(x) + Babx^2 + \frac{Bbcx^6}{3} + \frac{Bc^2x^8}{8} + \frac{Cc^2x^9}{9} + x^7\left(\frac{Ac^2}{7} + \frac{2Cbc}{7}\right) + x^5\left(\frac{2Abc}{5} + \frac{2Cac}{5} + \frac{Cb^2}{5}\right) + x^4\left(\frac{Bac}{2} + \frac{Bb^2}{4}\right) + x^3\left(\frac{2Aac}{3} + \frac{Ab^2}{3} + \frac{2Cab}{3}\right) + x\left(2Aab + Ca^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + B*a*b*x**2 + B*b*c*x**6/3 + B*c**2*x*
*8/8 + C*c**2*x**9/9 + x**7*(A*c**2/7 + 2*C*b*c/7) + x**5*(2*A*b*
c/5 + 2*C*a*c/5 + C*b**2/5) + x**4*(B*a*c/2 + B*b**2/4) + x**3*(2
*A*a*c/3 + A*b**2/3 + 2*C*a*b/3) + x*(2*A*a*b + C*a**2)
```

GIAC/XCAS [A] time = 0.286491, size = 198, normalized size = 1.37

$$\frac{1}{9}Cc^2x^9 + \frac{1}{8}Bc^2x^8 + \frac{2}{7}Cbcx^7 + \frac{1}{7}Ac^2x^7 + \frac{1}{3}Bbcx^6 + \frac{1}{5}Cb^2x^5 + \frac{2}{5}Cacx^5 + \frac{2}{5}Abcx^5 + \frac{1}{4}Bb^2x^4 + \frac{1}{2}Bacx^4 + \frac{2}{3}Cabx^3 + \frac{1}{3}Ab^2x^3 + \frac{2}{3}Aacx^3 + Babx^2 + Ca^2x + 2\,Aabx + Ba^2\ln(|x|) - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^2,x, algorithm="giac")
```

```
[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 2/7*C*b*c*x^7 + 1/7*A*c^2*x^7 + 1
/3*B*b*c*x^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 2/5*A*b*c*x^5 + 1/
4*B*b^2*x^4 + 1/2*B*a*c*x^4 + 2/3*C*a*b*x^3 + 1/3*A*b^2*x^3 + 2/3
*A*a*c*x^3 + B*a*b*x^2 + C*a^2*x + 2*A*a*b*x + B*a^2*ln(abs(x)) -
 A*a^2/x
```

3.16 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3} dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{4}x^4\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abC\right)+a\log(x)(aC+2Ab)+\frac{1}{3}Bx^3\left(2ac+b^2\right)+2abBx+\frac{1}{6}cx^6(Ac+2bC)+\frac{2}{5}bBcx^5+\frac{1}{7}Bc^2x^7+\frac{1}{8}c^2Cx^8$$

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + 2*a*b*B*x + ((A*(b^2 + 2*a*c) + 2*
a*b*C)*x^2)/2 + (B*(b^2 + 2*a*c)*x^3)/3 + ((2*A*b*c + (b^2 + 2*a*
c)*C)*x^4)/4 + (2*b*B*c*x^5)/5 + (c*(A*c + 2*b*C)*x^6)/6 + (B*c^2
*x^7)/7 + (c^2*C*x^8)/8 + a*(2*A*b + a*C)*Log[x]
```

Rubi [A] time = 0.308333, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{4}x^4\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abC\right)+a\log(x)(aC+2Ab)+\frac{1}{3}Bx^3\left(2ac+b^2\right)+2abBx+\frac{1}{6}cx^6(Ac+2bC)+\frac{2}{5}bBcx^5+\frac{1}{7}Bc^2x^7+\frac{1}{8}c^2Cx^8$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + 2*a*b*B*x + ((A*(b^2 + 2*a*c) + 2*
a*b*C)*x^2)/2 + (B*(b^2 + 2*a*c)*x^3)/3 + ((2*A*b*c + (b^2 + 2*a*
c)*C)*x^4)/4 + (2*b*B*c*x^5)/5 + (c*(A*c + 2*b*C)*x^6)/6 + (B*c^2
*x^7)/7 + (c^2*C*x^8)/8 + a*(2*A*b + a*C)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{2x^2}-\frac{Ba^2}{x}+2Babx+\frac{2Bbcx^5}{5}+\frac{Bc^2x^7}{7}+\frac{Bx^3\left(2ac+b^2\right)}{3}+\frac{Cc^2x^8}{8}+a\left(2Ab+Ca\right)\log\left(x\right)+\frac{cx^6\left(Ac+2Cb\right)}{6}+x^4\left(\frac{Abc}{2}+\frac{Cac}{2}+\frac{Cb^2}{4}\right)+\left(2Aac+Ab^2+2Cab\right)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) - B*a**2/x + 2*B*a*b*x + 2*B*b*c*x**5/5 + B*c**2
*x**7/7 + B*x**3*(2*a*c + b**2)/3 + C*c**2*x**8/8 + a*(2*A*b + C*
a)*log(x) + c*x**6*(A*c + 2*C*b)/6 + x**4*(A*b*c/2 + C*a*c/2 + C*
b**2/4) + (2*A*a*c + A*b**2 + 2*C*a*b)*Integral(x, x)
```

Mathematica [A] time = 0.217537, size = 139, normalized size = 0.93

$$-\frac{a^2(A+2Bx)}{2x^2}+\frac{1}{6}ax\left(cx\left(6A+4Bx+3Cx^2\right)+6b(2B+Cx)\right)+a\log(x)(aC+2Ab)+\frac{1}{840}x^2\left(140A\left(3b^2+3bcx^2+c^2x^4\right)+70b^2x(4B+3Cx)+56bcx^3(6B+5Cx)+15c^2x^5(8B+7Cx)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -(a^2*(A + 2*B*x))/(2*x^2) + (a*x*(6*b*(2*B + C*x) + c*x*(6*A + 4
*B*x + 3*C*x^2)))/6 + (x^2*(70*b^2*x*(4*B + 3*C*x) + 56*b*c*x^3*(
6*B + 5*C*x) + 15*c^2*x^5*(8*B + 7*C*x) + 140*A*(3*b^2 + 3*b*c*x^
2 + c^2*x^4)))/840 + a*(2*A*b + a*C)*Log[x]
```

Maple [A] time = 0.01, size = 148, normalized size = 1.

$$\frac{c^2Cx^8}{8}+\frac{Bc^2x^7}{7}+\frac{Ax^6c^2}{6}+\frac{bcCx^6}{3}+\frac{2\,bBcx^5}{5}+\frac{Ax^4bc}{2}+\frac{Cx^4ac}{2}+\frac{Cx^4b^2}{4}+\frac{2\,Bx^3ac}{3}+\frac{Bx^3b^2}{3}+Ax^2ac+\frac{Ax^2b^2}{2}+Cx^2ab+2\,abBx+2\,A\ln(x)\,ab+C\ln(x)\,a^2-\frac{Ba^2}{x}-\frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x)
```

```
[Out] 1/8*c^2*C*x^8+1/7*B*c^2*x^7+1/6*A*x^6*c^2+1/3*b*c*C*x^6+2/5*b*B*c
*x^5+1/2*A*x^4*b*c+1/2*C*x^4*a*c+1/4*C*x^4*b^2+2/3*B*x^3*a*c+1/3*
B*x^3*b^2+A*x^2*a*c+1/2*A*x^2*b^2+C*x^2*a*b+2*a*b*B*x+2*A*ln(x)*a
*b+C*ln(x)*a^2-a^2*B/x-1/2*a^2*A/x^2
```

Maxima [A] time = 0.695264, size = 188, normalized size = 1.26

$$\frac{1}{8}\,Cc^2x^8+\frac{1}{7}\,Bc^2x^7+\frac{2}{5}\,Bbcx^5+\frac{1}{6}\left(2\,Cbc+Ac^2\right)x^6+\frac{1}{4}\left(Cb^2+2\,(Ca+Ab)c\right)x^4+2\,Babx+\frac{1}{3}\left(Bb^2+2\,Bac\right)x^3+\frac{1}{2}\left(2\,Cab+Ab^2+2\,Aac\right)x^2+\left(Ca^2+2\,Aab\right)\log(x)-\frac{2\,Ba^2x+Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/6*(2*C*b*c + A*
c^2)*x^6 + 1/4*(C*b^2 + 2*(C*a + A*b)*c)*x^4 + 2*B*a*b*x + 1/3*(B
*b^2 + 2*B*a*c)*x^3 + 1/2*(2*C*a*b + A*b^2 + 2*A*a*c)*x^2 + (C*a^
2 + 2*A*a*b)*log(x) - 1/2*(2*B*a^2*x + A*a^2)/x^2
```

Fricas [A] time = 0.251666, size = 196, normalized size = 1.32

$$\frac{105\,Cc^2x^{10}+120\,Bc^2x^9+336\,Bbcx^7+140\left(2\,Cbc+Ac^2\right)x^8+210\left(Cb^2+2\,(Ca+Ab)c\right)x^6+1680\,Babx^3+280\left(Bb^2+2\,Bac\right)}{840\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*C*c^2*x^10 + 120*B*c^2*x^9 + 336*B*b*c*x^7 + 140*(2*C*
b*c + A*c^2)*x^8 + 210*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 1680*B*a*b
*x^3 + 280*(B*b^2 + 2*B*a*c)*x^5 + 420*(2*C*a*b + A*b^2 + 2*A*a*c
)*x^4 - 840*B*a^2*x + 840*(C*a^2 + 2*A*a*b)*x^2*log(x) - 420*A*a^
2)/x^2
```

Sympy [A] time = 2.30079, size = 151, normalized size = 1.01

$$2Babx + \frac{2Bbcx^5}{5} + \frac{Bc^2x^7}{7} + \frac{Cc^2x^8}{8} + a(2Ab + Ca)\log(x) + x^6\left(\frac{Ac^2}{6} + \frac{Cbc}{3}\right) + x^4\left(\frac{Abc}{2} + \frac{Cac}{2} + \frac{Cb^2}{4}\right) + x^3\left(\frac{2Bac}{3} + \frac{Bb^2}{3}\right) + x^2\left(Aac + \frac{Ab^2}{2} + Cab\right) - \frac{Aa^2 + 2Ba^2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] 2*B*a*b*x + 2*B*b*c*x**5/5 + B*c**2*x**7/7 + C*c**2*x**8/8 + a*(2
*A*b + C*a)*log(x) + x**6*(A*c**2/6 + C*b*c/3) + x**4*(A*b*c/2 +
C*a*c/2 + C*b**2/4) + x**3*(2*B*a*c/3 + B*b**2/3) + x**2*(A*a*c +
 A*b**2/2 + C*a*b) - (A*a**2 + 2*B*a**2*x)/(2*x**2)
```

GIAC/XCAS [A] time = 0.279804, size = 200, normalized size = 1.34

$$\frac{1}{8}Cc^2x^8 + \frac{1}{7}Bc^2x^7 + \frac{1}{3}Cbcx^6 + \frac{1}{6}Ac^2x^6 + \frac{2}{5}Bbcx^5 + \frac{1}{4}Cb^2x^4 + \frac{1}{2}Cacx^4 + \frac{1}{2}Abcx^4 + \frac{1}{3}Bb^2x^3 + \frac{2}{3}Bacx^3 + Cabx^2 + \frac{1}{2}Ab^2x^2 + Aacx^2 + 2\,Babx + \left(Ca^2 + 2\,Aab\right)\ln(|x|) - \frac{2\,Ba^2x + Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 1/3*C*b*c*x^6 + 1/6*A*c^2*x^6 + 2
/5*B*b*c*x^5 + 1/4*C*b^2*x^4 + 1/2*C*a*c*x^4 + 1/2*A*b*c*x^4 + 1/
3*B*b^2*x^3 + 2/3*B*a*c*x^3 + C*a*b*x^2 + 1/2*A*b^2*x^2 + A*a*c*x
^2 + 2*B*a*b*x + (C*a^2 + 2*A*a*b)*ln(abs(x)) - 1/2*(2*B*a^2*x +
A*a^2)/x^2
```

3.17 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log(x)+\frac{1}{5}cx^5(Ac+2bC)+\frac{1}{2}bBcx^4+\frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7$$

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + (A*(b^
2 + 2*a*c) + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + (
b^2 + 2*a*c)*C)*x^3)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5
+ (B*c^2*x^6)/6 + (c^2*C*x^7)/7 + 2*a*b*B*Log[x]
```

Rubi [A] time = 0.30541, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log(x)+\frac{1}{5}cx^5(Ac+2bC)+\frac{1}{2}bBcx^4+\frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + (A*(b^
2 + 2*a*c) + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + (
b^2 + 2*a*c)*C)*x^3)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5
+ (B*c^2*x^6)/6 + (c^2*C*x^7)/7 + 2*a*b*B*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{3x^3}-\frac{Ba^2}{2x^2}+2Bab\log(x)+\frac{Bbcx^4}{2}+\frac{Bc^2x^6}{6}+B\left(2ac+b^2\right)\int x\,dx+\frac{Cc^2x^7}{7}-\frac{a(2Ab+Ca)}{x}+\frac{cx^5(Ac+2Cb)}{5}+x^3\left(\frac{2Abc}{3}+\frac{2Cac}{3}+\frac{Cb^2}{3}\right)+\frac{\left(Ab^2+2a(Ac+Cb)\right)\int A\,dx}{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] -A*a**2/(3*x**3) - B*a**2/(2*x**2) + 2*B*a*b*log(x) + B*b*c*x**4/
2 + B*c**2*x**6/6 + B*(2*a*c + b**2)*Integral(x, x) + C*c**2*x**7
/7 - a*(2*A*b + C*a)/x + c*x**5*(A*c + 2*C*b)/5 + x**3*(2*A*b*c/3
 + 2*C*a*c/3 + C*b**2/3) + (A*b**2 + 2*a*(A*c + C*b))*Integral(A,
 x)/A
```

Mathematica [A] time = 0.174244, size = 151, normalized size = 1.01

$$\frac{a^2(-C)-2aAb}{x}-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(2acC+2Abc+b^2C\right)+x\left(2aAc+2abC+Ab^2\right)+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log(x)+\frac{1}{5}cx^5(Ac+2bC)+\frac{1}{2}bBcx^4+\frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) + (-2*a*A*b - a^2*C)/x + (A*b^
2 + 2*a*A*c + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c +
b^2*C + 2*a*c*C)*x^3)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5
 + (B*c^2*x^6)/6 + (c^2*C*x^7)/7 + 2*a*b*B*Log[x]
```

Maple [A] time = 0.01, size = 146, normalized size = 1.

$$\frac{c^2Cx^7}{7}+\frac{Bc^2x^6}{6}+\frac{Ax^5c^2}{5}+\frac{2\,Cx^5bc}{5}+\frac{bBcx^4}{2}+\frac{2\,Ax^3bc}{3}+\frac{2\,Cx^3ac}{3}+\frac{Cx^3b^2}{3}+Bx^2ac+\frac{Bx^2b^2}{2}+2\,Axac+Axb^2+2\,Cxab-\frac{Aa^2}{3\,x^3}+2\,abB\ln(x)-2\,\frac{abA}{x}-\frac{a^2C}{x}-\frac{Ba^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x)
```

```
[Out] 1/7*c^2*C*x^7+1/6*B*c^2*x^6+1/5*A*x^5*c^2+2/5*C*x^5*b*c+1/2*b*B*c
*x^4+2/3*A*x^3*b*c+2/3*C*x^3*a*c+1/3*C*x^3*b^2+B*x^2*a*c+1/2*B*x^
2*b^2+2*A*x*a*c+A*x*b^2+2*C*x*a*b-1/3*a^2*A/x^3+2*a*b*B*ln(x)-2*a
/x*A*b-a^2/x*C-1/2*a^2*B/x^2
```

Maxima [A] time = 0.689953, size = 189, normalized size = 1.27

$$\frac{1}{7}\,Cc^2x^7+\frac{1}{6}\,Bc^2x^6+\frac{1}{2}\,Bbcx^4+\frac{1}{5}\left(2\,Cbc+Ac^2\right)x^5+\frac{1}{3}\left(Cb^2+2\,(Ca+Ab)c\right)x^3+2\,Bab\log(x)+\frac{1}{2}\left(Bb^2+2\,Bac\right)x^2+\left(2\,Cab+Ab^2+2\,Aac\right)x-\frac{3\,Ba^2x+2\,Aa^2+6\left(Ca^2+2\,Aab\right)x^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/5*(2*C*b*c + A*
c^2)*x^5 + 1/3*(C*b^2 + 2*(C*a + A*b)*c)*x^3 + 2*B*a*b*log(x) + 1
/2*(B*b^2 + 2*B*a*c)*x^2 + (2*C*a*b + A*b^2 + 2*A*a*c)*x - 1/6*(3
*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Fricas [A] time = 0.248056, size = 196, normalized size = 1.32

$$\frac{30\,Cc^2x^{10}+35\,Bc^2x^9+105\,Bbcx^7+42\left(2\,Cbc+Ac^2\right)x^8+70\left(Cb^2+2\,(Ca+Ab)c\right)x^6+420\,Babx^3\log(x)+105\left(Bb^2+2\,Bac\right.}{210\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/210*(30*C*c^2*x^10 + 35*B*c^2*x^9 + 105*B*b*c*x^7 + 42*(2*C*b*c
 + A*c^2)*x^8 + 70*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 420*B*a*b*x^3*
log(x) + 105*(B*b^2 + 2*B*a*c)*x^5 + 210*(2*C*a*b + A*b^2 + 2*A*a
*c)*x^4 - 105*B*a^2*x - 70*A*a^2 - 210*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Sympy [A] time = 3.33591, size = 158, normalized size = 1.06

$$2Bab\log(x)+\frac{Bbcx^4}{2}+\frac{Bc^2x^6}{6}+\frac{Cc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Cbc}{5}\right)+x^3\left(\frac{2Abc}{3}+\frac{2Cac}{3}+\frac{Cb^2}{3}\right)+x^2\left(Bac+\frac{Bb^2}{2}\right)+x\left(2Aac+Ab^2+2Cab\right)-\frac{2Aa^2+3Ba^2x+x^2\left(12Aab+6Ca^2\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] 2*B*a*b*log(x) + B*b*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x
**5*(A*c**2/5 + 2*C*b*c/5) + x**3*(2*A*b*c/3 + 2*C*a*c/3 + C*b**2
/3) + x**2*(B*a*c + B*b**2/2) + x*(2*A*a*c + A*b**2 + 2*C*a*b) -
(2*A*a**2 + 3*B*a**2*x + x**2*(12*A*a*b + 6*C*a**2))/(6*x**3)
```

GIAC/XCAS [A] time = 0.285649, size = 197, normalized size = 1.32

$$\frac{1}{7}Cc^2x^7+\frac{1}{6}Bc^2x^6+\frac{2}{5}Cbcx^5+\frac{1}{5}Ac^2x^5+\frac{1}{2}Bbcx^4+\frac{1}{3}Cb^2x^3+\frac{2}{3}Cacx^3+\frac{2}{3}Abcx^3+\frac{1}{2}Bb^2x^2+Bacx^2+2Cabx+Ab^2x+2Aacx+2Bab\ln(|x|)-\frac{3Ba^2x+2Aa^2+6\left(Ca^2+2Aab\right)x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^4,x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*b*c*x^5 + 1/5*A*c^2*x^5 + 1
/2*B*b*c*x^4 + 1/3*C*b^2*x^3 + 2/3*C*a*c*x^3 + 2/3*A*b*c*x^3 + 1/
2*B*b^2*x^2 + B*a*c*x^2 + 2*C*a*b*x + A*b^2*x + 2*A*a*c*x + 2*B*a
*b*ln(abs(x)) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^
2)/x^3
```

3.18 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx$

Optimal. Leaf size=148

$$-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}+\frac{1}{2}x^2\left(C\left(2ac+b^2\right)+2Abc\right)+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{2x^2}+Bx\left(2ac+b^2\right)-\frac{2abB}{x}+\frac{1}{4}cx^4(Ac+2bC)+\frac{2}{3}bBcx^3+\frac{1}{5}Bc^2x^5+\frac{1}{6}c^2Cx^6$$

```
[Out] -(a^2*A)/(4*x^4) - (a^2*B)/(3*x^3) - (a*(2*A*b + a*C))/(2*x^2) -
(2*a*b*B)/x + B*(b^2 + 2*a*c)*x + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^
2)/2 + (2*b*B*c*x^3)/3 + (c*(A*c + 2*b*C)*x^4)/4 + (B*c^2*x^5)/5
+ (c^2*C*x^6)/6 + (A*(b^2 + 2*a*c) + 2*a*b*C)*Log[x]
```

Rubi [A] time = 0.327309, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}+\frac{1}{2}x^2\left(C\left(2ac+b^2\right)+2Abc\right)+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{2x^2}+Bx\left(2ac+b^2\right)-\frac{2abB}{x}+\frac{1}{4}cx^4(Ac+2bC)+\frac{2}{3}bBcx^3+\frac{1}{5}Bc^2x^5+\frac{1}{6}c^2Cx^6$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] -(a^2*A)/(4*x^4) - (a^2*B)/(3*x^3) - (a*(2*A*b + a*C))/(2*x^2) -
(2*a*b*B)/x + B*(b^2 + 2*a*c)*x + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^
2)/2 + (2*b*B*c*x^3)/3 + (c*(A*c + 2*b*C)*x^4)/4 + (B*c^2*x^5)/5
+ (c^2*C*x^6)/6 + (A*(b^2 + 2*a*c) + 2*a*b*C)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{4x^4}-\frac{Ba^2}{3x^3}-\frac{2Bab}{x}+\frac{2Bbcx^3}{3}+\frac{Bc^2x^5}{5}+\frac{Cc^2x^6}{6}-\frac{a\left(2Ab+Ca\right)}{2x^2}+\frac{cx^4\left(Ac+2Cb\right)}{4}+\left(2ac+b^2\right)\int B\,dx+\left(2Aac+Ab^2+2Cab\right)\log\left(x\right)+\left(2Abc+2Cac+Cb^2\right)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] -A*a**2/(4*x**4) - B*a**2/(3*x**3) - 2*B*a*b/x + 2*B*b*c*x**3/3 +
 B*c**2*x**5/5 + C*c**2*x**6/6 - a*(2*A*b + C*a)/(2*x**2) + c*x**
4*(A*c + 2*C*b)/4 + (2*a*c + b**2)*Integral(B, x) + (2*A*a*c + A*
b**2 + 2*C*a*b)*log(x) + (2*A*b*c + 2*C*a*c + C*b**2)*Integral(x,
 x)
```

Mathematica [A] time = 0.189995, size = 130, normalized size = 0.88

$$-\frac{a^2\left(3A+4Bx+6Cx^2\right)}{12x^4}+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)+\frac{a\left(-Ab-2bBx+cx^3(2B+Cx)\right)}{x^2}+\frac{1}{60}x\left(10bcx(6A+x(4B+3Cx))+c^2x^3(15A+2x(6B+5Cx))+30b^2(2B+Cx)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] -(a^2*(3*A + 4*B*x + 6*C*x^2))/(12*x^4) + (a*(-(A*b) - 2*b*B*x +
c*x^3*(2*B + C*x)))/x^2 + (x*(30*b^2*(2*B + C*x) + 10*b*c*x*(6*A
+ x*(4*B + 3*C*x)) + c^2*x^3*(15*A + 2*x*(6*B + 5*C*x))))/60 + (A
*(b^2 + 2*a*c) + 2*a*b*C)*Log[x]
```

Maple [A] time = 0.011, size = 144, normalized size = 1.

$$\frac{c^2Cx^6}{6}+\frac{Bc^2x^5}{5}+\frac{Ax^4c^2}{4}+\frac{Cx^4bc}{2}+\frac{2\,bBcx^3}{3}+Ax^2bc+Cx^2ac+\frac{Cx^2b^2}{2}+2\,Bxac+Bxb^2-\frac{Ba^2}{3\,x^3}+2\,A\ln(x)\,ac+A\ln(x)\,b^2+2\,C\ln(x)\,ab-2\,\frac{abB}{x}-\frac{abA}{x^2}-\frac{a^2C}{2\,x^2}-\frac{Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x)
```

```
[Out] 1/6*c^2*C*x^6+1/5*B*c^2*x^5+1/4*A*x^4*c^2+1/2*C*x^4*b*c+2/3*b*B*c
*x^3+A*x^2*b*c+C*x^2*a*c+1/2*C*x^2*b^2+2*B*x*a*c+B*x*b^2-1/3*a^2*
B/x^3+2*A*ln(x)*a*c+A*ln(x)*b^2+2*C*ln(x)*a*b-2*a*b*B/x-a/x^2*A*b
-1/2*a^2/x^2*C-1/4*a^2*A/x^4
```

Maxima [A] time = 0.693051, size = 188, normalized size = 1.27

$$\frac{1}{6}\,Cc^2x^6+\frac{1}{5}\,Bc^2x^5+\frac{2}{3}\,Bbcx^3+\frac{1}{4}\left(2\,Cbc+Ac^2\right)x^4+\frac{1}{2}\left(Cb^2+2\,(Ca+Ab)c\right)x^2+\left(Bb^2+2\,Bac\right)x+\left(2\,Cab+Ab^2+2\,Aac\right)\log(x)-\frac{24\,Babx^3+4\,Ba^2x+3\,Aa^2+6\left(Ca^2+2\,Aab\right)x^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/4*(2*C*b*c + A*
c^2)*x^4 + 1/2*(C*b^2 + 2*(C*a + A*b)*c)*x^2 + (B*b^2 + 2*B*a*c)*
x + (2*C*a*b + A*b^2 + 2*A*a*c)*log(x) - 1/12*(24*B*a*b*x^3 + 4*B
*a^2*x + 3*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

Fricas [A] time = 0.247075, size = 196, normalized size = 1.32

$$\frac{10\,Cc^2x^{10}+12\,Bc^2x^9+40\,Bbcx^7+15\left(2\,Cbc+Ac^2\right)x^8+30\left(Cb^2+2\,(Ca+Ab)c\right)x^6-120\,Babx^3+60\left(Bb^2+2\,Bac\right)x^5+60}{60\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^2*x^10 + 12*B*c^2*x^9 + 40*B*b*c*x^7 + 15*(2*C*b*c +
 A*c^2)*x^8 + 30*(C*b^2 + 2*(C*a + A*b)*c)*x^6 - 120*B*a*b*x^3 +
60*(B*b^2 + 2*B*a*c)*x^5 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4*log
(x) - 20*B*a^2*x - 15*A*a^2 - 30*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

Sympy [A] time = 9.81026, size = 151, normalized size = 1.02

$$\frac{2Bbcx^3}{3}+\frac{Bc^2x^5}{5}+\frac{Cc^2x^6}{6}+x^4\left(\frac{Ac^2}{4}+\frac{Cbc}{2}\right)+x^2\left(Abc+Cac+\frac{Cb^2}{2}\right)+x\left(2Bac+Bb^2\right)+\left(2Aac+Ab^2+2Cab\right)\log\left(x\right)-\frac{3Aa^2+4Ba^2x+24Babx^3+x^2\left(12Aab+6Ca^2\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] 2*B*b*c*x**3/3 + B*c**2*x**5/5 + C*c**2*x**6/6 + x**4*(A*c**2/4 +
 C*b*c/2) + x**2*(A*b*c + C*a*c + C*b**2/2) + x*(2*B*a*c + B*b**2
) + (2*A*a*c + A*b**2 + 2*C*a*b)*log(x) - (3*A*a**2 + 4*B*a**2*x
+ 24*B*a*b*x**3 + x**2*(12*A*a*b + 6*C*a**2))/(12*x**4)
```

GIAC/XCAS [A] time = 0.282811, size = 192, normalized size = 1.3

$$\frac{1}{6}Cc^2x^6+\frac{1}{5}Bc^2x^5+\frac{1}{2}Cbcx^4+\frac{1}{4}Ac^2x^4+\frac{2}{3}Bbcx^3+\frac{1}{2}Cb^2x^2+Cacx^2+Abcx^2+Bb^2x+2\,Bacx+\left(2\,Cab+Ab^2+2\,Aac\right)\ln\left(|x|\right)-\frac{24\,Babx^3+4\,Ba^2x+3\,Aa^2+6\left(Ca^2+2\,Aab\right)x^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^5,x, algorithm="giac")
```

```
[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 1/2*C*b*c*x^4 + 1/4*A*c^2*x^4 + 2
/3*B*b*c*x^3 + 1/2*C*b^2*x^2 + C*a*c*x^2 + A*b*c*x^2 + B*b^2*x +
2*B*a*c*x + (2*C*a*b + A*b^2 + 2*A*a*c)*ln(abs(x)) - 1/12*(24*B*a
*b*x^3 + 4*B*a^2*x + 3*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

3.19 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$

Optimal. Leaf size=143

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{A\left(2ac+b^2\right)+2abC}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac+2bC)+bBcx^2+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5$$

```
[Out] -(a^2*A)/(5*x^5) - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) -
(a*b*B)/x^2 - (A*(b^2 + 2*a*c) + 2*a*b*C)/x + (2*A*b*c + (b^2 + 2
*a*c)*C)*x + b*B*c*x^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4
+ (c^2*C*x^5)/5 + B*(b^2 + 2*a*c)*Log[x]
```

Rubi [A] time = 0.340421, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{A\left(2ac+b^2\right)+2abC}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac+2bC)+bBcx^2+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) -
(a*b*B)/x^2 - (A*(b^2 + 2*a*c) + 2*a*b*C)/x + (2*A*b*c + (b^2 + 2
*a*c)*C)*x + b*B*c*x^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4
+ (c^2*C*x^5)/5 + B*(b^2 + 2*a*c)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{5x^5}-\frac{Ba^2}{4x^4}-\frac{Bab}{x^2}+2Bbc\int x\,dx+\frac{Bc^2x^4}{4}+B\left(2ac+b^2\right)\log(x)+\frac{Cc^2x^5}{5}-\frac{a\,(2Ab+Ca)}{3x^3}+\frac{cx^3\,(Ac+2Cb)}{3}-\frac{2Aac+Ab^2+2Cab}{x}+\frac{\left(Cb^2+2c\,(Ab+Ca)\right)\int C\,dx}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] -A*a**2/(5*x**5) - B*a**2/(4*x**4) - B*a*b/x**2 + 2*B*b*c*Integra
l(x, x) + B*c**2*x**4/4 + B*(2*a*c + b**2)*log(x) + C*c**2*x**5/5
 - a*(2*A*b + C*a)/(3*x**3) + c*x**3*(A*c + 2*C*b)/3 - (2*A*a*c +
 A*b**2 + 2*C*a*b)/x + (C*b**2 + 2*c*(A*b + C*a))*Integral(C, x)/
C
```

Mathematica [A] time = 0.172236, size = 142, normalized size = 0.99

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{2aAc+2abC+Ab^2}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)+Cx\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac+2bC)+2Abcx+bBcx^2+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) -
(a*b*B)/x^2 - (A*b^2 + 2*a*A*c + 2*a*b*C)/x + 2*A*b*c*x + (b^2 +
2*a*c)*C*x + b*B*c*x^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4
+ (c^2*C*x^5)/5 + B*(b^2 + 2*a*c)*Log[x]
```

Maple [A] time = 0.012, size = 144, normalized size = 1.

$$\frac{c^2Cx^5}{5}+\frac{Bc^2x^4}{4}+\frac{Ac^2x^3}{3}+\frac{2\,Cx^3bc}{3}+bBcx^2+2\,Axbc+2\,Cxac+Cxb^2-\frac{2\,abA}{3\,x^3}-\frac{a^2C}{3\,x^3}+2\,B\ln(x)\,ac+B\ln(x)\,b^2-2\,\frac{aAc}{x}-\frac{Ab^2}{x}-2\,\frac{abC}{x}-\frac{abB}{x^2}-\frac{Aa^2}{5\,x^5}-\frac{Ba^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x)
```

```
[Out] 1/5*c^2*C*x^5+1/4*B*c^2*x^4+1/3*A*c^2*x^3+2/3*C*x^3*b*c+b*B*c*x^2
+2*A*x*b*c+2*C*x*a*c+C*x*b^2-2/3*a/x^3*A*b-1/3*a^2/x^3*C+2*B*ln(x
)*a*c+B*ln(x)*b^2-2/x*a*A*c-1/x*A*b^2-2/x*a*b*C-a*b*B/x^2-1/5*a^2
*A/x^5-1/4*a^2*B/x^4
```

Maxima [A] time = 0.693777, size = 186, normalized size = 1.3

$$\frac{1}{5}\,Cc^2x^5+\frac{1}{4}\,Bc^2x^4+Bbcx^2+\frac{1}{3}\,\left(2\,Cbc+Ac^2\right)x^3+\left(Cb^2+2\,(Ca+Ab)c\right)x+\left(Bb^2+2\,Bac\right)\log(x)-\frac{60\,Babx^3+60\,\left(2\,Cab+Ab^2+2\,Aac\right)x^4+15\,Ba^2x+12\,Aa^2+20\,\left(Ca^2+2\,Aab\right)x^2}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/3*(2*C*b*c + A*c^2)
*x^3 + (C*b^2 + 2*(C*a + A*b)*c)*x + (B*b^2 + 2*B*a*c)*log(x) - 1
/60*(60*B*a*b*x^3 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2
*x + 12*A*a^2 + 20*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

Fricas [A] time = 0.250123, size = 196, normalized size = 1.37

$$\frac{12\,Cc^2x^{10}+15\,Bc^2x^9+60\,Bbcx^7+20\,\left(2\,Cbc+Ac^2\right)x^8+60\,\left(Cb^2+2\,(Ca+Ab)c\right)x^6+60\,\left(Bb^2+2\,Bac\right)x^5\log(x)-60\,Babx^3}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c^2*x^10 + 15*B*c^2*x^9 + 60*B*b*c*x^7 + 20*(2*C*b*c +
 A*c^2)*x^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 60*(B*b^2 + 2*B*
a*c)*x^5*log(x) - 60*B*a*b*x^3 - 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x
^4 - 15*B*a^2*x - 12*A*a^2 - 20*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

Sympy [A] time = 31.2289, size = 151, normalized size = 1.06

$$Bbcx^2 + \frac{Bc^2x^4}{4} + B\left(2ac + b^2\right)\log\left(x\right) + \frac{Cc^2x^5}{5} + x^3\left(\frac{Ac^2}{3} + \frac{2Cbc}{3}\right) + x\left(2Abc + 2Cac + Cb^2\right) - \frac{12Aa^2 + 15Ba^2x + 60Babx^3 + x^4\left(120Aac + 60Ab^2 + 120Cab\right) + x^2\left(40Aab + 20Ca^2\right)}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] B*b*c*x**2 + B*c**2*x**4/4 + B*(2*a*c + b**2)*log(x) + C*c**2*x**
5/5 + x**3*(A*c**2/3 + 2*C*b*c/3) + x*(2*A*b*c + 2*C*a*c + C*b**2
) - (12*A*a**2 + 15*B*a**2*x + 60*B*a*b*x**3 + x**4*(120*A*a*c +
60*A*b**2 + 120*C*a*b) + x**2*(40*A*a*b + 20*C*a**2))/(60*x**5)
```

GIAC/XCAS [A] time = 0.279553, size = 189, normalized size = 1.32

$$\frac{1}{5}\,Cc^2x^5 + \frac{1}{4}\,Bc^2x^4 + \frac{2}{3}\,Cbcx^3 + \frac{1}{3}\,Ac^2x^3 + Bbcx^2 + Cb^2x + 2\,Cacx + 2\,Abcx + \left(Bb^2 + 2\,Bac\right)\ln\left(|x|\right) - \frac{60\,Babx^3 + 60\,\left(2\,Cab + Ab^2 + 2\,Aac\right)x^4 + 15\,Ba^2x + 12\,Aa^2 + 20\,\left(Ca^2 + 2\,Aab\right)x^2}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^6,x, algorithm="giac")
```

```
[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + 2/3*C*b*c*x^3 + 1/3*A*c^2*x^3 + B
*b*c*x^2 + C*b^2*x + 2*C*a*c*x + 2*A*b*c*x + (B*b^2 + 2*B*a*c)*ln
(abs(x)) - 1/60*(60*B*a*b*x^3 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^
4 + 15*B*a^2*x + 12*A*a^2 + 20*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

3.20 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{A(2ac+b^2)+2abC}{2x^2}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a(aC+2Ab)}{4x^4}-\frac{B(2ac+b^2)}{x}-\frac{2abB}{3x^3}+\frac{1}{2}cx^2(Ac+2bC)+2bBcx+\frac{1}{3}Bc^2x^3+\frac{1}{4}c^2Cx^4$$

```
[Out] -(a^2*A)/(6*x^6) - (a^2*B)/(5*x^5) - (a*(2*A*b + a*C))/(4*x^4) -
(2*a*b*B)/(3*x^3) - (A*(b^2 + 2*a*c) + 2*a*b*C)/(2*x^2) - (B*(b^2
 + 2*a*c))/x + 2*b*B*c*x + (c*(A*c + 2*b*C)*x^2)/2 + (B*c^2*x^3)/
3 + (c^2*C*x^4)/4 + (2*A*b*c + (b^2 + 2*a*c)*C)*Log[x]
```

Rubi [A] time = 0.316564, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{A(2ac+b^2)+2abC}{2x^2}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a(aC+2Ab)}{4x^4}-\frac{B(2ac+b^2)}{x}-\frac{2abB}{3x^3}+\frac{1}{2}cx^2(Ac+2bC)+2bBcx+\frac{1}{3}Bc^2x^3+\frac{1}{4}c^2Cx^4$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -(a^2*A)/(6*x^6) - (a^2*B)/(5*x^5) - (a*(2*A*b + a*C))/(4*x^4) -
(2*a*b*B)/(3*x^3) - (A*(b^2 + 2*a*c) + 2*a*b*C)/(2*x^2) - (B*(b^2
 + 2*a*c))/x + 2*b*B*c*x + (c*(A*c + 2*b*C)*x^2)/2 + (B*c^2*x^3)/
3 + (c^2*C*x^4)/4 + (2*A*b*c + (b^2 + 2*a*c)*C)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^2}{6x^6}-\frac{Ba^2}{5x^5}-\frac{2Bab}{3x^3}+2Bbcx+\frac{Bc^2x^3}{3}-\frac{B(2ac+b^2)}{x}+\frac{Cc^2x^4}{4}-\frac{a(2Ab+Ca)}{4x^4}+c(Ac+2Cb)\int x\,dx+\left(2Abc+2Cac+Cb^2\right)\log(x)-\frac{Aac+\frac{Ab^2}{2}+Cab}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] -A*a**2/(6*x**6) - B*a**2/(5*x**5) - 2*B*a*b/(3*x**3) + 2*B*b*c*x
 + B*c**2*x**3/3 - B*(2*a*c + b**2)/x + C*c**2*x**4/4 - a*(2*A*b
+ C*a)/(4*x**4) + c*(A*c + 2*C*b)*Integral(x, x) + (2*A*b*c + 2*C
*a*c + C*b**2)*log(x) - (A*a*c + A*b**2/2 + C*a*b)/x**2
```

Mathematica [A] time = 0.208139, size = 144, normalized size = 0.97

$$-\frac{a^2(10A+3x(4B+5Cx))}{60x^6}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a\left(3A\left(b+2cx^2\right)+2x\left(2bB+3bCx+6Bcx^2\right)\right)}{6x^4}+\frac{A\left(c^2x^4-b^2\right)}{2x^2}-\frac{b^2B}{x}+bcx(2B+Cx)+\frac{1}{12}c^2x^3(4B+3Cx)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -((b^2*B)/x) + b*c*x*(2*B + C*x) + (c^2*x^3*(4*B + 3*C*x))/12 + (
A*(-b^2 + c^2*x^4))/(2*x^2) - (a^2*(10*A + 3*x*(4*B + 5*C*x)))/(6
0*x^6) - (a*(3*A*(b + 2*c*x^2) + 2*x*(2*b*B + 3*b*C*x + 6*B*c*x^2
)))/(6*x^4) + (2*A*b*c + (b^2 + 2*a*c)*C)*Log[x]
```

Maple [A] time = 0.012, size = 148, normalized size = 1.

$$\frac{c^2Cx^4}{4}+\frac{Bc^2x^3}{3}+\frac{Ax^2c^2}{2}+Cx^2bc+2\,bBcx-\frac{2\,abB}{3\,x^3}-\frac{Aa^2}{6\,x^6}+2\,A\ln(x)\,bc+2\,C\ln(x)\,ac$$
$$+C\ln(x)\,b^2-2\,\frac{aBc}{x}-\frac{b^2B}{x}-\frac{aAc}{x^2}-\frac{Ab^2}{2\,x^2}-\frac{abC}{x^2}-\frac{Ba^2}{5\,x^5}-\frac{abA}{2\,x^4}-\frac{a^2C}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x)
```

```
[Out] 1/4*c^2*C*x^4+1/3*B*c^2*x^3+1/2*A*x^2*c^2+C*x^2*b*c+2*b*B*c*x-2/3
*a*b*B/x^3-1/6*a^2*A/x^6+2*A*ln(x)*b*c+2*C*ln(x)*a*c+C*ln(x)*b^2-
2*B/x*a*c-B/x*b^2-1/x^2*a*A*c-1/2/x^2*A*b^2-1/x^2*a*b*C-1/5*a^2*B
/x^5-1/2*a/x^4*A*b-1/4*a^2/x^4*C
```

Maxima [A] time = 0.704074, size = 189, normalized size = 1.27

$$\frac{1}{4}\,Cc^2x^4+\frac{1}{3}\,Bc^2x^3+2\,Bbcx+\frac{1}{2}\left(2\,Cbc+Ac^2\right)x^2+\left(Cb^2+2\,(Ca+Ab)c\right)\log(x)$$
$$-\frac{40\,Babx^3+60\left(Bb^2+2\,Bac\right)x^5+30\left(2\,Cab+Ab^2+2\,Aac\right)x^4+12\,Ba^2x+10\,Aa^2+15\left(Ca^2+2\,Aab\right)x^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + 2*B*b*c*x + 1/2*(2*C*b*c + A*c^2)
*x^2 + (C*b^2 + 2*(C*a + A*b)*c)*log(x) - 1/60*(40*B*a*b*x^3 + 60
*(B*b^2 + 2*B*a*c)*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*
B*a^2*x + 10*A*a^2 + 15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

Fricas [A] time = 0.250922, size = 196, normalized size = 1.32

$$\frac{15\,Cc^2x^{10}+20\,Bc^2x^9+120\,Bbcx^7+30\left(2\,Cbc+Ac^2\right)x^8+60\left(Cb^2+2\,(Ca+Ab)c\right)x^6\log(x)-40\,Babx^3-60\left(Bb^2+2\,Bac\right)\cdots}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(15*C*c^2*x^10 + 20*B*c^2*x^9 + 120*B*b*c*x^7 + 30*(2*C*b*c
+ A*c^2)*x^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6*log(x) - 40*B*a*b
*x^3 - 60*(B*b^2 + 2*B*a*c)*x^5 - 30*(2*C*a*b + A*b^2 + 2*A*a*c)*
x^4 - 12*B*a^2*x - 10*A*a^2 - 15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

Sympy [A] time = 98.9006, size = 153, normalized size = 1.03

$$2Bbcx + \frac{Bc^2x^3}{3} + \frac{Cc^2x^4}{4} + x^2\left(\frac{Ac^2}{2} + Cbc\right) + \left(2Abc + 2Cac + Cb^2\right)\log\left(x\right) - \frac{10Aa^2 + 12Ba^2x + 40Babx^3 + x^5\left(120Bac + 60Bb^2\right) + x^4\left(60Aac + 30Ab^2 + 60Cab\right) + x^2\left(30Aab + 15Ca^2\right)}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*B*b*c*x + B*c**2*x**3/3 + C*c**2*x**4/4 + x**2*(A*c**2/2 + C*b*
c) + (2*A*b*c + 2*C*a*c + C*b**2)*log(x) - (10*A*a**2 + 12*B*a**2
*x + 40*B*a*b*x**3 + x**5*(120*B*a*c + 60*B*b**2) + x**4*(60*A*a*
c + 30*A*b**2 + 60*C*a*b) + x**2*(30*A*a*b + 15*C*a**2))/(60*x**6
)
```

GIAC/XCAS [A] time = 0.282073, size = 190, normalized size = 1.28

$$\frac{1}{4}\,Cc^2x^4 + \frac{1}{3}\,Bc^2x^3 + Cbcx^2 + \frac{1}{2}\,Ac^2x^2 + 2\,Bbcx + \left(Cb^2 + 2\,Cac + 2\,Abc\right)\ln\left(|x|\right) - \frac{40\,Babx^3 + 60\,\left(Bb^2 + 2\,Bac\right)x^5 + 30\,\left(2\,Cab + Ab^2 + 2\,Aac\right)x^4 + 12\,Ba^2x + 10\,Aa^2 + 15\,\left(Ca^2 + 2\,Aab\right)x^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)/x^7,x, algorithm="giac")
```

```
[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + C*b*c*x^2 + 1/2*A*c^2*x^2 + 2*B*b
*c*x + (C*b^2 + 2*C*a*c + 2*A*b*c)*ln(abs(x)) - 1/60*(40*B*a*b*x^
3 + 60*(B*b^2 + 2*B*a*c)*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4
 + 12*B*a^2*x + 10*A*a^2 + 15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

3.21 $\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=339

$$-\frac{\left(-\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{bB\log(a+bx^2+cx^4)}{4c^2}+\frac{x(Ac-bC)}{c^2}+\frac{Bx^2}{2c}+\frac{Cx^3}{3c}$$

```
[Out] ((A*c - b*C)*x)/c^2 + (B*x^2)/(2*c) + (C*x^3)/(3*c) - ((A*b*c - b
^2*C + a*c*C - (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((A*b*c - b^2*C
+ a*c*C + (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*(b^2 - 2*a*c)*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) -
 (b*B*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 4.1874, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393

$$-\frac{\left(-\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{bB\log(a+bx^2+cx^4)}{4c^2}+\frac{x(Ac-bC)}{c^2}+\frac{Bx^2}{2c}+\frac{Cx^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((A*c - b*C)*x)/c^2 + (B*x^2)/(2*c) + (C*x^3)/(3*c) - ((A*b*c - b
^2*C + a*c*C - (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((A*b*c - b^2*C
+ a*c*C + (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*(b^2 - 2*a*c)*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) -
 (b*B*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi in Sympy [A] time = 163.925, size = 347, normalized size = 1.02

$$-\frac{Bb\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{Bx^2}{2c}-\frac{B\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\sqrt{-4ac+b^2}}+\frac{Cx^3}{3c}+\frac{x\left(Ac-Cb\right)}{c^2}$$
$$-\frac{\sqrt{2}\left(-2ac\left(Ac-Cb\right)+b\left(Cac+b\left(Ac-Cb\right)\right)+\sqrt{-4ac+b^2}\left(Cac+b\left(Ac-Cb\right)\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$
$$+\frac{\sqrt{2}\left(-2ac\left(Ac-Cb\right)+b\left(Cac+b\left(Ac-Cb\right)\right)-\sqrt{-4ac+b^2}\left(Cac+b\left(Ac-Cb\right)\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] -B*b*log(a + b*x**2 + c*x**4)/(4*c**2) + B*x**2/(2*c) - B*(-2*a*c
 + b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**2*sqrt(-
4*a*c + b**2)) + C*x**3/(3*c) + x*(A*c - C*b)/c**2 - sqrt(2)*(-2*
a*c*(A*c - C*b) + b*(C*a*c + b*(A*c - C*b)) + sqrt(-4*a*c + b**2)
*(C*a*c + b*(A*c - C*b)))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4
*a*c + b**2)))/(2*c**(5/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*
a*c + b**2)) + sqrt(2)*(-2*a*c*(A*c - C*b) + b*(C*a*c + b*(A*c -
C*b)) - sqrt(-4*a*c + b**2)*(C*a*c + b*(A*c - C*b)))*atan(sqrt(2)
*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c**(5/2)*sqrt(b - sq
rt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 1.23408, size = 460, normalized size = 1.36

$$\frac{\frac{6\sqrt{2}\left(Ac\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)+C\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{6\sqrt{2}\left(C\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)-Ac\left(b\sqrt{b^2-4\right.\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (12*Sqrt[c]*(A*c - b*C)*x + 6*B*c^(3/2)*x^2 + 4*c^(3/2)*C*x^3 + (
6*Sqrt[2]*(A*c*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]) + (-b^3 + 3*a*
b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (6*Sqrt[2]*(-(A*c*(b^2 - 2*a*c +
 b*Sqrt[b^2 - 4*a*c])) + (b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] -
 a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) - (3*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - (3*B*Sqrt[c]*(b^2
 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/Sqrt[b^2 - 4*a*c])/(12*c^(5/2))
```

Maple [B] time = 0.068, size = 1622, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2*(-4*a*c+b
^2)^(1/2)+1/2*B*x^2/c+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*A*b^2*(-4*a*c+b^2)^(1/2)+5/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*b^2*C*a-5/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b^2*C*a-1/2/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C
*(-4*a*c+b^2)^(1/2)*b^3-1/2/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^3+3/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a*b+3/2/c/(4*a*c-b^2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a*b-1/c^2*C*x*b+1/2
/c/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*a*(-4*a*c+b^2)
^(1/2)-1/4/c^2/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^
2*(-4*a*c+b^2)^(1/2)-1/c/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(
1/2)-b)*a*b-1/2/c/(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*
a*(-4*a*c+b^2)^(1/2)-2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C
*a^2+2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*a^2+1/4/c^2/
(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^2*(-4*a*c+b^2)^(
1/2)-1/c/(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a*b+1/c*A
*x+1/3*C*x^3/c+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b
^3-2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a*b-1/2/c^2/(4*a*
c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*C+1/4/c^2/(4*a*c-b^2)*B*l
n(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^3+1/4/c^2/(4*a*c-b^2)*B*ln(-2*c
*x^2+(-4*a*c+b^2)^(1/2)-b)*b^3-1/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*A*a*(-4*a*c+b^2)^(1/2)-1/(4*a*c-b^2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*A*a*(-4*a*c+b^2)^(1/2)+1/2/c^2/(4*a*c-b^2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^4*C-1/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*A*b^3+2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*A*a*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,Ccx^3 + 3\,Bcx^2 - 6\,(Cb - Ac)x}{6\,c^2} - \frac{\int \frac{Bbcx^3 + Bacx - Cab + Aac - \left(Cb^2 - (Ca + Ab)c\right)x^2}{cx^4 + bx^2 + a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*C*c*x^3 + 3*B*c*x^2 - 6*(C*b - A*c)*x)/c^2 - integrate((B*
b*c*x^3 + B*a*c*x - C*a*b + A*a*c - (C*b^2 - (C*a + A*b)*c)*x^2)/
(c*x^4 + b*x^2 + a), x)/c^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.58458, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.22 $\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=278

$$\frac{(2acC+Abc+b^2(-C))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(Ac-bC)\log(a+bx^2+cx^4)}{4c^2}-\frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}+\frac{Cx^2}{2c}$$

```
[Out] (B*x)/c + (C*x^2)/(2*c) - (B*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*(b + (b^2 - 2*a*c)/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*b*c
 - b^2*C + 2*a*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
c^2*Sqrt[b^2 - 4*a*c]) + ((A*c - b*C)*Log[a + b*x^2 + c*x^4])/(4*
c^2)
```

Rubi [A] time = 1.09246, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393

$$\frac{(2acC+Abc+b^2(-C))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(Ac-bC)\log(a+bx^2+cx^4)}{4c^2}-\frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}+\frac{Cx^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c + (C*x^2)/(2*c) - (B*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*(b + (b^2 - 2*a*c)/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*b*c
 - b^2*C + 2*a*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
c^2*Sqrt[b^2 - 4*a*c]) + ((A*c - b*C)*Log[a + b*x^2 + c*x^4])/(4*
c^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx}{c}-\frac{\sqrt{2}B\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}B\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\int^{x^2}C\,dx}{2c}+\frac{(Ac-Cb)\log(a+bx^2+cx^4)}{4c^2}+\frac{(2Cac+b(Ac-Cb))\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x/c - sqrt(2)*B*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sq
rt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b
 + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*B*(-2*a*c
+ b**2 - b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - s
qrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*sq
rt(-4*a*c + b**2)) + Integral(C, (x, x**2))/(2*c) + (A*c - C*b)*l
og(a + b*x**2 + c*x**4)/(4*c**2) + (2*C*a*c + b*(A*c - C*b))*atan
h((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**2*sqrt(-4*a*c + b**2)
)
```

Mathematica [A] time = 0.84283, size = 377, normalized size = 1.36

$$\frac{\frac{\left(Ac\left(\sqrt{b^2-4ac}-b\right)+C\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{\left(C\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-Ac\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{2\sqrt{2}B\sqrt{c}\cdots}{}}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*B*c*x + 2*c*C*x^2 - (2*Sqrt[2]*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt
[2]*B*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + ((A*c*(-b + Sqrt[b^2 - 4*a*c]) + (b^2
- 2*a*c - b*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2])/Sqrt[b^2 - 4*a*c] - ((-(A*c*(b + Sqrt[b^2 - 4*a*c])) + (b
^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] +
2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^2)
```

Maple [B] time = 0.048, size = 1171, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*C*x^2/c+B*x/c-1/4/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)
+b)*A*(-4*a*c+b^2)^(1/2)*b+1/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^
(1/2)+b)*a*A-1/4/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A
*b^2-1/2/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C*(-4*a*c
+b^2)^(1/2)*a+1/4/c^2/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b
)*C*(-4*a*c+b^2)^(1/2)*b^2-1/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2
)^(1/2)+b)*C*a*b+1/4/c^2/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2
)+b)*C*b^3-1/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*(-4*a*c+b
^2)^(1/2)*a+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*(-4*
a*c+b^2)^(1/2)*b^2-2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b
*B+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B+1/4/c/(4*
a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*(-4*a*c+b^2)^(1/2)*b
+1/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*a*A-1/4/c/(4*a*c
-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b^2+1/2/c/(4*a*c-b^2)*l
n(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*(-4*a*c+b^2)^(1/2)*a-1/4/c^2/(
4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*(-4*a*c+b^2)^(1/2)
```

```
*b^2-1/c/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*a*b+1/4/
c^2/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*b^3-1/(4*a*c-
b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*(-4*a*c+b^2)^(1/2)*a+1/2/c
/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*
x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*(-4*a*c+b^2)^(1/2)
*b^2+2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B-1/2/c/(4
*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Cx^2 + 2\,Bx}{2\,c} + \frac{-\int \frac{Bbx^2+(Cb-Ac)x^3+Cax+Ba}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/2*(C*x^2 + 2*B*x)/c + integrate(-(B*b*x^2 + (C*b - A*c)*x^3 + C
*a*x + B*a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.30489, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.23 $\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=270

$$\frac{\left(-\frac{Abc-C(b^2-2ac)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}+\frac{Cx}{c}$$

```
[Out] (C*x)/c + ((A*c - b*C - (A*b*c - (b^2 - 2*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((A*c - b*C + (A*b*c
- b^2*C + 2*a*c*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*
Sqrt[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rubi [A] time = 1.89827, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{\left(-\frac{Abc-C(b^2-2ac)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (C*x)/c + ((A*c - b*C - (A*b*c - (b^2 - 2*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((A*c - b*C + (A*b*c
- b^2*C + 2*a*c*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*
Sqrt[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rubi in Sympy [A] time = 104.165, size = 274, normalized size = 1.01

$$\frac{Bb\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c\sqrt{-4ac+b^2}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}+\frac{Cx}{c}+\frac{\sqrt{2}\left(2Cac+b\left(Ac-Cb\right)+\left(Ac-Cb\right)\sqrt{-4ac+b^2}\right)\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(2Cac+b\left(Ac-Cb\right)-\left(Ac-Cb\right)\sqrt{-4ac+b^2}\right)\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*b*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c*sqrt(-4*a*c +
b**2)) + B*log(a + b*x**2 + c*x**4)/(4*c) + C*x/c + sqrt(2)*(2*C*
a*c + b*(A*c - C*b) + (A*c - C*b)*sqrt(-4*a*c + b**2))*atan(sqrt(
2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b +
sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*(2*C*a*c + b*
(A*c - C*b) - (A*c - C*b)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(
c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b - sqrt(-4*
a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.764186, size = 360, normalized size = 1.33

$$-\frac{\frac{2\sqrt{2}\left(Ac\left(b-\sqrt{b^2-4ac}\right)+C\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-Ac\left(\sqrt{b^2-4ac}+b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+B\sqrt{c}\left(\sqrt{\cdots}\right.}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*Sqrt[c]*C*x - (2*Sqrt[2]*(A*c*(b - Sqrt[b^2 - 4*a*c]) + (-b^2
+ 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]) - (2*Sqrt[2]*(-(A*c*(b + Sqrt[b^2 - 4*a*c])) + (b^2 - 2*
a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + (B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*Sqrt[c]*(b + Sqrt[b^2 - 4
*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(
4*c^(3/2))
```

Maple [B] time = 0.05, size = 1327, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] C*x/c-1/4/c/(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b*(-4*
a*c+b^2)^(1/2)+1/(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a
-1/4/c/(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^2-1/2/(4*
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b*(-4*a*c+b^2)^(1/2)+2*c/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a-1/2/(4*a*c-b^2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*A*b^2+1/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*C*(-4*a*c+b^2)*b-1/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*C*(-4*a*c+b^2)^(1/2)*a+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2-1/(4*a*c-b^2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b*C*a+1/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b^3*C+1/4/c/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b
```

```
*(-4*a*c+b^2)^(1/2)+1/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2
)-b)*a-1/4/c/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^2-
1/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b*(-4*a*c+b^2)^
(1/2)-2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a+1/2/(4*
a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2-1/4/c/(4*a*c-b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)*b-1/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a+1/2/c/(4*a*c
-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2+1/
(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*C*a-1/4/c/(4*a*c-b^
2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Cx}{c}+\frac{\int \frac{Bcx^3-(Cb-Ac)x^2-Ca}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] C*x/c + integrate((B*c*x^3 - (C*b - A*c)*x^2 - C*a)/(c*x^4 + b*x^
2 + a), x)/c
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.43851, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.24 $\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=223

$$-\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}-\frac{B\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{B\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{C\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] -((B*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (
B*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - ((2*A
*c - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2
 - 4*a*c]) + (C*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rubi [A] time = 0.520649, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346

$$-\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}-\frac{B\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{B\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{C\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((B*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (
B*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - ((2*A
*c - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2
 - 4*a*c]) + (C*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rubi in Sympy [A] time = 64.4513, size = 209, normalized size = 0.94

$$-\frac{\sqrt{2}B\sqrt{b-\sqrt{-4ac+b^2}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}B\sqrt{b+\sqrt{-4ac+b^2}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}+\frac{C\log\left(a+bx^2+cx^4\right)}{4c}-\frac{(2Ac-Cb)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(2)*B*sqrt(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/s
qrt(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(-4*a*c + b**2)) + s
qrt(2)*B*sqrt(b + sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqr
t(b + sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(-4*a*c + b**2)) + C*l
```

```
og(a + b*x**2 + c*x**4)/(4*c) - (2*A*c - C*b)*atanh((b + 2*c*x**2
)/sqrt(-4*a*c + b**2))/(2*c*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.790396, size = 240, normalized size = 1.08

$$\frac{\left(C\left(\sqrt{b^2-4ac}-b\right)+2Ac\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-\left(2Ac-C\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)-2\sqrt{2}B}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*Sqrt[2]*B*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]] + 2*Sqrt[2]*B*Sqrt[c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]] + (2*A*c + (-b + Sqrt[b^2 - 4*a*c])*C)*Log[-b +
Sqrt[b^2 - 4*a*c] - 2*c*x^2] - (2*A*c - (b + Sqrt[b^2 - 4*a*c])*C
)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.035, size = 728, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*(-4*a*c+b^2)^(
1/2)-1/4/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C*(-4*a*c
+b^2)^(1/2)*b+1/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a*C-
1/4/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^2*C-1/2/(4*a
*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b+2*c/(
4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B-1/2/(4*a*c-b^2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b^2*B-1/2/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c
+b^2)^(1/2)-b)*A*(-4*a*c+b^2)^(1/2)+1/4/c/(4*a*c-b^2)*ln(-2*c*x^2
+(-4*a*c+b^2)^(1/2)-b)*C*(-4*a*c+b^2)^(1/2)*b+1/(4*a*c-b^2)*ln(-2
*c*x^2+(-4*a*c+b^2)^(1/2)-b)*a*C-1/4/c/(4*a*c-b^2)*ln(-2*c*x^2+(-
4*a*c+b^2)^(1/2)-b)*b^2*C-1/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b-2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*a*B+1/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^2*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2+Bx+A\right)x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 1.2154, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] Done
```

3.25 $\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=211

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((C - (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - (B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4
*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.597401, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((C - (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - (B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4
*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi in Sympy [A] time = 53.7049, size = 221, normalized size = 1.05

$$-\frac{B\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(2Ac-Cb-C\sqrt{-4ac+b^2}\right)\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(2Ac-Cb+C\sqrt{-4ac+b^2}\right)\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] -B*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2)
- sqrt(2)*(2*A*c - C*b - C*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt
(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(b + sqrt(-4*
a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*(2*A*c - C*b + C*sqrt
(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b*
*2)))/(2*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2
))
```

Mathematica [A] time = 0.402995, size = 234, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\left(C\left(\sqrt{b^2-4ac}-b\right)+2Ac\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(C\left(\sqrt{b^2-4ac}+b\right)-2Ac\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}+B\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-B\log\left(\sqrt{b\right.}}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*A*c + (-b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (Sqrt[2]*(-2*A*c + (b + Sqrt[b^2 - 4*a*c])*C)*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2
] - B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.025, size = 616, normalized size = 2.9

$$\begin{aligned}
&\frac{B}{8\,ac-2\,b^2}\sqrt{-4\,ac+b^2}\ln\left(2\,cx^2+\sqrt{-4\,ac+b^2}+b\right)\\
&+\frac{c\sqrt{2}A}{4\,ac-b^2}\sqrt{-4\,ac+b^2}\,\mathrm{arctan}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\\
&+2\,\frac{c\sqrt{2}Ca}{(4\,ac-b^2)\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\mathrm{arctan}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\\
&-\frac{\sqrt{2}Cb^2}{8\,ac-2\,b^2}\mathrm{arctan}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\\
&-\frac{b\sqrt{2}C}{8\,ac-2\,b^2}\sqrt{-4\,ac+b^2}\,\mathrm{arctan}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\\
&-\frac{B}{8\,ac-2\,b^2}\sqrt{-4\,ac+b^2}\ln\left(-2\,cx^2+\sqrt{-4\,ac+b^2}-b\right)\\
&+\frac{c\sqrt{2}A}{4\,ac-b^2}\sqrt{-4\,ac+b^2}\,Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\\
&-2\,\frac{c\sqrt{2}Ca}{(4\,ac-b^2)\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}Artanh\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\\
&+\frac{\sqrt{2}Cb^2}{8\,ac-2\,b^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\\
&-\frac{b\sqrt{2}C}{8\,ac-2\,b^2}\sqrt{-4\,ac+b^2}\,Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2
)+b)+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*A+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*a-1/2/(4*a*c-b
^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*C-1/2*(-4*a*c+b^2)^(1/2)/
(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+c*(-4*a*c+b^2)^(1
/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A-2*c/(4*a*c-b^2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*a+1/2/(4*a*c-b^2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*C*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2 + Bx + A}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.06884, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.26 $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=229

$$\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}+\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*b -
2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 -
4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rubi [A] time = 0.627816, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}+\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*b -
2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 -
4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rubi in Sympy [A] time = 76.694, size = 216, normalized size = 0.94

$$\frac{A\log\left(x^2\right)}{2a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}-\frac{\sqrt{2}B\sqrt{c}\,\text{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}B\sqrt{c}\,\text{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{(Ab-2Ca)\,\text{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] A*log(x**2)/(2*a) - A*log(a + b*x**2 + c*x**4)/(4*a) - sqrt(2)*B*
sqrt(c)*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(sq
rt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*B*sqrt
(c)*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(sqrt(b
 - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + (A*b - 2*C*a)*atan
```

```
h((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.932439, size = 285, normalized size = 1.24

$$-\frac{\left(A\left(\sqrt{b^2-4ac}+b\right)-2aC\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{4a\sqrt{b^2-4ac}}-\frac{\left(A\left(\sqrt{b^2-4ac}-b\right)+2aC\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{4a\sqrt{b^2-4ac}}+\frac{A\log(x)}{a}+\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (A*Log[x
])/a - ((A*(b + Sqrt[b^2 - 4*a*c]) - 2*a*C)*Log[-b + Sqrt[b^2 - 4
*a*c] - 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c]) - ((A*(-b + Sqrt[b^2 -
4*a*c]) + 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[
b^2 - 4*a*c])
```

Maple [B] time = 0.038, size = 488, normalized size = 2.1

$$-4\,\frac{c\ln\left(2\,cx^2+\sqrt{-4\,ac+b^2}+b\right)A}{16\,ac-4\,b^2}+\frac{Ab^2}{a\left(16\,ac-4\,b^2\right)}\ln\left(2\,cx^2+\sqrt{-4\,ac+b^2}+b\right)-\frac{Ab}{a\left(16\,ac-4\,b^2\right)}\sqrt{-4\,ac+b^2}\ln\left(2\,cx^2+\sqrt{-4\,ac+b^2}+b\right)+2\,\frac{\sqrt{-4\,ac+b^2}\ln\left(2\,cx^2+\sqrt{-4\,ac+b^2}+b\right)C}{16\,ac-4\,b^2}+4\,\frac{\sqrt{-4\,ac+b^2}cB\sqrt{2}}{\left(16\,ac-4\,b^2\right)\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)-4\,\frac{c\ln\left(-2\,cx^2+\sqrt{-4\,ac+b^2}-b\right)A}{16\,ac-4\,b^2}+\frac{Ab^2}{a\left(16\,ac-4\,b^2\right)}\ln\left(-2\,cx^2+\sqrt{-4\,ac+b^2}-b\right)+\frac{Ab}{a\left(16\,ac-4\,b^2\right)}\sqrt{-4\,ac+b^2}\ln\left(-2\,cx^2+\sqrt{-4\,ac+b^2}-b\right)-2\,\frac{\sqrt{-4\,ac+b^2}\ln\left(-2\,cx^2+\sqrt{-4\,ac+b^2}-b\right)C}{16\,ac-4\,b^2}+4\,\frac{\sqrt{-4\,ac+b^2}cB\sqrt{2}}{\left(16\,ac-4\,b^2\right)\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}Artanh\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)+\frac{A\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] -4*c/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A+1/a/(16*a*
c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b^2-1/a*(-4*a*c+b^2)^
(1/2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b+2*(-4*a
*c+b^2)^(1/2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C+4
*c*(-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*B*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))-4*c/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A+1/a/(16
*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b^2+1/a*(-4*a*c+b
^2)^(1/2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b-2*
(-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-
b)*C+4*c*(-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*B*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))+A*ln(x)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{A\log(x)}{a} - \frac{\int \frac{Acx^3 - Ba - (Ca - Ab)x}{cx^4 + bx^2 + a}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] A*log(x)/a - integrate((A*c*x^3 - B*a - (C*a - A*b)*x)/(c*x^4 + b
*x^2 + a), x)/a
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.866925, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] Done
```

3.27 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=260

$$-\frac{\sqrt{c}\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}+\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{B\log\left(a+bx^2+cx^4\right)}{4a}+\frac{B\log(x)}{a}$$

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*C)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + (B*Log[x])/a
 - (B*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rubi [A] time = 1.05935, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$-\frac{\sqrt{c}\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}+\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{B\log\left(a+bx^2+cx^4\right)}{4a}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*C)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + (B*Log[x])/a
 - (B*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rubi in Sympy [A] time = 100.472, size = 264, normalized size = 1.02

$$-\frac{A}{ax}+\frac{Bb\,\text{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a\sqrt{-4ac+b^2}}+\frac{B\log\left(x^2\right)}{2a}-\frac{B\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\sqrt{2}\sqrt{c}\left(Ab-A\sqrt{-4ac+b^2}-2Ca\right)\text{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\sqrt{c}\left(Ab+A\sqrt{-4ac+b^2}-2Ca\right)\text{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] -A/(a*x) + B*b*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a*sqr
t(-4*a*c + b**2)) + B*log(x**2)/(2*a) - B*log(a + b*x**2 + c*x**4
)/(4*a) + sqrt(2)*sqrt(c)*(A*b - A*sqrt(-4*a*c + b**2) - 2*C*a)*a
tan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*a*sqrt(b
+ sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*sqrt(c)*(A*
b + A*sqrt(-4*a*c + b**2) - 2*C*a)*atan(sqrt(2)*sqrt(c)*x/sqrt(b
- sqrt(-4*a*c + b**2)))/(2*a*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-
4*a*c + b**2))
```

Mathematica [A] time = 2.89054, size = 315, normalized size = 1.21

$$-\frac{\frac{2\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}+b\right)-2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{2\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{B\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}}+}{4a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((4*A)/x + (2*Sqrt[2]*Sqrt[c]*(A*(b + Sqrt[b^2 - 4*a*c]) - 2*a*C
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (2*Sqrt[2]*Sqrt[c]*(A
*(-b + Sqrt[b^2 - 4*a*c]) + 2*a*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 -
 4*a*c]]) - 4*B*Log[x] + (B*(b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt
[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*(-b + Sqrt[b^2 -
 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])
/(4*a)
```

Maple [B] time = 0.042, size = 811, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*B*(-4*a*c+b^
2)^(1/2)*b-4*c/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*B+
1/a/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^2*B-2*c/a/(
16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b
-8*c^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+2*c/a/(16*a*
c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+4*c/(16*a*c-4*b^2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)+1/a/(16*a*c-4*b^2
)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*B*(-4*a*c+b^2)^(1/2)*b-4*c/(1
6*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*B+1/a/(16*a*c-4*b^
2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^2*B-2*c/a/(16*a*c-4*b^2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b+8*c^2/(16*a
*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A-2*c/a/(16*a*c-4*b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+4*c/(16*a*c-4*b^2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)+1/a*ln(x)*B-A/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{B\log\left(x\right)}{a}-\frac{\int\frac{Bcx^{3}+Acx^{2}+Bbx-Ca+Ab}{cx^{4}+bx^{2}+a}\,dx}{a}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] B*log(x)/a - integrate((B*c*x^3 + A*c*x^2 + B*b*x - C*a + A*b)/(c
*x^4 + b*x^2 + a), x)/a - A/(a*x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.09492, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] Done
```

3.28 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=288

$$-\frac{\left(A\left(b^2-2ac\right)-abC\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B}{ax}$$

```
[Out] -A/(2*a*x^2) - B/(a*x) - (B*Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((A*(b^2 - 2*a*c) - a*b*C)*A
rcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]
) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2 + c*x^4
])/(4*a^2)
```

Rubi [A] time = 1.10163, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393

$$-\frac{\left(A\left(b^2-2ac\right)-abC\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(2*a*x^2) - B/(a*x) - (B*Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((A*(b^2 - 2*a*c) - a*b*C)*A
rcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]
) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2 + c*x^4
])/(4*a^2)
```

Rubi in Sympy [A] time = 117.703, size = 287, normalized size = 1.

$$-\frac{A}{2ax^2}+\frac{\sqrt{2}B\sqrt{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}B\sqrt{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{B}{ax}-\frac{(Ab-Ca)\log\left(x^2\right)}{2a^2}+\frac{(Ab-Ca)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(-2Aac+Ab^2-Cab\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] -A/(2*a*x**2) + sqrt(2)*B*sqrt(c)*(b - sqrt(-4*a*c + b**2))*atan(
sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*a*sqrt(b + sq
rt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*B*sqrt(c)*(b +
sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c
+ b**2)))/(2*a*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
 - B/(a*x) - (A*b - C*a)*log(x**2)/(2*a**2) + (A*b - C*a)*log(a +
 b*x**2 + c*x**4)/(4*a**2) - (-2*A*a*c + A*b**2 - C*a*b)*atanh((b
 + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a**2*sqrt(-4*a*c + b**2))
```

Mathematica [A]   time = 1.95423, size = 377, normalized size = 1.31

$$\frac{\frac{\left(A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-aC\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aC\left(b-\sqrt{b^2-4ac}\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+4\log(x)(}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*Sqrt[2]*a*B*Sqrt[c]*(b + Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*a*
B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 -
 4*a*c]]) + 4*(-(A*b) + a*C)*Log[x] + ((A*(b^2 - 2*a*c + b*Sqrt[b
^2 - 4*a*c]) - a*(b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4
*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((A*(-b^2 + 2*a*c + b*Sqrt[
b^2 - 4*a*c]) + a*(b - Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [B]   time = 0.057, size = 1054, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 8*c/a/(32*a*c-8*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b-2/a^2/(
32*a*c-8*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b^3-4*c/a/(32*a*
c-8*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*(-4*a*c+b^2)^(1/2)+2/
a^2/(32*a*c-8*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*(-4*a*c+b^2
)^(1/2)*b^2-2/a/(32*a*c-8*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C
*b*(-4*a*c+b^2)^(1/2)-8*c/(32*a*c-8*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^
(1/2)+b)*C+2/a/(32*a*c-8*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C*
b^2-4*c/a/(32*a*c-8*b^2)*B*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*(-4*a*c
+b^2)^(1/2)-16*c^2/(32*a*c-8*b^2)*B*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+
4*c/a/(32*a*c-8*b^2)*B*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+8*c/a/(32
*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b-2/a^2/(32*a*c-8
*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b^3+4*c/a/(32*a*c-8*b^2
)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*(-4*a*c+b^2)^(1/2)-2/a^2/(3
2*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*(-4*a*c+b^2)^(1/
2)*b^2+2/a/(32*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*b*(
-4*a*c+b^2)^(1/2)-8*c/(32*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/
2)-b)*C+2/a/(32*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*b^
2-4*c/a/(32*a*c-8*b^2)*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
```

```
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*(-4*a*
c+b^2)^(1/2)+16*c^2/(32*a*c-8*b^2)*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))-4*c/a/(32*a*c-8*b^2)*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-B/
a/x-1/2*A/a/x^2-1/a^2*ln(x)*A*b+1/a*ln(x)*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(Ca-Ab)\log(x)}{a^2}+\frac{-\int\frac{Bacx^2+(Ca-Ab)cx^3+Bab+(Cab-Ab^2+Aac)x}{cx^4+bx^2+a}\,dx}{a^2}-\frac{2\,Bx+A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] (C*a - A*b)*log(x)/a^2 + integrate(-(B*a*c*x^2 + (C*a - A*b)*c*x^
3 + B*a*b + (C*a*b - A*b^2 + A*a*c)*x)/(c*x^4 + b*x^2 + a), x)/a^
2 - 1/2*(2*B*x + A)/(a*x^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.29313, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] Done
```

3.29 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=412

$$\frac{\left(-\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x^3\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{x(2Ac-bC)}{2c\left(b^2-4ac\right)}+\frac{2aB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{Bx^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] ((2*A*c - b*C)*x)/(2*c*(b^2 - 4*a*c)) + (B*x^2*(2*a + b*x^2))/(2*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x^3*(A*b - 2*a*C + (2*A*c -
 b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((A*b*c + (b^
2 - 6*a*c)*C - (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
((A*b*c + (b^2 - 6*a*c)*C + (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*a*c)*
C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + (2*a*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^
2 - 4*a*c)^(3/2)
```

Rubi [A] time = 2.98195, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{\left(-\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x^3\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{x(2Ac-bC)}{2c\left(b^2-4ac\right)}+\frac{2aB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{Bx^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*A*c - b*C)*x)/(2*c*(b^2 - 4*a*c)) + (B*x^2*(2*a + b*x^2))/(2*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x^3*(A*b - 2*a*C + (2*A*c -
 b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((A*b*c + (b^
2 - 6*a*c)*C - (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
((A*b*c + (b^2 - 6*a*c)*C + (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*a*c)*
C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + (2*a*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^
2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 179.049, size = 376, normalized size = 0.91

$$\frac{2Ba\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{x\left(2Bacx+Bbcx^3+a\left(2Ac-Cb\right)-x^2\left(-Abc-2Cac+Cb^2\right)\right)}{2c\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(2ac\left(2Ac-Cb\right)+b\left(Abc-6Cac+Cb^2\right)+\sqrt{-4ac+b^2}\left(Abc-6Cac+Cb^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(2ac\left(2Ac-Cb\right)+b\left(Abc-6Cac+Cb^2\right)-\sqrt{-4ac+b^2}\left(Abc-6Cac+Cb^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*B*a*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**
(3/2) + x*(2*B*a*c*x + B*b*c*x**3 + a*(2*A*c - C*b) - x**2*(-A*b*
c - 2*C*a*c + C*b**2))/(2*c*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)
) + sqrt(2)*(2*a*c*(2*A*c - C*b) + b*(A*b*c - 6*C*a*c + C*b**2) +
 sqrt(-4*a*c + b**2)*(A*b*c - 6*C*a*c + C*b**2))*atan(sqrt(2)*sqr
t(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*c**(3/2)*sqrt(b + sqrt(-
4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) - sqrt(2)*(2*a*c*(2*A*c -
C*b) + b*(A*b*c - 6*C*a*c + C*b**2) - sqrt(-4*a*c + b**2)*(A*b*c
- 6*C*a*c + C*b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c
+ b**2)))/(4*c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**
2)**(3/2))
```

Mathematica [A] time = 2.80534, size = 444, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2\left(a(b(B+Cx)-2cx(A+x(B+Cx)))+bx^2(b(B+Cx)-Acx)\right)}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(C\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+8abc-b^3\right)-Ac\left(-b\sqrt{b^2-4ac}+4ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(Ac\left(b\sqrt{b^2-4ac}+4ac+b^2\right)+C\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}-8abc+b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4aB\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{4aB\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(b*x^2*(-(A*c*x) + b*(B + C*x)) + a*(b*(B + C*x) - 2*c*x*(A +
 x*(B + C*x)))))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2
]*(-(A*c*(b^2 + 4*a*c - b*Sqrt[b^2 - 4*a*c])) + (-b^3 + 8*a*b*c +
 b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(c^(3/2)*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(A*c*(b^2 + 4*a*
c + b*Sqrt[b^2 - 4*a*c]) + (b^3 - 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c]
 - 6*a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
```

```
+ Sqrt[b^2 - 4*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt
[b^2 - 4*a*c]]) - (4*a*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(
b^2 - 4*a*c)^(3/2) + (4*a*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])
/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.13, size = 5283, normalized size = 12.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(Cb^2 - (2\,Ca + Ab)c\right)x^3 + Bab + \left(Bb^2 - 2\,Bac\right)x^2 + (Cab - 2\,Aac)x}{2\left((b^2c^2 - 4\,ac^3)x^4 + ab^2c - 4\,a^2c^2 + (b^3c - 4\,abc^2)x^2\right)} + \frac{-\int \frac{4\,Bacx - Cab + 2\,Aac - \left(Cb^2 - (6\,Ca - Ab)c\right)x^2}{cx^4 + bx^2 + a}\,dx}{2\left(b^2c - 4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((C*b^2 - (2*C*a + A*b)*c)*x^3 + B*a*b + (B*b^2 - 2*B*a*c)*x
^2 + (C*a*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*
a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + 1/2*integrate(-(4*B*a*c*x -
C*a*b + 2*A*a*c - (C*b^2 - (6*C*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 +
 a), x)/(b^2*c - 4*a*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.30 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=347

$$-\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{Bx\left(2a+bx^2\right)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{B\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{B\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (B*x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (a*(2
*A*c - b*C) + (A*b*c - b^2*C + 2*a*c*C)*x^2)/(2*c*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) + (B*(b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (B*(b^2 + 4
*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]) - ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.36615, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$-\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{Bx\left(2a+bx^2\right)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{B\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{B\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (a*(2
*A*c - b*C) + (A*b*c - b^2*C + 2*a*c*C)*x^2)/(2*c*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) + (B*(b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (B*(b^2 + 4
*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]) - ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 136.213, size = 289, normalized size = 0.83

$$\frac{\sqrt{2}B\left(4ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}} - \frac{\sqrt{2}B\left(4ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}} + \frac{x\left(2Ba+Bbx^2-x^3\left(2Ac-Cb\right)-x\left(Ab-2Ca\right)\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)} - \frac{\left(Ab-2Ca\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] sqrt(2)*B*(4*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqr
t(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*sqrt(c)*sqrt(b + sqrt(-4
*a*c + b**2))*(-4*a*c + b**2)**(3/2)) - sqrt(2)*B*(4*a*c + b**2 -
 b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a
*c + b**2)))/(4*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b
**2)**(3/2)) + x*(2*B*a + B*b*x**2 - x**3*(2*A*c - C*b) - x*(A*b
- 2*C*a))/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)) - (A*b - 2*C*
a)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3/
2)
```

Mathematica [A] time = 1.84205, size = 358, normalized size = 1.03

$$\frac{1}{4}\left(-\frac{2\left(a(2Ac-bC+2cx(B+Cx))+bx^2(Ac-bC+Bcx)\right)}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} + \frac{2(Ab-2aC)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} - \frac{2(Ab-2aC)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\sqrt{2}B\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}B\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(b*x^2*(A*c - b*C + B*c*x) + a*(2*A*c - b*C + 2*c*x*(B + C*x
))))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*B*(-b^2 -
4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + (Sqrt[2]*B*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(A*b - 2*a*C
)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2
*(A*b - 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c
)^(3/2))/4
```

Maple [B] time = 0.101, size = 3041, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*b*B/(4*a*c-b^2)*x^3-1/2*(A*b*c+2*C*a*c-C*b^2)/(4*a*c-b^2)/c
*x^2-a*B/(4*a*c-b^2)*x-1/2*a*(2*A*c-C*b)/c/(4*a*c-b^2))/(c*x^4+b*
x^2+a)+c/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2*c
)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))
*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3+c/(4*a*c
-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2
-2*x^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))*A*(-64*a^3*c^3+4
8*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3+8/(4*a*c-b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*
b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*
a*b^4*c+b^6)^(1/2)*a^2*c^2-2*c/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-(
4*a*c-b^2)^3)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6
)^(1/2)*a*b^2-3*c/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/
((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan
(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4
*a*c-b^2)^3)^(1/2)))^(1/2))*B*a*b^5+1/4/(4*a*c-b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3
)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^
2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b^7+1/4/(4*a*
c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*
c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c
)^(1/2))*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^4-
4/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*b^2*c)*ln(8*x
^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))*A*(-64*a
^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+8/(4*a*c-b^2)
/(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*
x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*C*(-64*a^3*c^3+48*a
^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*a^2*c^2-2*c/(4*a*c-b^2)/(16*a^2*
c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c
-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^
2-12*a*b^4*c+b^6)^(1/2)*a*b^2-3*c/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2
*c+b^4)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2
)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^
3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*a*b^5+1/4/(4*a*
c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*
c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c
)^(1/2))*B*b^7-16*c^3/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1
/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*ar
ctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+
(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*a^3*b+12*c^2/(4*a*c-b^2)/(16*a^
2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c
-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4
*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*a^2*b^
3+4/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)
*a^2*c^2-1/4/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((4*a
*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*
(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-
b^2)^3)^(1/2)))^(1/2))*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b
^6)^(1/2)*b^4-4/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+
2*b^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)
^(1/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c
^2-16*c^3/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-4*a*b
*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(
-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2))*B*a^3*b+12*c^2/(4*a*c-b^2)/(16*a^2*c^2-8*
a*b^2*c+b^4)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*
```

```
c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b
*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*a^2*b^3-4/
(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-4*a*b*c+b^3+(-(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2
*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b
^2)*c)^(1/2))*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)
*a^2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbcx^3 + 2\,Bacx - Cab + 2\,Aac - \left(Cb^2 - (2\,Ca + Ab)c\right)x^2}{2\,((b^2c^2 - 4\,ac^3)x^4 + ab^2c - 4\,a^2c^2 + (b^3c - 4\,abc^2)x^2)} + \frac{\int \frac{Bbx^2 - 2\,Ba - 2\,(2\,Ca - Ab)x}{cx^4 + bx^2 + a}\,dx}{2\,(b^2 - 4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c*x^3 + 2*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (2*C*a +
A*b)*c)*x^2)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^
3*c - 4*a*b*c^2)*x^2) + 1/2*integrate((B*b*x^2 - 2*B*a - 2*(2*C*a
 - A*b)*x)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.31 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.9848, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.05628, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{(b^2-4ac)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}+4ac+b^2\right)-2Ac\left(\sqrt{b^2-4ac}+2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2bB\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 -
4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 -
 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqr
t[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[
b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.088, size = 4063, normalized size = 11.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -4/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1
/2)*b/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-
(4*a*c-b^2)^3)^(1/2))*a*c^2+(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*
b*B/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-a*B/(4*a*c-b^2)
)/(c*x^4+b*x^2+a)-4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c
-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*
x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1
/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+
1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3
+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c
^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a
*c-b^2)*c)^(1/2))*C*b^7+32/(4*a*c-b^2)^3*c^4*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^
2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
```

```
^(1/2)))^(1/2))*A*a^3+c/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*b^3/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x
^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(4*a*c-b^2)^3*
2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2
)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-
b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^6-1/4/(4*a*c-b^2)^3*2^(1/
2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arc
tan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a
*b^4*c+b^6)^(1/2)*b^4-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^
(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)
*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*a^2*c^2-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*
a*c-b^2)*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b/(-
8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*a*c^2-24/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c
^3*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^
(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*a^2*b^2+6/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a*b^4-16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^3*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*
arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^3*b-3*c/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1
/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*
a*b^5-c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c
+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8
*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*
(4*a*c-b^2)*c)^(1/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^
6)^(1/2)*b^3+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arc
tanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2
)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^2*b^3+4/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/(
(-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*(-6
4*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+1/4/(4*a*c
-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(
4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*b^7-24/(4*a*c-b^2)^
3*c^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*
a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*a^2*b^2+32/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^4*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a^3+1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/
((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arcta
nh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^
3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*
a*b^4*c+b^6)^(1/2)*b^4+6/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*A*a*b^4-16/(4*a*c-b^2)^3*c^3*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^3*b+12/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^2*b^3+4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4
*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b
^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*a^
2*c^2+c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*B*(-64*a^3*c^3+48*
a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3/(-8*a*c^2+2*b^2*c)*ln(-8*x^
2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a
```

```
*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^
2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)
*c)^(1/2))*A*b^6-3*c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*
c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)
*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(
1/2))*C*a*b^5+c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))
*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\,((b^2c - 4\,ac^2)x^4 + ab^2 - 4\,a^2c + (b^3 - 4\,abc)x^2)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\,(b^2 - 4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2
*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*
integrate(-(2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b
*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.32 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=317

$$\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aC+x^2(2Ac-bC)+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(B*x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (A*b
 - 2*a*C + (2*A*c - b*C)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (B*Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*A*c - b*
C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.99192, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$

$$\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aC+x^2(2Ac-bC)+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(B*x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (A*b
 - 2*a*C + (2*A*c - b*C)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (B*Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*A*c - b*
C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 98.3809, size = 291, normalized size = 0.92

$$-\frac{\sqrt{2}B\sqrt{c}\left(2b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt{2}B\sqrt{c}\left(2b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{(2Ac-Cb)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{x\left(-Bab-2Bacx^2+cx^3(Ab-2Ca)+x\left(-2Aac+Ab^2-Cab\right)\right)}{2a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(2)*B*sqrt(c)*(2*b + sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(
c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(b + sqrt(-4*a*c + b**
2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*B*sqrt(c)*(2*b - sqrt(-4*a*
c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/
(2*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + (2*A*c
 - C*b)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)
**(3/2) + x*(-B*a*b - 2*B*a*c*x**2 + c*x**3*(A*b - 2*C*a) + x*(-2
*A*a*c + A*b**2 - C*a*b))/(2*a*(-4*a*c + b**2)*(a + b*x**2 + c*x*
*4))
```

Mathematica [A] time = 3.315, size = 335, normalized size = 1.06

$$\frac{1}{2}\left(\frac{2aC-A\left(b+2cx^2\right)+x\left(-bB+bCx-2Bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(bC-2Ac)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{(2Ac-bC)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{\sqrt{2}B\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}B\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*C - A*(b + 2*c*x^2) + x*(-(b*B) + b*C*x - 2*B*c*x^2))/((b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*B*Sqrt[c]*(-2*b + Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2
]*B*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) + ((-2*A*c + b*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c
*x^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*c - b*C)*Log[b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/2
```

Maple [B] time = 0.162, size = 1344, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/
2)*b*B-1/2*c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B+1/4
/c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*C*b^3-1/2
/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*A*b^2-1/2/(
4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*A*b^2-c/(4*a*
c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*B-c/(4
*a*c-b^2)^2*A*(-4*a*c+b^2)^(1/2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b
)-1/2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*B*x*b^
2-1/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*C*(-4*a*
c+b^2)^(1/2)*a-1/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*
b/c)*C*a*b-1/2/(4*a*c-b^2)^2*C*(-4*a*c+b^2)^(1/2)*ln(2*c*x^2+(-4*
```

```
a*c+b^2)^(1/2)+b)*b-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+
b^2)^(1/2))*B*x*b^2+1/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^
2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a-1/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2
/c*(-4*a*c+b^2)^(1/2))*C*a*b+1/2/(4*a*c-b^2)^2*C*(-4*a*c+b^2)^(1/
2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b+2*c/(4*a*c-b^2)^2/(x^2+1/2
/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*a*A+1/4/c/(4*a*c-b^2)^2/(x^2+1/2/c
*(-4*a*c+b^2)^(1/2)+1/2*b/c)*C*b^3+c/(4*a*c-b^2)^2*A*(-4*a*c+b^2)
^(1/2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c/(4*a*c-b^2)^2/(x^2+1/
2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*a*A+2*c/(4*a*c-b^2)^2/(x^2+1/2/c*
(-4*a*c+b^2)^(1/2)+1/2*b/c)*B*a*x+1/4/c/(4*a*c-b^2)^2/(x^2+1/2/c*
(-4*a*c+b^2)^(1/2)+1/2*b/c)*C*(-4*a*c+b^2)^(1/2)*b^2+2*c/(4*a*c-b
^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*B*a*x-1/4/c/(4*a*c-b
^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)
*b^2-2*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B+1/2*
c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B+2*c^2/(4*a*
c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,Bcx^3+Bbx-(Cb-2\,Ac)x^2-2\,Ca+Ab}{2\,((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)}-\frac{\int\frac{2\,Bcx^2-Bb-2\,(Cb-2\,Ac)x}{cx^4+bx^2+a}\,dx}{2\,(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*B*c*x^3 + B*b*x - (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*i
ntegrate((2*B*c*x^2 - B*b - 2*(C*b - 2*A*c)*x)/(c*x^4 + b*x^2 + a
), x)/(b^2 - 4*a*c)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.33 $\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=368

$$\frac{x\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abC+Ab^2}{\sqrt{b^2-4ac}}-2aC+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2Bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{B\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(B*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(A*
(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*C + (A*(b^2 - 12*a*c)
+ 4*a*b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*C - (A*b^2 - 12*a*A*c + 4*a*b*
C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + (2*B*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*
a*c)^(3/2)
```

Rubi [A] time = 1.95437, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36

$$\frac{x\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abC+Ab^2}{\sqrt{b^2-4ac}}-2aC+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2Bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{B\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] -(B*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(A*
(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*C + (A*(b^2 - 12*a*c)
+ 4*a*b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*C - (A*b^2 - 12*a*A*c + 4*a*b*
C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + (2*B*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*
a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.88028, size = 393, normalized size = 1.07

$$\frac{1}{4}\left(\frac{4acx(A+x(B+Cx))+2ab(B+Cx)-2Abx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-2aC\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(A\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)+2aC\left(\sqrt{b^2-4ac}+2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{4Bc\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{4Bc\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*(B + C*x) - 2*A*b*x*(b + c*x^2) + 4*a*c*x*(A + x*(B + C*x
)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*
(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) - 2*a*(-2*b + Sqrt[b^2 - 4*a
*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sq
rt[c]*(A*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c]) + 2*a*(2*b + Sqrt[b
^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (4*
B*c*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) +
(4*B*c*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))
/4
```

Maple [B] time = 0.141, size = 2851, normalized size = 7.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*A*b^3-8*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a^2/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*C+3/2*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*C*b^4-2*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*A*b^3+12*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^
2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)+8*c^2/(4*a*c-b^
2)^2*2^(1/2)/(4*a*c+3*b^2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)
```

```
^(1/2)*b^2+4*c^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*A*b-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/a/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*A*b^5-3*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^3-4*c^2/(4*a*c-b
^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b^2+12*c^
3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-
4*a*c+b^2)^(1/2)+8*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^2-4*c^3/(4*a*c-b^2)^2*2^(1/
2)/(4*a*c+3*b^2)*a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+3/4*c/(4*a*c-b^2)^2*2
^(1/2)/(4*a*c+3*b^2)/a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^5-3*c/(4*a*c-b^2)
^2*2^(1/2)/(4*a*c+3*b^2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2
)*b^3+4*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*C*b^2-4*c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(4
*a*c+3*b^2)/a/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^4-4*
c^2/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*C*(-4*a*c+b^2)^(1/2)*b-3/4*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)
/a/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^4+1/4/(4*a*c-b^2)^
2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))/a*x*A*(-4*a*c+b^2)^(1/2)
*b^2-1/4/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)/a*x
*A*(-4*a*c+b^2)^(1/2)*b^2-4*c^2/(4*a*c-b^2)^2/(4*a*c+3*b^2)*a*ln(
(4*a*c+3*b^2)*a*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b))*B*(-4*a*c+b^2)^(
1/2)+3*c/(4*a*c-b^2)^2/(4*a*c+3*b^2)*ln((4*a*c+3*b^2)*a*(2*c*x^2+
(-4*a*c+b^2)^(1/2)+b))*B*(-4*a*c+b^2)^(1/2)*b^2-3*c/(4*a*c-b^2)^2
/(4*a*c+3*b^2)*ln((4*a*c+3*b^2)*a*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)
)*B*(-4*a*c+b^2)^(1/2)*b^2+4*c^2/(4*a*c-b^2)^2/(4*a*c+3*b^2)*a*ln
((4*a*c+3*b^2)*a*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b))*B*(-4*a*c+b^2)^(
1/2)-1/2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*B*b
^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*B*b^2
+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*B*a-1/2
/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*x*C*b^2-1/2
/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*x*C*b^2+8*c
^3/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)*a^2/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C
-3/2*c/(4*a*c-b^2)^2*2^(1/2)/(4*a*c+3*b^2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C
*b^4+2*c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*B*a
-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*x*A*(-4*a
*c+b^2)^(1/2)+1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(
1/2))/a*x*A*b^3+c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2
*b/c)*x*A*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(
-4*a*c+b^2)^(1/2))*A*x*b+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4
*a*c+b^2)^(1/2))*C*x*a-c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1
/2)+1/2*b/c)*A*x*b+1/4/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2
)+1/2*b/c)/a*x*A*b^3+2*c/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1
/2)+1/2*b/c)*C*x*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,Bacx^2+(2\,Ca-Ab)cx^3+Bab+\left(Cab-Ab^2+2\,Aac\right)x}{2\,((ab^2c-4\,a^2c^2)x^4+a^2b^2-4\,a^3c+(ab^3-4\,a^2bc)x^2)}+\frac{-\int\frac{4\,Bacx+(2\,Ca-Ab)cx^2-Cab-Ab^2+6\,Aac}{cx^4+bx^2+a}\,dx}{2\,(ab^2-4\,a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*B*a*c*x^2 + (2*C*a - A*b)*c*x^3 + B*a*b + (C*a*b - A*b^2
+ 2*A*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a
*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate(-(4*B*a*c*x + (2*C*a - A*b
)*c*x^2 - C*a*b - A*b^2 + 6*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2
 - 4*a^2*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.34 $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=403

$$\frac{\left(4a^2cC+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{Bx\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (B*x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + (B*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) - (B*Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*(b^3 - 6*a*b*c)
+ 4*a^2*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^
2 - 4*a*c)^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(
4*a^2)
```

Rubi [A] time = 2.00849, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{\left(4a^2cC+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{Bx\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] (B*x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + (B*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) - (B*Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*(b^3 - 6*a*b*c)
+ 4*a^2*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^
2 - 4*a*c)^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(
4*a^2)
```

Rubi in Sympy [A] time = 167.269, size = 382, normalized size = 0.95

$$\frac{A\log\left(x^{2}\right)}{2a^{2}}-\frac{A\log\left(a+bx^{2}+cx^{4}\right)}{4a^{2}}-\frac{\sqrt{2}B\sqrt{c}\left(-12ac+b^{2}-b\sqrt{-4ac+b^{2}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^{2}}}}\right)}{4a\sqrt{b+\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}+\frac{\sqrt{2}B\sqrt{c}\left(-12ac+b^{2}+b\sqrt{-4ac+b^{2}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^{2}}}}\right)}{4a\sqrt{b-\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}+\frac{Bx\left(-2ac+b^{2}+bcx^{2}\right)}{2a\left(-4ac+b^{2}\right)\left(a+bx^{2}+cx^{4}\right)}+\frac{-2Aac+Ab^{2}-Cab+cx^{2}\left(Ab-2Ca\right)}{2a\left(-4ac+b^{2}\right)\left(a+bx^{2}+cx^{4}\right)}+\frac{\left(-6Aabc+Ab^{3}+4Ca^{2}c\right)\operatorname{atanh}\left(\frac{b+2cx^{2}}{\sqrt{-4ac+b^{2}}}\right)}{2a^{2}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*log(x**2)/(2*a**2) - A*log(a + b*x**2 + c*x**4)/(4*a**2) - sqrt
(2)*B*sqrt(c)*(-12*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(sqrt(
2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a*sqrt(b + sqrt(-4
*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*B*sqrt(c)*(-12*a*
c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b -
 sqrt(-4*a*c + b**2)))/(4*a*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c
 + b**2)**(3/2)) + B*x*(-2*a*c + b**2 + b*c*x**2)/(2*a*(-4*a*c +
b**2)*(a + b*x**2 + c*x**4)) + (-2*A*a*c + A*b**2 - C*a*b + c*x**
2*(A*b - 2*C*a))/(2*a*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)) + (-
6*A*a*b*c + A*b**3 + 4*C*a**2*c)*atanh((b + 2*c*x**2)/sqrt(-4*a*c
 + b**2))/(2*a**2*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 3.00325, size = 458, normalized size = 1.14

$$\frac{-\frac{\left(4a^{2}cC+A\left(b^{2}\sqrt{b^{2}-4ac}-4ac\sqrt{b^{2}-4ac}-6abc+b^{3}\right)\right)\log\left(\sqrt{b^{2}-4ac}-b-2cx^{2}\right)}{\left(b^{2}-4ac\right)^{3/2}}-\frac{\left(A\left(b^{2}\sqrt{b^{2}-4ac}-4ac\sqrt{b^{2}-4ac}+6abc-b^{3}\right)-4a^{2}cC\right)\log\left(\sqrt{b^{2}-4ac}+b+2cx^{2}\right)}{\left(b^{2}-4ac\right)^{3/2}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*b*C + 2*a*c*x*(B + C*x) - b*B*x*(b + c*x^2) - A*(b^2 -
2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]
*a*B*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*B*Sqrt[c]*(-b^2 + 12*a*c
+ b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ 4*A*Log[x] - ((A*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c
*Sqrt[b^2 - 4*a*c]) + 4*a^2*c*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c
*x^2])/(b^2 - 4*a*c)^(3/2) - ((A*(-b^3 + 6*a*b*c + b^2*Sqrt[b^2 -
 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c]) - 4*a^2*c*C)*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [B] time = 0.097, size = 4871, normalized size = 12.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -8/a/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2*c)*ln
(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*A*b
^6*c^2+4/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-4*a*b*
c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-
8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))
*(4*a*c-b^2)*c)^(1/2))*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b
^6)^(1/2)*c^2*b^2+12*a/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*
arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*b^3*c^3-12*a/(4*a*c-b^2)/(
16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)
^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))
*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*c^3+5/a/(4*a
*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c
^2-2*x^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))*A*(-64*a^3*c^3
+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3*c^2+1/4/a*c/(4*a*c-b^2)
/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^
3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2
^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2
))*B*b^7+1/4/a*c/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(
1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*
a*c-b^2)^3)^(1/2)))^(1/2))*B*b^7-16*a^2/(4*a*c-b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3
)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^
2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b*c^4+12*a/(4
*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((4*a*c-b^2)*c*(4*a*
b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*
c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))
^(1/2))*B*b^3*c^3+12*a/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(
1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*a
rctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12
*a*b^4*c+b^6)^(1/2)*c^3+5/a/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4
)/(-8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4
*a*c-b^2)^3)^(1/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)
^(1/2)*b^3*c^2-16*a^2/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1
/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*a
rctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b
^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*b*c^4-4/(4*a*c-b^2)/(16*a^2
*c^2-8*a*b^2*c+b^4)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-
b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*
a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*(-64*a^
3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*c^2*b^2-3/(4*a*c-b^2)/
(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-
(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/
2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*
b^5*c^2-12/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2
*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2
))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*c^3-2/(4
*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2*c)*ln(-8*x^2
*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*C*(-64*a^3
*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*c^2*b^2-3/(4*a*c-b^2)/(
16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)
^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))
*B*b^5*c^2-12/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*b
^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/
2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*c^3-1/2
/a^2*c/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*b^2*c)*l
n(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))*A*b
^8+1/2/a^2*c/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b
^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1
/2))*A*b^8+8/a/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*
b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1
/2))*A*b^6*c^2+8*a/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(8*a*c^
2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3
)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*c^3-
```

```
128*a/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2*c)*l
n(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*A*
c^4*b^2-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*b^2+48/(4*a*c-b^2)/
(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*x
^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*A*b^4*c^3+128*a^2/(4
*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2*c)*ln(-8*x^2
*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*A*c^5-1/2/
a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b-128*a^2/(4*a*c-b^2)/(16*a
^2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*
c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))*A*c^5-48/(4*a*c-b^2)/(16*a^
2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c
+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))*A*b^4*c^3-1/2/a^2*c/(4*a*c-b
^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2
*x^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))*A*(-64*a^3*c^3+48*
a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^5-1/2/a^2*c/(4*a*c-b^2)/(16*a
^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^
2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*A*(-64*a^3*c^3+48*a^2*b^2
*c^2-12*a*b^4*c+b^6)^(1/2)*b^5+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*
c-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*B+1/4/a*c/(4*a*c-b^2)
/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1
/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B
*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^4-1/4/a*c/(4
*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-4*a*b*c+b^3+(-(4*
a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b
^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2
)*c)^(1/2))*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b
^4+8*a/(4*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(-8*a*c^2+2*b^2*c)*
ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*C
*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*c^3+128*a/(4*a
*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c
^2-2*x^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))*A*c^4*b^2-2/(4
*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(8*a*c^2-2*b^2*c)*ln(8*x^2*a
*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))*C*(-64*a^3*c
^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*c^2*b^2+1/(c*x^4+b*x^2+a)
/(4*a*c-b^2)*A*c+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*C+A*ln(x)/a^2+
1/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*C-1/2/a/(c*x^4+b*x^2+a)/(4*a*
c-b^2)*A*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbcx^3 - (2\,Ca - Ab)cx^2 - Cab + Ab^2 - 2\,Aac + \left(Bb^2 - 2\,Bac\right)x}{2\,((ab^2c - 4\,a^2c^2)x^4 + a^2b^2 - 4\,a^3c + (ab^3 - 4\,a^2bc)x^2)} + \frac{\int \frac{Babcx^2 + Bab^2 - 6\,Ba^2c - 2\,(Ab^2c - 4\,Aac^2)\,x^3 - 2\,(Ab^3 + (2\,Ca^2 - 5\,Aab)\,c)\,x}{cx^4 + bx^2 + a}\,dx}{2\,(a^2b^2 - 4\,a^3c)} + \frac{A\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c*x^3 - (2*C*a - A*b)*c*x^2 - C*a*b + A*b^2 - 2*A*a*c +
(B*b^2 - 2*B*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3
*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((B*a*b*c*x^2 + B*a*
b^2 - 6*B*a^2*c - 2*(A*b^2*c - 4*A*a*c^2)*x^3 - 2*(A*b^3 + (2*C*a
^2 - 5*A*a*b)*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) +
 A*log(x)/a^2
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.35 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=514

$$-\frac{-10aAc-abC+3Ab^2}{2a^2x(b^2-4ac)}-\frac{\sqrt{c}\left(A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\frac{A(3b^3-16abc)-aC(b^2-12ac)}{\sqrt{b^2-4ac}}-10aAc-abC+3Ab^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bB(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}-\frac{B\log(a+bx^2+cx^4)}{4a^2}+\frac{B\log(x)}{a^2}+\frac{A(b^2-2ac)+cx^2(Ab-2aC)-abC}{2ax(b^2-4ac)(a+bx^2+cx^4)}+\frac{B(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

```
[Out] -(3*A*b^2 - 10*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 -
 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*(
b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x*
(a + b*x^2 + c*x^4)) - (Sqrt[c]*(A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt
[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) - a*(b^2 - 12*a*c + b*S
qrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - 10*a*A*c - a*b*C - (A*(3*b^3 -
16*a*b*c) - a*(b^2 - 12*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 -
4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*(b^2 - 6*a*c)*ArcTanh[
(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (
B*Log[x])/a^2 - (B*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 3.79182, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$

$$-\frac{-10aAc-abC+3Ab^2}{2a^2x(b^2-4ac)}-\frac{\sqrt{c}\left(A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\frac{A(3b^3-16abc)-aC(b^2-12ac)}{\sqrt{b^2-4ac}}-10aAc-abC+3Ab^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bB(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}-\frac{B\log(a+bx^2+cx^4)}{4a^2}+\frac{B\log(x)}{a^2}+\frac{A(b^2-2ac)+cx^2(Ab-2aC)-abC}{2ax(b^2-4ac)(a+bx^2+cx^4)}+\frac{B(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(3*A*b^2 - 10*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 -
 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*(
b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x*
(a + b*x^2 + c*x^4)) - (Sqrt[c]*(A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt
[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) - a*(b^2 - 12*a*c + b*S
qrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
```

```
 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - 10*a*A*c - a*b*C - (A*(3*b^3 -
16*a*b*c) - a*(b^2 - 12*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 -
4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*(b^2 - 6*a*c)*ArcTanh[
(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (
B*Log[x])/a^2 - (B*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.33749, size = 559, normalized size = 1.09

$$\frac{\frac{-4a^2c(B+Cx)+2a\left(bcx(3A+x(B+Cx))+2Ac^2x^3+b^2(B+Cx)\right)-2Ab^2x\left(b+cx^2\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)+aC\left(b\sqrt{b^2-4ac}-12ac+\right.\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*A)/x + (-4*a^2*c*(B + C*x) - 2*A*b^2*x*(b + c*x^2) + 2*a*(2*
A*c^2*x^3 + b^2*(B + C*x) + b*c*x*(3*A + x*(B + C*x))))/((b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(-3*b^3 + 16*a*b
*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(b^2
 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(A*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt
[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(-b^2 + 12*a*c + b*
Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) +
 4*B*Log[x] - (B*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*S
qrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4
*a*c)^(3/2) - (B*(-b^3 + 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*
Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4
*a*c)^(3/2))/(4*a^2)
```

Maple [B] time = 0.119, size = 6960, normalized size = 13.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Babcx^3 + \left(10\,Aac^2 + \left(Cab - 3\,Ab^2\right)c\right)x^4 - 2\,Aab^2 + 8\,Aa^2c + \left(Cab^2 - 3\,Ab^3 - \left(2\,Ca^2 - 11\,Aab\right)c\right)x^2 + \left(Bab^2 - 2\,Ba^2c\right)x}{2\left(\left(a^2b^2c - 4\,a^3c^2\right)x^5 + \left(a^2b^3 - 4\,a^3bc\right)x^3 + \left(a^3b^2 - 4\,a^4c\right)x\right)} + \frac{\int \frac{Cab^2 - 3\,Ab^3 - 2\left(Bb^2c - 4\,Bac^2\right)x^3 + \left(10\,Aac^2 + \left(Cab - 3\,Ab^2\right)c\right)x^2 - \left(6\,Ca^2 - 13\,Aab\right)c - 2\left(Bb^3 - 5\,Babc\right)x}{cx^4 + bx^2 + a}\,dx}{2\left(a^2b^2 - 4\,a^3c\right)} + \frac{B\log\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(B*a*b*c*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^4 - 2*A*a
*b^2 + 8*A*a^2*c + (C*a*b^2 - 3*A*b^3 - (2*C*a^2 - 11*A*a*b)*c)*x
^2 + (B*a*b^2 - 2*B*a^2*c)*x)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2
*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate((C
*a*b^2 - 3*A*b^3 - 2*(B*b^2*c - 4*B*a*c^2)*x^3 + (10*A*a*c^2 + (C
*a*b - 3*A*b^2)*c)*x^2 - (6*C*a^2 - 13*A*a*b)*c - 2*(B*b^3 - 5*B*
a*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + B*log(x)/
a^2
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.36 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=534

$$\frac{(2Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{-6aAc-abC+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}-\frac{B\left(3b^2-10ac\right)}{2a^2x\left(b^2-4ac\right)}-\frac{B\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{B\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\left(2A\left(6a^2c^2-6ab^2c+b^4\right)-abC\left(b^2-6ac\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\left(-2ac+b^2+bcx^2\right)}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(2*A*b^2 - 6*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x^2) - (B*(3*b^
2 - 10*a*c))/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b*c*x^2)
)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) -
a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 +
c*x^4)) - (B*Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) + (B*Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2) - a*b*(b^2 - 6*a*c)*C)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) -
 ((2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[a + b*x^2 + c*x^
4])/(4*a^3)
```

Rubi [A] time = 4.17279, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$

$$\frac{(2Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{-6aAc-abC+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}-\frac{B\left(3b^2-10ac\right)}{2a^2x\left(b^2-4ac\right)}-\frac{B\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{B\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\left(2A\left(6a^2c^2-6ab^2c+b^4\right)-abC\left(b^2-6ac\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\left(-2ac+b^2+bcx^2\right)}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(2*A*b^2 - 6*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x^2) - (B*(3*b^
2 - 10*a*c))/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b*c*x^2)
)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) -
a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 +
```

```
c*x^4)) - (B*Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) + (B*Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2) - a*b*(b^2 - 6*a*c)*C)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) -
 ((2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[a + b*x^2 + c*x^
4])/(4*a^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.20855, size = 655, normalized size = 1.23

$$-\frac{\frac{2a\left(2a^2cC+A\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)-a\left(b^2C+bcx(3B+Cx)+2Bc^2x^3\right)+b^2Bx\left(b+cx^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(2A\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)+aC\left(-b\cdots\right.\right.}{(b^2-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*a*(2*a^2*c*C + b^2*B*x*(b + c*x^2)
 + A*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2) - a*(b^2*C + 2*B*c
^2*x^3 + b*c*x*(3*B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + (Sqrt[2]*a*B*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a
*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
4*a*c]]) + (Sqrt[2]*a*B*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^
2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt
[b^2 - 4*a*c]]) + 4*(-2*A*b + a*C)*Log[x] + ((2*A*(b^4 - 6*a*b^2*
c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]
) + a*(-b^3 + 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 -
4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(
3/2) + ((2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c
] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(b^3 - 6*a*b*c - b^2*Sqrt[b^2
- 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

Maple [B] time = 0.122, size = 6930, normalized size = 13.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(3\,Bb^2c-10\,Bac^2\right)x^5-\left(6\,Aac^2+\left(Cab-2\,Ab^2\right)c\right)x^4+Aab^2-4\,Aa^2c+\left(3\,Bb^3-11\,Babc\right)x^3-\left(Cab^2-2\,Ab^3-\left(2\,Ca^2-\right.\right.}{2\left((a^2b^2c-4\,a^3c^2)x^6+(a^2b^3-4\,a^3bc)x^4+(a^3b^2-4\,a^4c)x^2\right)}$$

$$-\frac{\int\frac{3\,Bab^3-13\,Ba^2bc-2\left(4\left(Ca^2-2\,Aab\right)c^2-\left(Cab^2-2\,Ab^3\right)c\right)x^3+\left(3\,Bab^2c-10\,Ba^2c^2\right)x^2+2\left(Cab^3-2\,Ab^4-6\,Aa^2c^2-5\left(Ca^2b-2\,Aab^2\right)c\right)x}{cx^4+bx^2+a}\,dx}{2\left(a^3b^2-4\,a^4c\right)}$$

$$+\frac{(Ca-2\,Ab)\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*((3*B*b^2*c - 10*B*a*c^2)*x^5 - (6*A*a*c^2 + (C*a*b - 2*A*b^
2)*c)*x^4 + A*a*b^2 - 4*A*a^2*c + (3*B*b^3 - 11*B*a*b*c)*x^3 - (C
*a*b^2 - 2*A*b^3 - (2*C*a^2 - 7*A*a*b)*c)*x^2 + 2*(B*a*b^2 - 4*B*
a^2*c)*x)/((a^2*b^2*c - 4*a^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^
4 + (a^3*b^2 - 4*a^4*c)*x^2) - 1/2*integrate((3*B*a*b^3 - 13*B*a^
2*b*c - 2*(4*(C*a^2 - 2*A*a*b)*c^2 - (C*a*b^2 - 2*A*b^3)*c)*x^3 +
 (3*B*a*b^2*c - 10*B*a^2*c^2)*x^2 + 2*(C*a*b^3 - 2*A*b^4 - 6*A*a^
2*c^2 - 5*(C*a^2*b - 2*A*a*b^2)*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^
3*b^2 - 4*a^4*c) + (C*a - 2*A*b)*log(x)/a^3
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.37 $\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=399

$$\frac{a^3A(dx)^{m+1}}{d(m+1)}+\frac{a^3B(dx)^{m+2}}{d^2(m+2)}+\frac{a^2(dx)^{m+3}(aC+3Ab)}{d^3(m+3)}+\frac{3a^2bB(dx)^{m+4}}{d^4(m+4)} +\frac{3c(dx)^{m+11}\left(C\left(ac+b^2\right)+Abc\right)}{d^{11}(m+11)}+\frac{(dx)^{m+9}\left(3Ac\left(ac+b^2\right)+bC\left(6ac+b^2\right)\right)}{d^9(m+9)} +\frac{3a(dx)^{m+5}\left(A\left(ac+b^2\right)+abC\right)}{d^5(m+5)}+\frac{(dx)^{m+7}\left(A\left(6abc+b^3\right)+3aC\left(ac+b^2\right)\right)}{d^7(m+7)} +\frac{3Bc\left(ac+b^2\right)(dx)^{m+10}}{d^{10}(m+10)}+\frac{bB\left(6ac+b^2\right)(dx)^{m+8}}{d^8(m+8)}+\frac{3aB\left(ac+b^2\right)(dx)^{m+6}}{d^6(m+6)} +\frac{c^2(dx)^{m+13}(Ac+3bC)}{d^{13}(m+13)}+\frac{3bBc^2(dx)^{m+12}}{d^{12}(m+12)}+\frac{Bc^3(dx)^{m+14}}{d^{14}(m+14)}+\frac{c^3C(dx)^{m+15}}{d^{15}(m+15)}$$

```
[Out] (a^3*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^3*B*(d*x)^(2 + m))/(d^2*(2
 + m)) + (a^2*(3*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a^2
*b*B*(d*x)^(4 + m))/(d^4*(4 + m)) + (3*a*(A*(b^2 + a*c) + a*b*C)*
(d*x)^(5 + m))/(d^5*(5 + m)) + (3*a*B*(b^2 + a*c)*(d*x)^(6 + m))/
(d^6*(6 + m)) + ((A*(b^3 + 6*a*b*c) + 3*a*(b^2 + a*c)*C)*(d*x)^(7
 + m))/(d^7*(7 + m)) + (b*B*(b^2 + 6*a*c)*(d*x)^(8 + m))/(d^8*(8
+ m)) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*(d*x)^(9 + m))/(
d^9*(9 + m)) + (3*B*c*(b^2 + a*c)*(d*x)^(10 + m))/(d^10*(10 + m))
 + (3*c*(A*b*c + (b^2 + a*c)*C)*(d*x)^(11 + m))/(d^11*(11 + m)) +
 (3*b*B*c^2*(d*x)^(12 + m))/(d^12*(12 + m)) + (c^2*(A*c + 3*b*C)*
(d*x)^(13 + m))/(d^13*(13 + m)) + (B*c^3*(d*x)^(14 + m))/(d^14*(1
4 + m)) + (c^3*C*(d*x)^(15 + m))/(d^15*(15 + m))
```

Rubi [A] time = 0.945443, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$

$$\frac{a^3A(dx)^{m+1}}{d(m+1)}+\frac{a^3B(dx)^{m+2}}{d^2(m+2)}+\frac{a^2(dx)^{m+3}(aC+3Ab)}{d^3(m+3)}+\frac{3a^2bB(dx)^{m+4}}{d^4(m+4)} +\frac{3c(dx)^{m+11}\left(C\left(ac+b^2\right)+Abc\right)}{d^{11}(m+11)}+\frac{(dx)^{m+9}\left(3Ac\left(ac+b^2\right)+bC\left(6ac+b^2\right)\right)}{d^9(m+9)} +\frac{3a(dx)^{m+5}\left(A\left(ac+b^2\right)+abC\right)}{d^5(m+5)}+\frac{(dx)^{m+7}\left(A\left(6abc+b^3\right)+3aC\left(ac+b^2\right)\right)}{d^7(m+7)} +\frac{3Bc\left(ac+b^2\right)(dx)^{m+10}}{d^{10}(m+10)}+\frac{bB\left(6ac+b^2\right)(dx)^{m+8}}{d^8(m+8)}+\frac{3aB\left(ac+b^2\right)(dx)^{m+6}}{d^6(m+6)} +\frac{c^2(dx)^{m+13}(Ac+3bC)}{d^{13}(m+13)}+\frac{3bBc^2(dx)^{m+12}}{d^{12}(m+12)}+\frac{Bc^3(dx)^{m+14}}{d^{14}(m+14)}+\frac{c^3C(dx)^{m+15}}{d^{15}(m+15)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] (a^3*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^3*B*(d*x)^(2 + m))/(d^2*(2
 + m)) + (a^2*(3*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a^2
*b*B*(d*x)^(4 + m))/(d^4*(4 + m)) + (3*a*(A*(b^2 + a*c) + a*b*C)*
(d*x)^(5 + m))/(d^5*(5 + m)) + (3*a*B*(b^2 + a*c)*(d*x)^(6 + m))/
(d^6*(6 + m)) + ((A*(b^3 + 6*a*b*c) + 3*a*(b^2 + a*c)*C)*(d*x)^(7
 + m))/(d^7*(7 + m)) + (b*B*(b^2 + 6*a*c)*(d*x)^(8 + m))/(d^8*(8
+ m)) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*(d*x)^(9 + m))/(
d^9*(9 + m)) + (3*B*c*(b^2 + a*c)*(d*x)^(10 + m))/(d^10*(10 + m))
 + (3*c*(A*b*c + (b^2 + a*c)*C)*(d*x)^(11 + m))/(d^11*(11 + m)) +
 (3*b*B*c^2*(d*x)^(12 + m))/(d^12*(12 + m)) + (c^2*(A*c + 3*b*C)*
(d*x)^(13 + m))/(d^13*(13 + m)) + (B*c^3*(d*x)^(14 + m))/(d^14*(1
4 + m)) + (c^3*C*(d*x)^(15 + m))/(d^15*(15 + m))
```

Rubi in Sympy [A] time = 120.922, size = 396, normalized size = 0.99

$$\frac{Aa^3 (dx)^{m+1}}{d(m+1)} + \frac{Ba^3 (dx)^{m+2}}{d^2(m+2)} + \frac{3Ba^2b (dx)^{m+4}}{d^4(m+4)} + \frac{3Ba (dx)^{m+6} (ac+b^2)}{d^6(m+6)} + \frac{3Bbc^2 (dx)^{m+12}}{d^{12}(m+12)} + \frac{Bb (dx)^{m+8} (6ac+b^2)}{d^8(m+8)} + \frac{Bc^3 (dx)^{m+14}}{d^{14}(m+14)} + \frac{3Bc (dx)^{m+10} (ac+b^2)}{d^{10}(m+10)} + \frac{Cc^3 (dx)^{m+15}}{d^{15}(m+15)} + \frac{a^2 (dx)^{m+3} (3Ab+Ca)}{d^3(m+3)} + \frac{3a (dx)^{m+5} (Aac+Ab^2+Cab)}{d^5(m+5)} + \frac{c^2 (dx)^{m+13} (Ac+3Cb)}{d^{13}(m+13)} + \frac{3c (dx)^{m+11} (Abc+Cac+Cb^2)}{d^{11}(m+11)} + \frac{(dx)^{m+7} (6Aabc+Ab^3+3Ca^2c+3Cab^2)}{d^7(m+7)} + \frac{(dx)^{m+9} (3Aac^2+3Ab^2c+6Cabc+Cb^3)}{d^9(m+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*(d*x)**(m + 1)/(d*(m + 1)) + B*a**3*(d*x)**(m + 2)/(d**2*(
m + 2)) + 3*B*a**2*b*(d*x)**(m + 4)/(d**4*(m + 4)) + 3*B*a*(d*x)*
*(m + 6)*(a*c + b**2)/(d**6*(m + 6)) + 3*B*b*c**2*(d*x)**(m + 12)
/(d**12*(m + 12)) + B*b*(d*x)**(m + 8)*(6*a*c + b**2)/(d**8*(m +
8)) + B*c**3*(d*x)**(m + 14)/(d**14*(m + 14)) + 3*B*c*(d*x)**(m +
 10)*(a*c + b**2)/(d**10*(m + 10)) + C*c**3*(d*x)**(m + 15)/(d**1
5*(m + 15)) + a**2*(d*x)**(m + 3)*(3*A*b + C*a)/(d**3*(m + 3)) +
3*a*(d*x)**(m + 5)*(A*a*c + A*b**2 + C*a*b)/(d**5*(m + 5)) + c**2
*(d*x)**(m + 13)*(A*c + 3*C*b)/(d**13*(m + 13)) + 3*c*(d*x)**(m +
 11)*(A*b*c + C*a*c + C*b**2)/(d**11*(m + 11)) + (d*x)**(m + 7)*(
6*A*a*b*c + A*b**3 + 3*C*a**2*c + 3*C*a*b**2)/(d**7*(m + 7)) + (d
*x)**(m + 9)*(3*A*a*c**2 + 3*A*b**2*c + 6*C*a*b*c + C*b**3)/(d**9
*(m + 9))
```

Mathematica [A] time = 4.45753, size = 296, normalized size = 0.74

$$x(dx)^m \left(\frac{a^3A}{m+1} + \frac{a^3Bx}{m+2} + \frac{a^2x^2(aC+3Ab)}{m+3} + \frac{3a^2bBx^3}{m+4} + \frac{3cx^{10}\left(C\left(ac+b^2\right)+Abc\right)}{m+11} + \frac{x^8\left(3Ac\left(ac+b^2\right)+bC\left(6ac+b^2\right)\right)}{m+9} + \frac{3ax^4\left(A\left(ac+b^2\right)+abC\right)}{m+5} + \frac{x^6\left(A\left(6abc+b^3\right)+3aC\left(ac+b^2\right)\right)}{m+7} + \frac{3Bcx^9\left(ac+b^2\right)}{m+10} + \frac{bBx^7\left(6ac+b^2\right)}{m+8} + \frac{3aBx^5\left(ac+b^2\right)}{m+6} + \frac{c^2x^{12}(Ac+3bC)}{m+13} + \frac{3bBc^2x^{11}}{m+12} + \frac{Bc^3x^{13}}{m+14} + \frac{c^3Cx^{14}}{m+15} \right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(d*x)^m*((a^3*A)/(1 + m) + (a^3*B*x)/(2 + m) + (a^2*(3*A*b + a*
C)*x^2)/(3 + m) + (3*a^2*b*B*x^3)/(4 + m) + (3*a*(A*(b^2 + a*c) +
 a*b*C)*x^4)/(5 + m) + (3*a*B*(b^2 + a*c)*x^5)/(6 + m) + ((A*(b^3
 + 6*a*b*c) + 3*a*(b^2 + a*c)*C)*x^6)/(7 + m) + (b*B*(b^2 + 6*a*c
)*x^7)/(8 + m) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*x^8)/(9
 + m) + (3*B*c*(b^2 + a*c)*x^9)/(10 + m) + (3*c*(A*b*c + (b^2 + a
*c)*C)*x^10)/(11 + m) + (3*b*B*c^2*x^11)/(12 + m) + (c^2*(A*c + 3
*b*C)*x^12)/(13 + m) + (B*c^3*x^13)/(14 + m) + (c^3*C*x^14)/(15 +
 m))
```

Maple [B] time = 0.017, size = 5520, normalized size = 13.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(C*x^2 + B*x + A)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.353297, size = 5262, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(C*x^2 + B*x + A)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((C*c^3*m^14 + 105*C*c^3*m^13 + 5005*C*c^3*m^12 + 143325*C*c^3*m^
11 + 2749747*C*c^3*m^10 + 37312275*C*c^3*m^9 + 368411615*C*c^3*m^
8 + 2681453775*C*c^3*m^7 + 14409322928*C*c^3*m^6 + 56663366760*C*
c^3*m^5 + 159721605680*C*c^3*m^4 + 310989260400*C*c^3*m^3 + 39215
6797824*C*c^3*m^2 + 283465647360*C*c^3*m + 87178291200*C*c^3)*x^1
5 + (B*c^3*m^14 + 106*B*c^3*m^13 + 5096*B*c^3*m^12 + 147056*B*c^3
*m^11 + 2840838*B*c^3*m^10 + 38786748*B*c^3*m^9 + 385081268*B*c^3
*m^8 + 2816490248*B*c^3*m^7 + 15200266081*B*c^3*m^6 + 59999485546
*B*c^3*m^5 + 169679309436*B*c^3*m^4 + 331303013496*B*c^3*m^3 + 41
8753514880*B*c^3*m^2 + 303268406400*B*c^3*m + 93405312000*B*c^3)*
x^14 + ((3*C*b*c^2 + A*c^3)*m^14 + 107*(3*C*b*c^2 + A*c^3)*m^13 +
 5189*(3*C*b*c^2 + A*c^3)*m^12 + 150943*(3*C*b*c^2 + A*c^3)*m^11
+ 2937363*(3*C*b*c^2 + A*c^3)*m^10 + 40372761*(3*C*b*c^2 + A*c^3)
*m^9 + 403249847*(3*C*b*c^2 + A*c^3)*m^8 + 2965379989*(3*C*b*c^2
+ A*c^3)*m^7 + 16081189696*(3*C*b*c^2 + A*c^3)*m^6 + 63747744632*
(3*C*b*c^2 + A*c^3)*m^5 + 180951426864*(3*C*b*c^2 + A*c^3)*m^4 +
301771008000*C*b*c^2 + 100590336000*A*c^3 + 354444796368*(3*C*b*c
^2 + A*c^3)*m^3 + 449213351040*(3*C*b*c^2 + A*c^3)*m^2 + 32604405
1200*(3*C*b*c^2 + A*c^3)*m)*x^13 + 3*(B*b*c^2*m^14 + 108*B*b*c^2*
m^13 + 5284*B*b*c^2*m^12 + 154992*B*b*c^2*m^11 + 3039718*B*b*c^2*
m^10 + 42081864*B*b*c^2*m^9 + 423113372*B*b*c^2*m^8 + 3130267536*
B*b*c^2*m^7 + 17067919121*B*b*c^2*m^6 + 67988181228*B*b*c^2*m^5 +
 193813932344*B*b*c^2*m^4 + 381046157472*B*b*c^2*m^3 + 4844418141
60*B*b*c^2*m^2 + 352515844800*B*b*c^2*m + 108972864000*B*b*c^2)*x
^12 + 3*((C*b^2*c + (C*a + A*b)*c^2)*m^14 + 109*(C*b^2*c + (C*a +
 A*b)*c^2)*m^13 + 5381*(C*b^2*c + (C*a + A*b)*c^2)*m^12 + 159209*
(C*b^2*c + (C*a + A*b)*c^2)*m^11 + 3148323*(C*b^2*c + (C*a + A*b)
*c^2)*m^10 + 43926927*(C*b^2*c + (C*a + A*b)*c^2)*m^9 + 444899543
*(C*b^2*c + (C*a + A*b)*c^2)*m^8 + 3313733027*(C*b^2*c + (C*a + A
*b)*c^2)*m^7 + 18180066256*(C*b^2*c + (C*a + A*b)*c^2)*m^6 + 7282
2481864*(C*b^2*c + (C*a + A*b)*c^2)*m^5 + 208624806576*(C*b^2*c +
 (C*a + A*b)*c^2)*m^4 + 118879488000*C*b^2*c + 411940473264*(C*b^
2*c + (C*a + A*b)*c^2)*m^3 + 118879488000*(C*a + A*b)*c^2 + 52565
0497920*(C*b^2*c + (C*a + A*b)*c^2)*m^2 + 383662137600*(C*b^2*c +
 (C*a + A*b)*c^2)*m)*x^11 + 3*((B*b^2*c + B*a*c^2)*m^14 + 110*(B*
b^2*c + B*a*c^2)*m^13 + 5480*(B*b^2*c + B*a*c^2)*m^12 + 163600*(B
*b^2*c + B*a*c^2)*m^11 + 3263622*(B*b^2*c + B*a*c^2)*m^10 + 45922
260*(B*b^2*c + B*a*c^2)*m^9 + 468873140*(B*b^2*c + B*a*c^2)*m^8 +
```

3518896600*(B*b^2*c + B*a*c^2)*m^7 + 19442163553*(B*b^2*c + B*a*
c^2)*m^6 + 78381575150*(B*b^2*c + B*a*c^2)*m^5 + 225856355580*(B*
b^2*c + B*a*c^2)*m^4 + 130767436800*B*b^2*c + 130767436800*B*a*c^
2 + 448249789800*(B*b^2*c + B*a*c^2)*m^3 + 574497805824*(B*b^2*c
+ B*a*c^2)*m^2 + 420839556480*(B*b^2*c + B*a*c^2)*m)*x^10 + ((C*b
^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^14 + 111*(C*b^3 + 3*A*a
c^2 + 3(2*C*a*b + A*b^2)*c)*m^13 + 5581*(C*b^3 + 3*A*a*c^2 + 3*
(2*C*a*b + A*b^2)*c)*m^12 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*
b + A*b^2)*c)*m^11 + 3386083*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*
b^2)*c)*m^10 + 48083733*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*
c)*m^9 + 495342143*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^
8 + 3749548713*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^7 +
20885191136*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^6 + 848
36490456*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^5 + 246143
692976*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^4 + 14529715
2000*C*b^3 + 435891456000*A*a*c^2 + 491520108816*(C*b^3 + 3*A*a*c
^2 + 3*(2*C*a*b + A*b^2)*c)*m^3 + 633314724480*(C*b^3 + 3*A*a*c^2
+ 3*(2*C*a*b + A*b^2)*c)*m^2 + 435891456000*(2*C*a*b + A*b^2)*c
+ 465985094400*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m)*x^9
+ ((B*b^3 + 6*B*a*b*c)*m^14 + 112*(B*b^3 + 6*B*a*b*c)*m^13 + 568
4*(B*b^3 + 6*B*a*b*c)*m^12 + 172928*(B*b^3 + 6*B*a*b*c)*m^11 + 35
16198*(B*b^3 + 6*B*a*b*c)*m^10 + 50428896*(B*b^3 + 6*B*a*b*c)*m^9
+ 524664572*(B*b^3 + 6*B*a*b*c)*m^8 + 4010311424*(B*b^3 + 6*B*a*
b*c)*m^7 + 22548638161*(B*b^3 + 6*B*a*b*c)*m^6 + 92414105392*(B*b
^3 + 6*B*a*b*c)*m^5 + 270359263944*(B*b^3 + 6*B*a*b*c)*m^4 + 1634
59296000*B*b^3 + 980755776000*B*a*b*c + 543939234048*(B*b^3 + 6*B
*a*b*c)*m^3 + 705481831440*(B*b^3 + 6*B*a*b*c)*m^2 + 521962963200
*(B*b^3 + 6*B*a*b*c)*m)*x^8 + ((3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*
A*a*b)*c)*m^14 + 113*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*
m^13 + 5789*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^12 + 17
7877*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^11 + 3654483*(
3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^10 + 52977099*(3*C*a
*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^9 + 557256047*(3*C*a*b^2
+ A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^8 + 4306835671*(3*C*a*b^2 + A*
b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^7 + 24483279856*(3*C*a*b^2 + A*b^3
+ 3*(C*a^2 + 2*A*a*b)*c)*m^6 + 101420251688*(3*C*a*b^2 + A*b^3 +
3*(C*a^2 + 2*A*a*b)*c)*m^5 + 299730345264*(3*C*a*b^2 + A*b^3 + 3
*(C*a^2 + 2*A*a*b)*c)*m^4 + 560431872000*C*a*b^2 + 186810624000*A
b^3 + 608700928752(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m
^3 + 796089202560*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^2
+ 560431872000*(C*a^2 + 2*A*a*b)*c + 593193196800*(3*C*a*b^2 + A
b^3 + 3(C*a^2 + 2*A*a*b)*c)*m)*x^7 + 3*((B*a*b^2 + B*a^2*c)*m^1
4 + 114*(B*a*b^2 + B*a^2*c)*m^13 + 5896*(B*a*b^2 + B*a^2*c)*m^12
+ 183024*(B*a*b^2 + B*a^2*c)*m^11 + 3801478*(B*a*b^2 + B*a^2*c)*m
^10 + 55749612*(B*a*b^2 + B*a^2*c)*m^9 + 593598068*(B*a*b^2 + B*a
^2*c)*m^8 + 4646039592*(B*a*b^2 + B*a^2*c)*m^7 + 26754892001*(B*a
*b^2 + B*a^2*c)*m^6 + 112273858674*(B*a*b^2 + B*a^2*c)*m^5 + 3360
28955036*(B*a*b^2 + B*a^2*c)*m^4 + 217945728000*B*a*b^2 + 2179457
28000*B*a^2*c + 690639615384*(B*a*b^2 + B*a^2*c)*m^3 + 9131580115
20*(B*a*b^2 + B*a^2*c)*m^2 + 686869545600*(B*a*b^2 + B*a^2*c)*m)*
x^6 + 3*((C*a^2*b + A*a*b^2 + A*a^2*c)*m^14 + 115*(C*a^2*b + A*a*
b^2 + A*a^2*c)*m^13 + 6005*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^12 + 1
88375*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^11 + 3957747*(C*a^2*b + A*a
*b^2 + A*a^2*c)*m^10 + 58769745*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^9
+ 634247015*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^8 + 5036392925*(C*a^
2*b + A*a*b^2 + A*a^2*c)*m^7 + 29449164928*(C*a^2*b + A*a*b^2 + A
*a^2*c)*m^6 + 125557386040*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^5 + 38
1885176880*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^4 + 261534873600*C*a^2
*b + 261534873600*A*a*b^2 + 261534873600*A*a^2*c + 797387461200*(
C*a^2*b + A*a*b^2 + A*a^2*c)*m^3 + 1070058397824*(C*a^2*b + A*a*b
^2 + A*a^2*c)*m^2 + 815525625600*(C*a^2*b + A*a*b^2 + A*a^2*c)*m)
x^5 + 3(B*a^2*b*m^14 + 116*B*a^2*b*m^13 + 6116*B*a^2*b*m^12 + 1
93936*B*a^2*b*m^11 + 4123878*B*a^2*b*m^10 + 62062968*B*a^2*b*m^9
+ 679843868*B*a^2*b*m^8 + 5488252528*B*a^2*b*m^7 + 32678119441*B*
a^2*b*m^6 + 142090732916*B*a^2*b*m^5 + 441309175416*B*a^2*b*m^4 +
941576643936*B*a^2*b*m^3 + 1290689128080*B*a^2*b*m^2 + 100306110
2400*B*a^2*b*m + 326918592000*B*a^2*b)*x^4 + ((C*a^3 + 3*A*a^2*b)
m^14 + 117(C*a^3 + 3*A*a^2*b)*m^13 + 6229*(C*a^3 + 3*A*a^2*b)*m
^12 + 199713*(C*a^3 + 3*A*a^2*b)*m^11 + 4300483*(C*a^3 + 3*A*a^2*
b)*m^10 + 65657031*(C*a^3 + 3*A*a^2*b)*m^9 + 731124647*(C*a^3 + 3
*A*a^2*b)*m^8 + 6014254059*(C*a^3 + 3*A*a^2*b)*m^7 + 36588367376*
(C*a^3 + 3*A*a^2*b)*m^6 + 163038108552*(C*a^3 + 3*A*a^2*b)*m^5 +

```
520557781424*(C*a^3 + 3*A*a^2*b)*m^4 + 435891456000*C*a^3 + 13076
74368000*A*a^2*b + 1145140001328*(C*a^3 + 3*A*a^2*b)*m^3 + 162157
5699840*(C*a^3 + 3*A*a^2*b)*m^2 + 1301090515200*(C*a^3 + 3*A*a^2*
b)*m)*x^3 + (B*a^3*m^14 + 118*B*a^3*m^13 + 6344*B*a^3*m^12 + 2057
12*B*a^3*m^11 + 4488198*B*a^3*m^10 + 69582084*B*a^3*m^9 + 7889315
72*B*a^3*m^8 + 6629764856*B*a^3*m^7 + 41371599841*B*a^3*m^6 + 190
060010998*B*a^3*m^5 + 629552085084*B*a^3*m^4 + 1447709175432*B*a^
3*m^3 + 2161577352960*B*a^3*m^2 + 1842662908800*B*a^3*m + 6538371
84000*B*a^3)*x^2 + (A*a^3*m^14 + 119*A*a^3*m^13 + 6461*A*a^3*m^12
 + 211939*A*a^3*m^11 + 4687683*A*a^3*m^10 + 73870797*A*a^3*m^9 +
854224943*A*a^3*m^8 + 7353403057*A*a^3*m^7 + 47277726496*A*a^3*m^
6 + 225525484184*A*a^3*m^5 + 784146622896*A*a^3*m^4 + 19226667227
04*A*a^3*m^3 + 3134328981120*A*a^3*m^2 + 3031488633600*A*a^3*m +
1307674368000*A*a^3)*x)*(d*x)^m/(m^15 + 120*m^14 + 6580*m^13 + 21
8400*m^12 + 4899622*m^11 + 78558480*m^10 + 928095740*m^9 + 820762
8000*m^8 + 54631129553*m^7 + 272803210680*m^6 + 1009672107080*m^5
 + 2706813345600*m^4 + 5056995703824*m^3 + 6165817614720*m^2 + 43
39163001600*m + 1307674368000)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.347214, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(C*x^2 + B*x + A)*(d*x)^m,x, algorithm="giac")
```

```
[Out] Done
```

3.38 $\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=260

$$\frac{a^2A(dx)^{m+1}}{d(m+1)}+\frac{a^2B(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+7}\left(C\left(2ac+b^2\right)+2Abc\right)}{d^7(m+7)}+\frac{(dx)^{m+5}\left(A\left(2ac+b^2\right)+2abC\right)}{d^5(m+5)}+\frac{a(dx)^{m+3}(aC+2Ab)}{d^3(m+3)}+\frac{B\left(2ac+b^2\right)(dx)^{m+6}}{d^6(m+6)}+\frac{2abB(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+9}(Ac+2bC)}{d^9(m+9)}+\frac{2bBc(dx)^{m+8}}{d^8(m+8)}+\frac{Bc^2(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2C(dx)^{m+11}}{d^{11}(m+11)}$$

```
[Out] (a^2*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^2*B*(d*x)^(2 + m))/(d^2*(2
 + m)) + (a*(2*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*a*b*B
*(d*x)^(4 + m))/(d^4*(4 + m)) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*(d*x
)^(5 + m))/(d^5*(5 + m)) + (B*(b^2 + 2*a*c)*(d*x)^(6 + m))/(d^6*(
6 + m)) + ((2*A*b*c + (b^2 + 2*a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m
)) + (2*b*B*c*(d*x)^(8 + m))/(d^8*(8 + m)) + (c*(A*c + 2*b*C)*(d*
x)^(9 + m))/(d^9*(9 + m)) + (B*c^2*(d*x)^(10 + m))/(d^10*(10 + m)
) + (c^2*C*(d*x)^(11 + m))/(d^11*(11 + m))
```

Rubi [A] time = 0.487742, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$\frac{a^2A(dx)^{m+1}}{d(m+1)}+\frac{a^2B(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+7}\left(C\left(2ac+b^2\right)+2Abc\right)}{d^7(m+7)}+\frac{(dx)^{m+5}\left(A\left(2ac+b^2\right)+2abC\right)}{d^5(m+5)}+\frac{a(dx)^{m+3}(aC+2Ab)}{d^3(m+3)}+\frac{B\left(2ac+b^2\right)(dx)^{m+6}}{d^6(m+6)}+\frac{2abB(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+9}(Ac+2bC)}{d^9(m+9)}+\frac{2bBc(dx)^{m+8}}{d^8(m+8)}+\frac{Bc^2(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2C(dx)^{m+11}}{d^{11}(m+11)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^2*B*(d*x)^(2 + m))/(d^2*(2
 + m)) + (a*(2*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*a*b*B
*(d*x)^(4 + m))/(d^4*(4 + m)) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*(d*x
)^(5 + m))/(d^5*(5 + m)) + (B*(b^2 + 2*a*c)*(d*x)^(6 + m))/(d^6*(
6 + m)) + ((2*A*b*c + (b^2 + 2*a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m
)) + (2*b*B*c*(d*x)^(8 + m))/(d^8*(8 + m)) + (c*(A*c + 2*b*C)*(d*
x)^(9 + m))/(d^9*(9 + m)) + (B*c^2*(d*x)^(10 + m))/(d^10*(10 + m)
) + (c^2*C*(d*x)^(11 + m))/(d^11*(11 + m))
```

Rubi in Sympy [A] time = 75.8458, size = 248, normalized size = 0.95

$$\frac{Aa^2\left(dx\right)^{m+1}}{d\left(m+1\right)}+\frac{Ba^2\left(dx\right)^{m+2}}{d^2\left(m+2\right)}+\frac{2Bab\left(dx\right)^{m+4}}{d^4\left(m+4\right)}+\frac{2Bbc\left(dx\right)^{m+8}}{d^8\left(m+8\right)}+\frac{Bc^2\left(dx\right)^{m+10}}{d^{10}\left(m+10\right)}+\frac{B\left(dx\right)^{m+6}\left(2ac+b^2\right)}{d^6\left(m+6\right)}+\frac{Cc^2\left(dx\right)^{m+11}}{d^{11}\left(m+11\right)}+\frac{a\left(dx\right)^{m+3}\left(2Ab+Ca\right)}{d^3\left(m+3\right)}+\frac{c\left(dx\right)^{m+9}\left(Ac+2Cb\right)}{d^9\left(m+9\right)}+\frac{\left(dx\right)^{m+5}\left(2Aac+Ab^2+2Cab\right)}{d^5\left(m+5\right)}+\frac{\left(dx\right)^{m+7}\left(2Abc+2Cac+Cb^2\right)}{d^7\left(m+7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*(d*x)**(m + 1)/(d*(m + 1)) + B*a**2*(d*x)**(m + 2)/(d**2*(
m + 2)) + 2*B*a*b*(d*x)**(m + 4)/(d**4*(m + 4)) + 2*B*b*c*(d*x)**
(m + 8)/(d**8*(m + 8)) + B*c**2*(d*x)**(m + 10)/(d**10*(m + 10))
+ B*(d*x)**(m + 6)*(2*a*c + b**2)/(d**6*(m + 6)) + C*c**2*(d*x)**
(m + 11)/(d**11*(m + 11)) + a*(d*x)**(m + 3)*(2*A*b + C*a)/(d**3*
(m + 3)) + c*(d*x)**(m + 9)*(A*c + 2*C*b)/(d**9*(m + 9)) + (d*x)*
*(m + 5)*(2*A*a*c + A*b**2 + 2*C*a*b)/(d**5*(m + 5)) + (d*x)**(m
+ 7)*(2*A*b*c + 2*C*a*c + C*b**2)/(d**7*(m + 7))
```

Mathematica [A] time = 1.56496, size = 187, normalized size = 0.72

$$(dx)^m\left(\frac{a^2Ax}{m+1}+\frac{a^2Bx^2}{m+2}+\frac{x^7\left(2acC+2Abc+b^2C\right)}{m+7}+\frac{x^5\left(2aAc+2abC+Ab^2\right)}{m+5}+\frac{ax^3(aC+2Ab)}{m+3}+\frac{Bx^6\left(2ac+b^2\right)}{m+6}+\frac{2abBx^4}{m+4}+\frac{cx^9(Ac+2bC)}{m+9}+\frac{2bBcx^8}{m+8}+\frac{Bc^2x^{10}}{m+10}+\frac{c^2Cx^{11}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (d*x)^m*((a^2*A*x)/(1 + m) + (a^2*B*x^2)/(2 + m) + (a*(2*A*b + a*
C)*x^3)/(3 + m) + (2*a*b*B*x^4)/(4 + m) + ((A*b^2 + 2*a*A*c + 2*a
*b*C)*x^5)/(5 + m) + (B*(b^2 + 2*a*c)*x^6)/(6 + m) + ((2*A*b*c +
b^2*C + 2*a*c*C)*x^7)/(7 + m) + (2*b*B*c*x^8)/(8 + m) + (c*(A*c +
 2*b*C)*x^9)/(9 + m) + (B*c^2*x^10)/(10 + m) + (c^2*C*x^11)/(11 +
 m))
```

Maple [B] time = 0.013, size = 2187, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(C*c^2*m^10*x^10+B*c^2*m^10*x^9+55*C*c^2*m^9*x^10+A*c^2*m^10*x^
8+56*B*c^2*m^9*x^9+2*C*b*c*m^10*x^8+1320*C*c^2*m^8*x^10+57*A*c^2*
m^9*x^8+2*B*b*c*m^10*x^7+1365*B*c^2*m^8*x^9+114*C*b*c*m^9*x^8+181
50*C*c^2*m^7*x^10+2*A*b*c*m^10*x^6+1412*A*c^2*m^8*x^8+116*B*b*c*m
^9*x^7+19020*B*c^2*m^7*x^9+2*C*a*c*m^10*x^6+C*b^2*m^10*x^6+2824*C
*b*c*m^8*x^8+157773*C*c^2*m^6*x^10+118*A*b*c*m^9*x^6+19962*A*c^2*
m^7*x^8+2*B*a*c*m^10*x^5+B*b^2*m^10*x^5+2922*B*b*c*m^8*x^7+167223
*B*c^2*m^6*x^9+118*C*a*c*m^9*x^6+59*C*b^2*m^9*x^6+39924*C*b*c*m^7
*x^8+902055*C*c^2*m^5*x^10+2*A*a*c*m^10*x^4+A*b^2*m^10*x^4+3024*A
*b*c*m^8*x^6+177765*A*c^2*m^6*x^8+120*B*a*c*m^9*x^5+60*B*b^2*m^9*
x^5+41964*B*b*c*m^7*x^7+965328*B*c^2*m^5*x^9+2*C*a*b*m^10*x^4+302
4*C*a*c*m^8*x^6+1512*C*b^2*m^8*x^6+355530*C*b*c*m^6*x^8+3416930*C
*c^2*m^4*x^10+122*A*a*c*m^9*x^4+61*A*b^2*m^9*x^4+44172*A*b*c*m^7*
x^6+1037673*A*c^2*m^5*x^8+2*B*a*b*m^10*x^3+3130*B*a*c*m^8*x^5+156
5*B*b^2*m^8*x^5+379134*B*b*c*m^6*x^7+3686255*B*c^2*m^4*x^9+122*C*
a*b*m^9*x^4+44172*C*a*c*m^7*x^6+22086*C*b^2*m^7*x^6+2075346*C*b*c
*m^5*x^8+8409500*C*c^2*m^3*x^10+2*A*a*b*m^10*x^2+3240*A*a*c*m^8*x
^4+1620*A*b^2*m^8*x^4+405642*A*b*c*m^6*x^6+4000478*A*c^2*m^4*x^8+
124*B*a*b*m^9*x^3+46560*B*a*c*m^7*x^5+23280*B*b^2*m^7*x^5+2242044
*B*b*c*m^5*x^7+9133180*B*c^2*m^3*x^9+C*a^2*m^10*x^2+3240*C*a*b*m^
8*x^4+405642*C*a*c*m^6*x^6+202821*C*b^2*m^6*x^6+8000956*C*b*c*m^4
*x^8+12753576*C*c^2*m^2*x^10+126*A*a*b*m^9*x^2+49140*A*a*c*m^7*x^
4+24570*A*b^2*m^7*x^4+2435622*A*b*c*m^5*x^6+9991428*A*c^2*m^3*x^8
+B*a^2*m^10*x+3354*B*a*b*m^8*x^3+435486*B*a*c*m^6*x^5+217743*B*b^
2*m^6*x^5+8742718*B*b*c*m^4*x^7+13926276*B*c^2*m^2*x^9+63*C*a^2*m
^9*x^2+49140*C*a*b*m^7*x^4+2435622*C*a*c*m^5*x^6+1217811*C*b^2*m^
5*x^6+19982856*C*b*c*m^3*x^8+10628640*C*c^2*m*x^10+A*a^2*m^10+347
```

```
2*A*a*b*m^8*x^2+469146*A*a*c*m^6*x^4+234573*A*b^2*m^6*x^4+9629716
*A*b*c*m^4*x^6+15335224*A*c^2*m^2*x^8+64*B*a^2*m^9*x+51924*B*a*b*
m^7*x^3+2662200*B*a*c*m^5*x^5+1331100*B*b^2*m^5*x^5+22049716*B*b*
c*m^3*x^7+11655216*B*c^2*m*x^9+1736*C*a^2*m^8*x^2+469146*C*a*b*m^
6*x^4+9629716*C*a*c*m^4*x^6+4814858*C*b^2*m^4*x^6+30670448*C*b*c*
m^2*x^8+3628800*C*c^2*x^10+65*A*a^2*m^9+54924*A*a*b*m^7*x^2+29293
86*A*a*c*m^5*x^4+1464693*A*b^2*m^5*x^4+24583448*A*b*c*m^3*x^6+129
00960*A*c^2*m*x^8+1797*B*a^2*m^8*x+507150*B*a*b*m^6*x^3+10705870*
B*a*c*m^4*x^5+5352935*B*b^2*m^4*x^5+34118424*B*b*c*m^2*x^7+399168
0*B*c^2*x^9+27462*C*a^2*m^7*x^2+2929386*C*a*b*m^5*x^4+24583448*C*
a*c*m^3*x^6+12291724*C*b^2*m^3*x^6+25801920*C*b*c*m*x^8+1860*A*a^
2*m^8+550074*A*a*b*m^6*x^2+12032140*A*a*c*m^4*x^4+6016070*A*b^2*m
^4*x^4+38432016*A*b*c*m^2*x^6+4435200*A*c^2*x^8+29076*B*a^2*m^7*x
+3246516*B*a*b*m^5*x^3+27756240*B*a*c*m^3*x^5+13878120*B*b^2*m^3*
x^5+28888560*B*b*c*m*x^7+275037*C*a^2*m^6*x^2+12032140*C*a*b*m^4*
x^4+38432016*C*a*c*m^2*x^6+19216008*C*b^2*m^2*x^6+8870400*C*b*c*x
^8+30810*A*a^2*m^7+3624894*A*a*b*m^5*x^2+31830760*A*a*c*m^3*x^4+1
5915380*A*b^2*m^3*x^4+32811840*A*b*c*m*x^6+299271*B*a^2*m^6*x+136
93006*B*a*b*m^4*x^3+43978712*B*a*c*m^2*x^5+21989356*B*b^2*m^2*x^5
+9979200*B*b*c*x^7+1812447*C*a^2*m^5*x^2+31830760*C*a*b*m^3*x^4+3
2811840*C*a*c*m*x^6+16405920*C*b^2*m*x^6+326613*A*a^2*m^6+1580438
8*A*a*b*m^4*x^2+51362352*A*a*c*m^2*x^4+25681176*A*b^2*m^2*x^4+114
04800*A*b*c*x^6+2039016*B*a^2*m^5*x+37219436*B*a*b*m^3*x^3+379636
80*B*a*c*m*x^5+18981840*B*b^2*m*x^5+7902194*C*a^2*m^4*x^2+5136235
2*C*a*b*m^2*x^4+11404800*C*a*c*x^6+5702400*C*b^2*x^6+2310945*A*a^
2*m^5+44578296*A*a*b*m^3*x^2+45024192*A*a*c*m*x^4+22512096*A*b^2*
m*x^4+9261503*B*a^2*m^4*x+61638408*B*a*b*m^2*x^3+13305600*B*a*c*x
^5+6652800*B*b^2*x^5+22289148*C*a^2*m^3*x^2+45024192*C*a*b*m*x^4+
11028590*A*a^2*m^4+76781264*A*a*b*m^2*x^2+15966720*A*a*c*x^4+7983
360*A*b^2*x^4+27472724*B*a^2*m^3*x+55282320*B*a*b*m*x^3+38390632*
C*a^2*m^2*x^2+15966720*C*a*b*x^4+34967140*A*a^2*m^3+71492160*A*a*
b*m*x^2+50312628*B*a^2*m^2*x+19958400*B*a*b*x^3+35746080*C*a^2*m*
x^2+70290936*A*a^2*m^2+26611200*A*a*b*x^2+50292720*B*a^2*m*x+1330
5600*C*a^2*x^2+80627040*A*a^2*m+19958400*B*a^2*x+39916800*A*a^2)*
(d*x)^m/(11+m)/(10+m)/(9+m)/(8+m)/(7+m)/(6+m)/(5+m)/(4+m)/(3+m)/(
2+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.345777, size = 2164, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((C*c^2*m^10 + 55*C*c^2*m^9 + 1320*C*c^2*m^8 + 18150*C*c^2*m^7 +
157773*C*c^2*m^6 + 902055*C*c^2*m^5 + 3416930*C*c^2*m^4 + 8409500
*C*c^2*m^3 + 12753576*C*c^2*m^2 + 10628640*C*c^2*m + 3628800*C*c^
2)*x^11 + (B*c^2*m^10 + 56*B*c^2*m^9 + 1365*B*c^2*m^8 + 19020*B*c
^2*m^7 + 167223*B*c^2*m^6 + 965328*B*c^2*m^5 + 3686255*B*c^2*m^4
+ 9133180*B*c^2*m^3 + 13926276*B*c^2*m^2 + 11655216*B*c^2*m + 399
1680*B*c^2)*x^10 + ((2*C*b*c + A*c^2)*m^10 + 57*(2*C*b*c + A*c^2)
*m^9 + 1412*(2*C*b*c + A*c^2)*m^8 + 19962*(2*C*b*c + A*c^2)*m^7 +
```

```
 177765*(2*C*b*c + A*c^2)*m^6 + 1037673*(2*C*b*c + A*c^2)*m^5 + 4
000478*(2*C*b*c + A*c^2)*m^4 + 9991428*(2*C*b*c + A*c^2)*m^3 + 88
70400*C*b*c + 4435200*A*c^2 + 15335224*(2*C*b*c + A*c^2)*m^2 + 12
900960*(2*C*b*c + A*c^2)*m)*x^9 + 2*(B*b*c*m^10 + 58*B*b*c*m^9 +
1461*B*b*c*m^8 + 20982*B*b*c*m^7 + 189567*B*b*c*m^6 + 1121022*B*b
*c*m^5 + 4371359*B*b*c*m^4 + 11024858*B*b*c*m^3 + 17059212*B*b*c*
m^2 + 14444280*B*b*c*m + 4989600*B*b*c)*x^8 + ((C*b^2 + 2*(C*a +
A*b)*c)*m^10 + 59*(C*b^2 + 2*(C*a + A*b)*c)*m^9 + 1512*(C*b^2 + 2
*(C*a + A*b)*c)*m^8 + 22086*(C*b^2 + 2*(C*a + A*b)*c)*m^7 + 20282
1*(C*b^2 + 2*(C*a + A*b)*c)*m^6 + 1217811*(C*b^2 + 2*(C*a + A*b)*
c)*m^5 + 4814858*(C*b^2 + 2*(C*a + A*b)*c)*m^4 + 12291724*(C*b^2
+ 2*(C*a + A*b)*c)*m^3 + 5702400*C*b^2 + 19216008*(C*b^2 + 2*(C*a
 + A*b)*c)*m^2 + 11404800*(C*a + A*b)*c + 16405920*(C*b^2 + 2*(C*
a + A*b)*c)*m)*x^7 + ((B*b^2 + 2*B*a*c)*m^10 + 60*(B*b^2 + 2*B*a*
c)*m^9 + 1565*(B*b^2 + 2*B*a*c)*m^8 + 23280*(B*b^2 + 2*B*a*c)*m^7
 + 217743*(B*b^2 + 2*B*a*c)*m^6 + 1331100*(B*b^2 + 2*B*a*c)*m^5 +
 5352935*(B*b^2 + 2*B*a*c)*m^4 + 13878120*(B*b^2 + 2*B*a*c)*m^3 +
 6652800*B*b^2 + 13305600*B*a*c + 21989356*(B*b^2 + 2*B*a*c)*m^2
+ 18981840*(B*b^2 + 2*B*a*c)*m)*x^6 + ((2*C*a*b + A*b^2 + 2*A*a*c
)*m^10 + 61*(2*C*a*b + A*b^2 + 2*A*a*c)*m^9 + 1620*(2*C*a*b + A*b
^2 + 2*A*a*c)*m^8 + 24570*(2*C*a*b + A*b^2 + 2*A*a*c)*m^7 + 23457
3*(2*C*a*b + A*b^2 + 2*A*a*c)*m^6 + 1464693*(2*C*a*b + A*b^2 + 2*
A*a*c)*m^5 + 6016070*(2*C*a*b + A*b^2 + 2*A*a*c)*m^4 + 15915380*(
2*C*a*b + A*b^2 + 2*A*a*c)*m^3 + 15966720*C*a*b + 7983360*A*b^2 +
 15966720*A*a*c + 25681176*(2*C*a*b + A*b^2 + 2*A*a*c)*m^2 + 2251
2096*(2*C*a*b + A*b^2 + 2*A*a*c)*m)*x^5 + 2*(B*a*b*m^10 + 62*B*a*
b*m^9 + 1677*B*a*b*m^8 + 25962*B*a*b*m^7 + 253575*B*a*b*m^6 + 162
3258*B*a*b*m^5 + 6846503*B*a*b*m^4 + 18609718*B*a*b*m^3 + 3081920
4*B*a*b*m^2 + 27641160*B*a*b*m + 9979200*B*a*b)*x^4 + ((C*a^2 + 2
*A*a*b)*m^10 + 63*(C*a^2 + 2*A*a*b)*m^9 + 1736*(C*a^2 + 2*A*a*b)*
m^8 + 27462*(C*a^2 + 2*A*a*b)*m^7 + 275037*(C*a^2 + 2*A*a*b)*m^6
+ 1812447*(C*a^2 + 2*A*a*b)*m^5 + 7902194*(C*a^2 + 2*A*a*b)*m^4 +
 22289148*(C*a^2 + 2*A*a*b)*m^3 + 13305600*C*a^2 + 26611200*A*a*b
 + 38390632*(C*a^2 + 2*A*a*b)*m^2 + 35746080*(C*a^2 + 2*A*a*b)*m)
*x^3 + (B*a^2*m^10 + 64*B*a^2*m^9 + 1797*B*a^2*m^8 + 29076*B*a^2*
m^7 + 299271*B*a^2*m^6 + 2039016*B*a^2*m^5 + 9261503*B*a^2*m^4 +
27472724*B*a^2*m^3 + 50312628*B*a^2*m^2 + 50292720*B*a^2*m + 1995
8400*B*a^2)*x^2 + (A*a^2*m^10 + 65*A*a^2*m^9 + 1860*A*a^2*m^8 + 3
0810*A*a^2*m^7 + 326613*A*a^2*m^6 + 2310945*A*a^2*m^5 + 11028590*
A*a^2*m^4 + 34967140*A*a^2*m^3 + 70290936*A*a^2*m^2 + 80627040*A*
a^2*m + 39916800*A*a^2)*x)*(d*x)^m/(m^11 + 66*m^10 + 1925*m^9 + 3
2670*m^8 + 357423*m^7 + 2637558*m^6 + 13339535*m^5 + 45995730*m^4
 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 39916800)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.302109, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(C*x^2 + B*x + A)*(d*x)^m,x, algorithm="giac")
```

```
[Out] Done
```

3.39 $\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=137

$$\frac{(dx)^{m+3}(aC+Ab)}{d^3(m+3)}+\frac{aA(dx)^{m+1}}{d(m+1)}+\frac{aB(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}(Ac+bC)}{d^5(m+5)}+\frac{bB(dx)^{m+4}}{d^4(m+4)}+\frac{Bc(dx)^{m+6}}{d^6(m+6)}+\frac{cC(dx)^{m+7}}{d^7(m+7)}$$

```
[Out] (a*A*(d*x)^(1 + m))/(d*(1 + m)) + (a*B*(d*x)^(2 + m))/(d^2*(2 + m
)) + ((A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (b*B*(d*x)^(4 +
m))/(d^4*(4 + m)) + ((A*c + b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (
B*c*(d*x)^(6 + m))/(d^6*(6 + m)) + (c*C*(d*x)^(7 + m))/(d^7*(7 +
m))
```

Rubi [A] time = 0.198099, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{(dx)^{m+3}(aC+Ab)}{d^3(m+3)}+\frac{aA(dx)^{m+1}}{d(m+1)}+\frac{aB(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}(Ac+bC)}{d^5(m+5)}+\frac{bB(dx)^{m+4}}{d^4(m+4)}+\frac{Bc(dx)^{m+6}}{d^6(m+6)}+\frac{cC(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*(d*x)^(1 + m))/(d*(1 + m)) + (a*B*(d*x)^(2 + m))/(d^2*(2 + m
)) + ((A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (b*B*(d*x)^(4 +
m))/(d^4*(4 + m)) + ((A*c + b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (
B*c*(d*x)^(6 + m))/(d^6*(6 + m)) + (c*C*(d*x)^(7 + m))/(d^7*(7 +
m))
```

Rubi in Sympy [A] time = 39.764, size = 122, normalized size = 0.89

$$\frac{Aa\left(dx\right)^{m+1}}{d\left(m+1\right)}+\frac{Ba\left(dx\right)^{m+2}}{d^2\left(m+2\right)}+\frac{Bb\left(dx\right)^{m+4}}{d^4\left(m+4\right)}+\frac{Bc\left(dx\right)^{m+6}}{d^6\left(m+6\right)}+\frac{Cc\left(dx\right)^{m+7}}{d^7\left(m+7\right)}+\frac{\left(dx\right)^{m+3}\left(Ab+Ca\right)}{d^3\left(m+3\right)}+\frac{\left(dx\right)^{m+5}\left(Ac+Cb\right)}{d^5\left(m+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*(d*x)**(m + 1)/(d*(m + 1)) + B*a*(d*x)**(m + 2)/(d**2*(m + 2)
) + B*b*(d*x)**(m + 4)/(d**4*(m + 4)) + B*c*(d*x)**(m + 6)/(d**6*
(m + 6)) + C*c*(d*x)**(m + 7)/(d**7*(m + 7)) + (d*x)**(m + 3)*(A*
b + C*a)/(d**3*(m + 3)) + (d*x)**(m + 5)*(A*c + C*b)/(d**5*(m + 5
))
```

Mathematica [A] time = 0.248383, size = 92, normalized size = 0.67

$$(dx)^m\left(\frac{x^3(aC+Ab)}{m+3}+\frac{aAx}{m+1}+\frac{aBx^2}{m+2}+\frac{x^5(Ac+bC)}{m+5}+\frac{bBx^4}{m+4}+\frac{Bcx^6}{m+6}+\frac{cCx^7}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d*x)^m*((a*A*x)/(1 + m) + (a*B*x^2)/(2 + m) + ((A*b + a*C)*x^3)/
(3 + m) + (b*B*x^4)/(4 + m) + ((A*c + b*C)*x^5)/(5 + m) + (B*c*x^
```

```
6)/(6 + m) + (c*C*x^7)/(7 + m))
```

Maple [B] time = 0.007, size = 585, normalized size = 4.3

$$\frac{\left(Ccm^6x^6 + Bcm^6x^5 + 21\,Ccm^5x^6 + Acm^6x^4 + 22\,Bcm^5x^5 + Cbm^6x^4 + 175\,Ccm^4x^6 + 23\,Acm^5x^4 + Bbm^6x^3 + 190\,Bcm^4x^5 + 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(C*c*m^6*x^6+B*c*m^6*x^5+21*C*c*m^5*x^6+A*c*m^6*x^4+22*B*c*m^5*
x^5+C*b*m^6*x^4+175*C*c*m^4*x^6+23*A*c*m^5*x^4+B*b*m^6*x^3+190*B*
c*m^4*x^5+23*C*b*m^5*x^4+735*C*c*m^3*x^6+A*b*m^6*x^2+207*A*c*m^4*
x^4+24*B*b*m^5*x^3+820*B*c*m^3*x^5+C*a*m^6*x^2+207*C*b*m^4*x^4+16
24*C*c*m^2*x^6+25*A*b*m^5*x^2+925*A*c*m^3*x^4+B*a*m^6*x+226*B*b*m
^4*x^3+1849*B*c*m^2*x^5+25*C*a*m^5*x^2+925*C*b*m^3*x^4+1764*C*c*m
*x^6+A*a*m^6+247*A*b*m^4*x^2+2144*A*c*m^2*x^4+26*B*a*m^5*x+1056*B
*b*m^3*x^3+2038*B*c*m*x^5+247*C*a*m^4*x^2+2144*C*b*m^2*x^4+720*C*
c*x^6+27*A*a*m^5+1219*A*b*m^3*x^2+2412*A*c*m*x^4+270*B*a*m^4*x+25
45*B*b*m^2*x^3+840*B*c*x^5+1219*C*a*m^3*x^2+2412*C*b*m*x^4+295*A*
a*m^4+3112*A*b*m^2*x^2+1008*A*c*x^4+1420*B*a*m^3*x+2952*B*b*m*x^3
+3112*C*a*m^2*x^2+1008*C*b*x^4+1665*A*a*m^3+3796*A*b*m*x^2+3929*B
*a*m^2*x+1260*B*b*x^3+3796*C*a*m*x^2+5104*A*a*m^2+1680*A*b*x^2+52
74*B*a*m*x+1680*C*a*x^2+8028*A*a*m+2520*B*a*x+5040*A*a)*(d*x)^m/(
7+m)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.312039, size = 599, normalized size = 4.37

$$\frac{\left(\left(Ccm^6 + 21\,Ccm^5 + 175\,Ccm^4 + 735\,Ccm^3 + 1624\,Ccm^2 + 1764\,Ccm + 720\,Cc\right)x^7 + \left(Bcm^6 + 22\,Bcm^5 + 190\,Bcm^4 + 820\,Bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((C*c*m^6 + 21*C*c*m^5 + 175*C*c*m^4 + 735*C*c*m^3 + 1624*C*c*m^2
 + 1764*C*c*m + 720*C*c)*x^7 + (B*c*m^6 + 22*B*c*m^5 + 190*B*c*m^
4 + 820*B*c*m^3 + 1849*B*c*m^2 + 2038*B*c*m + 840*B*c)*x^6 + ((C*
b + A*c)*m^6 + 23*(C*b + A*c)*m^5 + 207*(C*b + A*c)*m^4 + 925*(C*
b + A*c)*m^3 + 2144*(C*b + A*c)*m^2 + 1008*C*b + 1008*A*c + 2412*
(C*b + A*c)*m)*x^5 + (B*b*m^6 + 24*B*b*m^5 + 226*B*b*m^4 + 1056*B
*b*m^3 + 2545*B*b*m^2 + 2952*B*b*m + 1260*B*b)*x^4 + ((C*a + A*b)
*m^6 + 25*(C*a + A*b)*m^5 + 247*(C*a + A*b)*m^4 + 1219*(C*a + A*b
)*m^3 + 3112*(C*a + A*b)*m^2 + 1680*C*a + 1680*A*b + 3796*(C*a +
A*b)*m)*x^3 + (B*a*m^6 + 26*B*a*m^5 + 270*B*a*m^4 + 1420*B*a*m^3
+ 3929*B*a*m^2 + 5274*B*a*m + 2520*B*a)*x^2 + (A*a*m^6 + 27*A*a*m
^5 + 295*A*a*m^4 + 1665*A*a*m^3 + 5104*A*a*m^2 + 8028*A*a*m + 504
```

```
0*A*a)*x)*(d*x)^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 +
 13132*m^2 + 13068*m + 5040)
```

Sympy [A] time = 7.63541, size = 3735, normalized size = 27.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-A*a/(6*x**6) - A*b/(4*x**4) - A*c/(2*x**2) - B*a/(5*
x**5) - B*b/(3*x**3) - B*c/x - C*a/(4*x**4) - C*b/(2*x**2) + C*c*
log(x))/d**7, Eq(m, -7)), ((-A*a/(5*x**5) - A*b/(3*x**3) - A*c/x
- B*a/(4*x**4) - B*b/(2*x**2) + B*c*log(x) - C*a/(3*x**3) - C*b/x
 + C*c*x)/d**6, Eq(m, -6)), ((-A*a/(4*x**4) - A*b/(2*x**2) + A*c*
log(x) - B*a/(3*x**3) - B*b/x + B*c*x - C*a/(2*x**2) + C*b*log(x)
 + C*c*x**2/2)/d**5, Eq(m, -5)), ((-A*a/(3*x**3) - A*b/x + A*c*x
- B*a/(2*x**2) + B*b*log(x) + B*c*x**2/2 - C*a/x + C*b*x + C*c*x*
*3/3)/d**4, Eq(m, -4)), ((-A*a/(2*x**2) + A*b*log(x) + A*c*x**2/2
 - B*a/x + B*b*x + B*c*x**3/3 + C*a*log(x) + C*b*x**2/2 + C*c*x**
4/4)/d**3, Eq(m, -3)), ((-A*a/x + A*b*x + A*c*x**3/3 + B*a*log(x)
 + B*b*x**2/2 + B*c*x**4/4 + C*a*x + C*b*x**3/3 + C*c*x**5/5)/d**
2, Eq(m, -2)), ((A*a*log(x) + A*b*x**2/2 + A*c*x**4/4 + B*a*x + B
*b*x**3/3 + B*c*x**5/5 + C*a*x**2/2 + C*b*x**4/4 + C*c*x**6/6)/d,
 Eq(m, -1)), (A*a*d**m*m**6*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1
960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 27*A*a*d**m
*m**5*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
 13132*m**2 + 13068*m + 5040) + 295*A*a*d**m*m**4*x*x**m/(m**7 +
28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
 + 5040) + 1665*A*a*d**m*m**3*x*x**m/(m**7 + 28*m**6 + 322*m**5 +
 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5104*A*a*
d**m*m**2*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m*
*3 + 13132*m**2 + 13068*m + 5040) + 8028*A*a*d**m*m*x*x**m/(m**7
+ 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068
*m + 5040) + 5040*A*a*d**m*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + A*b*d**m*m**
6*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
13132*m**2 + 13068*m + 5040) + 25*A*b*d**m*m**5*x**3*x**m/(m**7 +
 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*
m + 5040) + 247*A*b*d**m*m**4*x**3*x**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1219*A
*b*d**m*m**3*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6
769*m**3 + 13132*m**2 + 13068*m + 5040) + 3112*A*b*d**m*m**2*x**3
*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*
m**2 + 13068*m + 5040) + 3796*A*b*d**m*m*x**3*x**m/(m**7 + 28*m**
6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) + 1680*A*b*d**m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m*
*4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + A*c*d**m*m**6*x**
5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132
*m**2 + 13068*m + 5040) + 23*A*c*d**m*m**5*x**5*x**m/(m**7 + 28*m
**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040) + 207*A*c*d**m*m**4*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1
960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 925*A*c*d**
m*m**3*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m*
*3 + 13132*m**2 + 13068*m + 5040) + 2144*A*c*d**m*m**2*x**5*x**m/
(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
 13068*m + 5040) + 2412*A*c*d**m*m*x**5*x**m/(m**7 + 28*m**6 + 32
2*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1
008*A*c*d**m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6
769*m**3 + 13132*m**2 + 13068*m + 5040) + B*a*d**m*m**6*x**2*x**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
+ 13068*m + 5040) + 26*B*a*d**m*m**5*x**2*x**m/(m**7 + 28*m**6 +
322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) +
 270*B*a*d**m*m**4*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m*
*4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1420*B*a*d**m*m**
3*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
```

```
13132*m**2 + 13068*m + 5040) + 3929*B*a*d**m*m**2*x**2*x**m/(m**7
 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1306
8*m + 5040) + 5274*B*a*d**m*m*x**2*x**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2520*B
*a*d**m*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m
**3 + 13132*m**2 + 13068*m + 5040) + B*b*d**m*m**6*x**4*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 130
68*m + 5040) + 24*B*b*d**m*m**5*x**4*x**m/(m**7 + 28*m**6 + 322*m
**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 226*
B*b*d**m*m**4*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1056*B*b*d**m*m**3*x**
4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132
*m**2 + 13068*m + 5040) + 2545*B*b*d**m*m**2*x**4*x**m/(m**7 + 28
*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
 5040) + 2952*B*b*d**m*m*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1
960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1260*B*b*d*
*m*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
 13132*m**2 + 13068*m + 5040) + B*c*d**m*m**6*x**6*x**m/(m**7 + 2
8*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
+ 5040) + 22*B*c*d**m*m**5*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 +
 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 190*B*c*d
**m*m**4*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*
m**3 + 13132*m**2 + 13068*m + 5040) + 820*B*c*d**m*m**3*x**6*x**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
+ 13068*m + 5040) + 1849*B*c*d**m*m**2*x**6*x**m/(m**7 + 28*m**6
+ 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
 + 2038*B*c*d**m*m*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m*
*4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 840*B*c*d**m*x**6
*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*
m**2 + 13068*m + 5040) + C*a*d**m*m**6*x**3*x**m/(m**7 + 28*m**6
+ 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
 + 25*C*a*d**m*m**5*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m
**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 247*C*a*d**m*m**
4*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
13132*m**2 + 13068*m + 5040) + 1219*C*a*d**m*m**3*x**3*x**m/(m**7
 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1306
8*m + 5040) + 3112*C*a*d**m*m**2*x**3*x**m/(m**7 + 28*m**6 + 322*
m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 379
6*C*a*d**m*m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6
769*m**3 + 13132*m**2 + 13068*m + 5040) + 1680*C*a*d**m*x**3*x**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
+ 13068*m + 5040) + C*b*d**m*m**6*x**5*x**m/(m**7 + 28*m**6 + 322
*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 23
*C*b*d**m*m**5*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 207*C*b*d**m*m**4*x**
5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132
*m**2 + 13068*m + 5040) + 925*C*b*d**m*m**3*x**5*x**m/(m**7 + 28*
m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
5040) + 2144*C*b*d**m*m**2*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 +
 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2412*C*b*
d**m*m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m*
*3 + 13132*m**2 + 13068*m + 5040) + 1008*C*b*d**m*x**5*x**m/(m**7
 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1306
8*m + 5040) + C*c*d**m*m**6*x**7*x**m/(m**7 + 28*m**6 + 322*m**5
+ 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 21*C*c*d
**m*m**5*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*
m**3 + 13132*m**2 + 13068*m + 5040) + 175*C*c*d**m*m**4*x**7*x**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
+ 13068*m + 5040) + 735*C*c*d**m*m**3*x**7*x**m/(m**7 + 28*m**6 +
 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
+ 1624*C*c*d**m*m**2*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*
m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1764*C*c*d**m*m
*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 1
3132*m**2 + 13068*m + 5040) + 720*C*c*d**m*x**7*x**m/(m**7 + 28*m
**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040), True))
```

GIAC/XCAS [A] time = 0.296954, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(C*x^2 + B*x + A)*(d*x)^m,x, algorithm="giac")
```

```
[Out] Done
```

3.40 $\int \frac{(dx)^m (A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=368

$$\frac{(dx)^{m+1}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(dx)^{m+1}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\left(\sqrt{b^2-4ac}+b\right)}+\frac{2Bc(dx)^{m+2}{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d^2(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2Bc(dx)^{m+2}{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d^2(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometr
ic2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
])/((b - Sqrt[b^2 - 4*a*c])*d*(1 + m)) + ((C - (2*A*c - b*C)/Sqrt
[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4*a*
c])*d*(1 + m)) + (2*B*c*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m
)/2, (4 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 -
4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d^2*(2 + m)) - (2*B*c*(d*x)^(2 + m
)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d^2*(
2 + m))
```

Rubi [A] time = 1.48125, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{(dx)^{m+1}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(dx)^{m+1}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\left(\sqrt{b^2-4ac}+b\right)}+\frac{2Bc(dx)^{m+2}{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d^2(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2Bc(dx)^{m+2}{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d^2(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometr
ic2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
])/((b - Sqrt[b^2 - 4*a*c])*d*(1 + m)) + ((C - (2*A*c - b*C)/Sqrt
[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4*a*
c])*d*(1 + m)) + (2*B*c*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m
)/2, (4 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 -
4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d^2*(2 + m)) - (2*B*c*(d*x)^(2 + m
)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d^2*(
2 + m))
```

Rubi in Sympy [A] time = 117.687, size = 335, normalized size = 0.91

$$-\frac{2Bc\,(dx)^{m+2}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{d^2\left(b+\sqrt{-4ac+b^2}\right)(m+2)\sqrt{-4ac+b^2}}+\frac{2Bc\,(dx)^{m+2}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|-\frac{2cx^2}{b-\sqrt{-4ac+b^2}}\right)}{d^2\left(b-\sqrt{-4ac+b^2}\right)(m+2)\sqrt{-4ac+b^2}}-\frac{(dx)^{m+1}\left(2Ac-Cb-C\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{d\left(b+\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}+\frac{(dx)^{m+1}\left(2Ac-Cb+C\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{2cx^2}{b-\sqrt{-4ac+b^2}}\right)}{d\left(b-\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] -2*B*c*(d*x)**(m + 2)*hyper((1, m/2 + 1), (m/2 + 2,), -2*c*x**2/(
b + sqrt(-4*a*c + b**2)))/(d**2*(b + sqrt(-4*a*c + b**2))*(m + 2)
*sqrt(-4*a*c + b**2)) + 2*B*c*(d*x)**(m + 2)*hyper((1, m/2 + 1),
(m/2 + 2,), -2*c*x**2/(b - sqrt(-4*a*c + b**2)))/(d**2*(b - sqrt(
-4*a*c + b**2))*(m + 2)*sqrt(-4*a*c + b**2)) - (d*x)**(m + 1)*(2*
A*c - C*b - C*sqrt(-4*a*c + b**2))*hyper((1, m/2 + 1/2), (m/2 + 3
/2,), -2*c*x**2/(b + sqrt(-4*a*c + b**2)))/(d*(b + sqrt(-4*a*c +
b**2))*(m + 1)*sqrt(-4*a*c + b**2)) + (d*x)**(m + 1)*(2*A*c - C*b
 + C*sqrt(-4*a*c + b**2))*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -2*
c*x**2/(b - sqrt(-4*a*c + b**2)))/(d*(b - sqrt(-4*a*c + b**2))*(m
 + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.428581, size = 438, normalized size = 1.19

$$(dx)^m\left(A\left(m^2+3m+2\right)\text{RootSum}\left[\#1^4c+\#1^2b+a\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^3c+\#1b}\&\right]+B(m+2)\text{RootSum}\left[\#1^4c+\#1^2b\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In]   Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d*x)^m*(A*(2 + 3*m + m^2)*RootSum[a + b*#1^2 + c*#1^4 & , Hyper
geometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1
 + 2*c*#1^3)) & ] + B*(2 + m)*RootSum[a + b*#1^2 + c*#1^4 & , (m*
x + (Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1)/(x/(x -
 #1))^m + (m*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1)
/(x/(x - #1))^m)/(b*#1 + 2*c*#1^3) & ] + C*RootSum[a + b*#1^2 + c
*#1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeomet
ric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m
*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #
1))^m + (m^2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^
2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(b*#1 + 2*c*#1^3) & ]))/(2
*m*(1 + m)*(2 + m))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int\frac{(dx)^m\left(Cx^2+Bx+A\right)}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)(dx)^m}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)(dx)^m}{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d*x)**m*(A + B*x + C*x**2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)(dx)^m}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.41 $\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=685

$$\frac{c(dx)^{m+1}\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$
$$-\frac{c(dx)^{m+1}\left(A\left(-b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left((1-m)\sqrt{b^2-4ac}+2b\right)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{(dx)^{m+1}\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$-\frac{Bc(dx)^{m+2}\left(bm\left(\sqrt{b^2-4ac}+b\right)+4ac(2-m)\right){}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad^2(m+2)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$
$$+\frac{Bc(dx)^{m+2}\left(bm\left(b-\sqrt{b^2-4ac}\right)+4ac(2-m)\right){}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2ad^2(m+2)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{B(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (B*(d*x)^(2 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d^2*
(a + b*x^2 + c*x^4)) + ((d*x)^(1 + m)*(A*(b^2 - 2*a*c) - a*b*C +
c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c*x^4)) +
 (c*(2*a*C*(2*b - Sqrt[b^2 - 4*a*c]*(1 - m)) + A*(b^2*(1 - m) + b
*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m)))*(d*x)^(1 + m)*Hyperg
eometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 +
m)) - (c*(2*a*C*(2*b + Sqrt[b^2 - 4*a*c]*(1 - m)) + A*(b^2*(1 - m
) - b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m)))*(d*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*
(1 + m)) - (B*c*(4*a*c*(2 - m) + b*(b + Sqrt[b^2 - 4*a*c])*m)*(d*
x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2
- 4*a*c])*d^2*(2 + m)) + (B*c*(4*a*c*(2 - m) + b*(b - Sqrt[b^2 -
4*a*c])*m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*
(b + Sqrt[b^2 - 4*a*c])*d^2*(2 + m))
```

Rubi [A] time = 5.21198, antiderivative size = 670, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{c(dx)^{m+1}\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$
$$-\frac{c(dx)^{m+1}\left(-(1-m)\sqrt{b^2-4ac}(Ab-2aC)-4aAc(3-m)+4abC+Ab^2(1-m)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{(dx)^{m+1}\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$-\frac{Bc(dx)^{m+2}\left(bm\left(\sqrt{b^2-4ac}+b\right)+4ac(2-m)\right){}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad^2(m+2)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$
$$+\frac{Bc(dx)^{m+2}\left(bm\left(b-\sqrt{b^2-4ac}\right)+4ac(2-m)\right){}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2ad^2(m+2)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}$$
$$+\frac{B(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(d*x)^(2 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d^2*
(a + b*x^2 + c*x^4)) + ((d*x)^(1 + m)*(A*(b^2 - 2*a*c) - a*b*C +
c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c*x^4)) +
 (c*(2*a*C*(2*b - Sqrt[b^2 - 4*a*c]*(1 - m)) + A*(b^2*(1 - m) + b
*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m)))*(d*x)^(1 + m)*Hyperg
eometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 +
m)) - (c*(4*a*b*C + A*b^2*(1 - m) - Sqrt[b^2 - 4*a*c]*(A*b - 2*a*
C)*(1 - m) - 4*a*A*c*(3 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*(
b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m)) - (B*c*(4*a
*c*(2 - m) + b*(b + Sqrt[b^2 - 4*a*c])*m)*(d*x)^(2 + m)*Hypergeom
etric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d^2*(2 + m
)) + (B*c*(4*a*c*(2 - m) + b*(b - Sqrt[b^2 - 4*a*c])*m)*(d*x)^(2
+ m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*
c])*d^2*(2 + m))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 9.0187, size = 1136, normalized size = 1.66

$$\frac{aA(m+3)x\left(2cx^2+b-\sqrt{b^2-4ac}\right)\left(2cx^2+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{m+1}{2};2,2\ldots\right.}{4c(m+1)\left(cx^4+bx^2+a\right)^3\left(a(m+3)F_1\left(\frac{m+1}{2};2,2;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-2x^2\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{m+3}{2};2,3;\frac{m+5}{2};-\ldots\right.\right.\right.}+\frac{aB(m+4)x^2\left(2cx^2+b-\sqrt{b^2-4ac}\right)\left(2cx^2+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{m+2}{2};2\ldots\right.}{4c(m+2)\left(cx^4+bx^2+a\right)^3\left(a(m+4)F_1\left(\frac{m+2}{2};2,2;\frac{m+4}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-2x^2\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{m+4}{2};2,3;\frac{m+6}{2};\ldots\right.\right.\right.}+\frac{aC(m+5)x^3\left(2cx^2+b-\sqrt{b^2-4ac}\right)\left(2cx^2+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{m+3}{2};2\ldots\right.}{4c(m+3)\left(cx^4+bx^2+a\right)^3\left(a(m+5)F_1\left(\frac{m+3}{2};2,2;\frac{m+5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-2x^2\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{m+5}{2};2,3;\frac{m+7}{2};\ldots\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*A*(3 + m)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])])/(4*c*(1 + m)*(a + b*x^2 + c*x^4)^3*(a*(3 + m)*AppellF1[(1 +
 m)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 2*x^2*((b + Sqrt[b^2 - 4*a*c])*A
ppellF1[(3 + m)/2, 2, 3, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[(3 + m)/2, 3, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (a*B*(4 + m)
*x^2*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)*AppellF1[(2 + m)/2, 2, 2, (4 + m)/2, (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(4*c
*(2 + m)*(a + b*x^2 + c*x^4)^3*(a*(4 + m)*AppellF1[(2 + m)/2, 2,
2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] - 2*x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(4
 + m)/2, 2, 3, (6 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appell
F1[(4 + m)/2, 3, 2, (6 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (a*C*(5 + m)*x^3*(d*x)
^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)*AppellF1[(3 + m)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(4*c*(3 + m)*(
a + b*x^2 + c*x^4)^3*(a*(5 + m)*AppellF1[(3 + m)/2, 2, 2, (5 + m)
/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])] - 2*x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(5 + m)/2, 2
, 3, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(5 + m)
/2, 3, 2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{(dx)^m\left(Cx^2+Bx+A\right)}{\left(cx^4+bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Cx^2 + Bx + A)(dx)^m}{(cx^4 + bx^2 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)(dx)^m}{c^2x^8 + 2\,bcx^6 + (b^2 + 2\,ac)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 +
2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.42 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 2.02407, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.1013, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}+4ac+b^2\right)-2Ac\left(\sqrt{b^2-4ac}+2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2bB\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{2bB\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 -
4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 -
 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqr
t[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[
b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0., size = 4063, normalized size = 11.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -4/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1
/2)*b/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-
(4*a*c-b^2)^3)^(1/2))*a*c^2+(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*
b*B/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-a*B/(4*a*c-b^2)
)/(c*x^4+b*x^2+a)-4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c
-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*
x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1
/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+
1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3
+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c
^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a
*c-b^2)*c)^(1/2))*C*b^7+32/(4*a*c-b^2)^3*c^4*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^
2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
```

```
^(1/2)))^(1/2))*A*a^3+c/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*b^3/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x
^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(4*a*c-b^2)^3*
2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2
)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-
b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^6-1/4/(4*a*c-b^2)^3*2^(1/
2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arc
tan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a
*b^4*c+b^6)^(1/2)*b^4-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^
(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)
*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*a^2*c^2-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*
a*c-b^2)*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b/(-
8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*a*c^2-24/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c
^3*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^
(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*a^2*b^2+6/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a*b^4-16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^3*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*
arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^3*b-3*c/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1
/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*
a*b^5-c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c
+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8
*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*
(4*a*c-b^2)*c)^(1/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^
6)^(1/2)*b^3+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arc
tanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2
)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^2*b^3+4/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/(
(-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*(-6
4*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+1/4/(4*a*c
-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(
4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*b^7-24/(4*a*c-b^2)^
3*c^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*
a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*a^2*b^2+32/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^4*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a^3+1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/
((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arcta
nh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^
3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*
a*b^4*c+b^6)^(1/2)*b^4+6/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*A*a*b^4-16/(4*a*c-b^2)^3*c^3*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^3*b+12/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^2*b^3+4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4
*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b
^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*a^
2*c^2+c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*B*(-64*a^3*c^3+48*
a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3/(-8*a*c^2+2*b^2*c)*ln(-8*x^
2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a
```

```
*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^
2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)
*c)^(1/2))*A*b^6-3*c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*
c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)
*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(
1/2))*C*a*b^5+c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))
*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\,((b^2c - 4\,ac^2)x^4 + ab^2 - 4\,a^2c + (b^3 - 4\,abc)x^2)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\,(b^2 - 4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2
*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*
integrate(-(2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b
*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2 + B*x + A)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.43 $\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.19751, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(C*x**3+B*x**2+A*x)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.281284, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}+4ac+b^2\right)-2Ac\left(\sqrt{b^2-4ac}+2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2bB\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{2bB\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 -
4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 -
 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqr
t[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[
b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.007, size = 4063, normalized size = 11.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -4/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1
/2)*b/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-
(4*a*c-b^2)^3)^(1/2))*a*c^2+(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*
b*B/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-a*B/(4*a*c-b^2)
)/(c*x^4+b*x^2+a)-4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c
-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*
x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1
/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+
1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3
+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c
^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a
*c-b^2)*c)^(1/2))*C*b^7+32/(4*a*c-b^2)^3*c^4*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^
2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
```

```
^(1/2)))^(1/2))*A*a^3+c/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*b^3/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x
^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(4*a*c-b^2)^3*
2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2
)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-
b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^6-1/4/(4*a*c-b^2)^3*2^(1/
2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arc
tan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a
*b^4*c+b^6)^(1/2)*b^4-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^
(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)
*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*a^2*c^2-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*
a*c-b^2)*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b/(-
8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*a*c^2-24/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c
^3*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^
(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*a^2*b^2+6/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a*b^4-16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^3*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*
arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^3*b-3*c/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1
/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*
a*b^5-c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c
+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8
*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*
(4*a*c-b^2)*c)^(1/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^
6)^(1/2)*b^3+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arc
tanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2
)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^2*b^3+4/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/(
(-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*(-6
4*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+1/4/(4*a*c
-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(
4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*b^7-24/(4*a*c-b^2)^
3*c^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*
a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*a^2*b^2+32/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^4*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a^3+1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/
((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arcta
nh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^
3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*
a*b^4*c+b^6)^(1/2)*b^4+6/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*A*a*b^4-16/(4*a*c-b^2)^3*c^3*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^3*b+12/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^2*b^3+4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4
*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b
^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*a^
2*c^2+c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*B*(-64*a^3*c^3+48*
a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3/(-8*a*c^2+2*b^2*c)*ln(-8*x^
2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a
```

```
*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^
2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)
*c)^(1/2))*A*b^6-3*c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*
c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)
*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(
1/2))*C*a*b^5+c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))
*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\,((b^2c - 4\,ac^2)x^4 + ab^2 - 4\,a^2c + (b^3 - 4\,abc)x^2)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\,(b^2 - 4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^3 + B*x^2 + A*x)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2
*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*
integrate(-(2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b
*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^3 + B*x^2 + A*x)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(C*x**3+B*x**2+A*x)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^3 + B*x^2 + A*x)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.44 $\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.10976, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**4+B*x**3+A*x**2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.272812, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{(b^2-4ac)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}+4ac+b^2\right)-2Ac\left(\sqrt{b^2-4ac}+2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2bB\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{2bB\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 -
4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 -
 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqr
t[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[
b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.007, size = 4063, normalized size = 11.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -4/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1
/2)*b/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-
(4*a*c-b^2)^3)^(1/2))*a*c^2+(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*
b*B/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-a*B/(4*a*c-b^2)
)/(c*x^4+b*x^2+a)-4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c
-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*
x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1
/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+
1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3
+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c
^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a
*c-b^2)*c)^(1/2))*C*b^7+32/(4*a*c-b^2)^3*c^4*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^
2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
```

^(1/2)))^(1/2))*A*a^3+c/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*b^3/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x
^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(4*a*c-b^2)^3*
2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2
)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-
b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^6-1/4/(4*a*c-b^2)^3*2^(1/
2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arc
tan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a
*b^4*c+b^6)^(1/2)*b^4-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^
(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)
arctanh(1/2(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*a^2*c^2-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*
a*c-b^2)*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b/(-
8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*a*c^2-24/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c
^3*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^
(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*a^2*b^2+6/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a*b^4-16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^3*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*
arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^3*b-3*c/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1
/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*
a*b^5-c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c
+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8
*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*
(4*a*c-b^2)*c)^(1/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^
6)^(1/2)*b^3+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arc
tanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2
)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^2*b^3+4/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/(
(-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*(-6
4*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+1/4/(4*a*c
-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(
4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*b^7-24/(4*a*c-b^2)^
3*c^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*
a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*a^2*b^2+32/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^4*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a^3+1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/
((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arcta
nh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^
3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*
a*b^4*c+b^6)^(1/2)*b^4+6/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*A*a*b^4-16/(4*a*c-b^2)^3*c^3*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^3*b+12/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^2*b^3+4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4
*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b
^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*a^
2*c^2+c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*B*(-64*a^3*c^3+48*
a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3/(-8*a*c^2+2*b^2*c)*ln(-8*x^
2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a

```
*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^
2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)
*c)^(1/2))*A*b^6-3*c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*
c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)
*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(
1/2))*C*a*b^5+c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))
*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\,((b^2c - 4\,ac^2)x^4 + ab^2 - 4\,a^2c + (b^3 - 4\,abc)x^2)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\,(b^2 - 4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4 + B*x^3 + A*x^2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2
*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*
integrate(-(2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b
*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4 + B*x^3 + A*x^2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**4+B*x**3+A*x**2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4 + B*x^3 + A*x^2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.45 $\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.11942, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**5+B*x**4+A*x**3)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.269288, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}+4ac+b^2\right)-2Ac\left(\sqrt{b^2-4ac}+2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2bB\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{2bB\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 -
4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 -
 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqr
t[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[
b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.007, size = 4063, normalized size = 11.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -4/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1
/2)*b/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-
(4*a*c-b^2)^3)^(1/2))*a*c^2+(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*
b*B/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-a*B/(4*a*c-b^2)
)/(c*x^4+b*x^2+a)-4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c
-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*
x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1
/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+
1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3
+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c
^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a
*c-b^2)*c)^(1/2))*C*b^7+32/(4*a*c-b^2)^3*c^4*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^
2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
```

```
^(1/2)))^(1/2))*A*a^3+c/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*b^3/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x
^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(4*a*c-b^2)^3*
2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2
)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-
b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^6-1/4/(4*a*c-b^2)^3*2^(1/
2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arc
tan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a
*b^4*c+b^6)^(1/2)*b^4-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^
(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)
*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*a^2*c^2-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*
a*c-b^2)*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b/(-
8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*a*c^2-24/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c
^3*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^
(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*a^2*b^2+6/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a*b^4-16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^3*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*
arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^3*b-3*c/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1
/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*
a*b^5-c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c
+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8
*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*
(4*a*c-b^2)*c)^(1/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^
6)^(1/2)*b^3+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arc
tanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2
)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^2*b^3+4/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/(
(-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*(-6
4*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+1/4/(4*a*c
-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(
4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*b^7-24/(4*a*c-b^2)^
3*c^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*
a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*a^2*b^2+32/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^4*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a^3+1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/
((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arcta
nh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^
3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*
a*b^4*c+b^6)^(1/2)*b^4+6/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*A*a*b^4-16/(4*a*c-b^2)^3*c^3*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^3*b+12/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^2*b^3+4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4
*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b
^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*a^
2*c^2+c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*B*(-64*a^3*c^3+48*
a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3/(-8*a*c^2+2*b^2*c)*ln(-8*x^
2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a
```

```
*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^
2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)
*c)^(1/2))*A*b^6-3*c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*
c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)
*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(
1/2))*C*a*b^5+c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))
*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2Ac)x^3 + 2Ba + (2Ca - Ab)x}{2\left((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4abc)x^2\right)} - \frac{-\int \frac{2Bbx + (Cb - 2Ac)x^2 - 2Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5 + B*x^4 + A*x^3)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="maxima"
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2
*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*
integrate(-(2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b
*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5 + B*x^4 + A*x^3)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="fricas"
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**5+B*x**4+A*x**3)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5 + B*x^4 + A*x^3)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.46 $\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.23333, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b
 - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*A
*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((C*x**6+B*x**5+A*x**4)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.268409, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}+4ac+b^2\right)-2Ac\left(\sqrt{b^2-4ac}+2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2bB\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 -
4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 -
 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqr
t[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[
b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.007, size = 4063, normalized size = 11.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -4/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1
/2)*b/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x^2*b^2*c+4*a*b*c-b^3+(-
(4*a*c-b^2)^3)^(1/2))*a*c^2+(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*
b*B/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-a*B/(4*a*c-b^2)
)/(c*x^4+b*x^2+a)-4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c
-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*
x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1
/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+
1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3
+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c
^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a
*c-b^2)*c)^(1/2))*C*b^7+32/(4*a*c-b^2)^3*c^4*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^
2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
```

```
^(1/2)))^(1/2))*A*a^3+c/(4*a*c-b^2)^3*B*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*b^3/(8*a*c^2-2*b^2*c)*ln(8*x^2*a*c^2-2*x
^2*b^2*c+4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(4*a*c-b^2)^3*
2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2
)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-
b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^6-1/4/(4*a*c-b^2)^3*2^(1/
2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arc
tan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a
*b^4*c+b^6)^(1/2)*b^4-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^
(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)
*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c
^2-12*a*b^4*c+b^6)^(1/2)*a^2*c^2-4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*
a*c-b^2)*B*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b/(-
8*a*c^2+2*b^2*c)*ln(-8*x^2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*a*c^2-24/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c
^3*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^
(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*a^2*b^2+6/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a*b^4-16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^3*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*
arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^3*b-3*c/(16*a^2*c^2-8*a*
b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1
/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*
a*b^5-c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c
+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8
*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*
(4*a*c-b^2)*c)^(1/2))*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^
6)^(1/2)*b^3+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^2*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arc
tanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2
)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*a^2*b^3+4/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/(
(-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*(-6
4*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b*a*c^2+1/4/(4*a*c
-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(
4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*C*b^7-24/(4*a*c-b^2)^
3*c^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*
a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*a^2*b^2+32/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*c^4*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*A*a^3+1/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/
((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arcta
nh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^
3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*
a*b^4*c+b^6)^(1/2)*b^4+6/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*A*a*b^4-16/(4*a*c-b^2)^3*c^3*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^3*b+12/(4*a*c-b^2)^3*c^2*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*C*a^2*b^3+4/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4
*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b
^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2))*C*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*a^
2*c^2+c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*B*(-64*a^3*c^3+48*
a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3/(-8*a*c^2+2*b^2*c)*ln(-8*x^
2*a*c^2+2*x^2*b^2*c-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))-1/2*c/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a
```

```
*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^
2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)
*c)^(1/2))*A*b^6-3*c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*
c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)
*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(
1/2))*C*a*b^5+c/(4*a*c-b^2)^3*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))
*A*(-64*a^3*c^3+48*a^2*b^2*c^2-12*a*b^4*c+b^6)^(1/2)*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\,((b^2c - 4\,ac^2)x^4 + ab^2 - 4\,a^2c + (b^3 - 4\,abc)x^2)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\,(b^2 - 4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6 + B*x^5 + A*x^4)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="maxim
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2
*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*
integrate(-(2*B*b*x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b
*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6 + B*x^5 + A*x^4)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="frica
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((C*x**6+B*x**5+A*x**4)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6 + B*x^5 + A*x^4)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="giac"
```

```
[Out] Exception raised: TypeError
```

3.47 $\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=273

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e-b^3c(cd-5af)-4ab^2c^2e+abc^2(3cd-5af)+b^5(-f)+b^4ce\right)}{2c^5\sqrt{b^2-4ac}}+\frac{x^4\left(-c(af+be)+b^2f+c^2d\right)}{4c^3}+\frac{x^2\left(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{2c^4}-\frac{\log\left(a+bx^2+cx^4\right)\left(-b^2c(cd-3af)-2abc^2e+ac^2(cd-af)+b^4(-f)+b^3ce\right)}{4c^5}+\frac{x^6(ce-bf)}{6c^2}+\frac{fx^8}{8c}$$

```
[Out] ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*x^2)/(2*c^4) + (
(c^2*d + b^2*f - c*(b*e + a*f))*x^4)/(4*c^3) + ((c*e - b*f)*x^6)/
(6*c^2) + (f*x^8)/(8*c) - ((b^4*c*e - 4*a*b^2*c^2*e + 2*a^2*c^3*e
 - b^5*f - b^3*c*(c*d - 5*a*f) + a*b*c^2*(3*c*d - 5*a*f))*ArcTanh
[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^5*Sqrt[b^2 - 4*a*c]) - ((
b^3*c*e - 2*a*b*c^2*e - b^4*f - b^2*c*(c*d - 3*a*f) + a*c^2*(c*d
- a*f))*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Rubi [A] time = 1.63638, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e-b^3c(cd-5af)-4ab^2c^2e+abc^2(3cd-5af)+b^5(-f)+b^4ce\right)}{2c^5\sqrt{b^2-4ac}}+\frac{x^4\left(-c(af+be)+b^2f+c^2d\right)}{4c^3}+\frac{x^2\left(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{2c^4}-\frac{\log\left(a+bx^2+cx^4\right)\left(-b^2c(cd-3af)-2abc^2e+ac^2(cd-af)+b^4(-f)+b^3ce\right)}{4c^5}+\frac{x^6(ce-bf)}{6c^2}+\frac{fx^8}{8c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```


```
[Out] ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*x^2)/(2*c^4) + (
(c^2*d + b^2*f - c*(b*e + a*f))*x^4)/(4*c^3) + ((c*e - b*f)*x^6)/
(6*c^2) + (f*x^8)/(8*c) - ((b^4*c*e - 4*a*b^2*c^2*e + 2*a^2*c^3*e
 - b^5*f - b^3*c*(c*d - 5*a*f) + a*b*c^2*(3*c*d - 5*a*f))*ArcTanh
[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^5*Sqrt[b^2 - 4*a*c]) - ((
b^3*c*e - 2*a*b*c^2*e - b^4*f - b^2*c*(c*d - 3*a*f) + a*c^2*(c*d
- a*f))*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.393585, size = 260, normalized size = 0.95

$$-\frac{12\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-2a^2c^3e+b^3c(cd-5af)+4ab^2c^2e+abc^2(5af-3cd)+b^5f-b^4ce\right)}{\sqrt{4ac-b^2}}+6c^2x^4\left(-c(af+be)+b^2f+c^2d\right)-12cx^2\left(bc(cd-2a\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-12*c*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*x^2 + 6
*c^2*(c^2*d + b^2*f - c*(b*e + a*f))*x^4 + 4*c^3*(c*e - b*f)*x^6
+ 3*c^4*f*x^8 - (12*(-(b^4*c*e) + 4*a*b^2*c^2*e - 2*a^2*c^3*e + b
^5*f + b^3*c*(c*d - 5*a*f) + a*b*c^2*(-3*c*d + 5*a*f))*ArcTan[(b
+ 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 6*(-(b^3*c*e
) + 2*a*b*c^2*e + b^4*f + b^2*c*(c*d - 3*a*f) + a*c^2*(-(c*d) + a
*f))*Log[a + b*x^2 + c*x^4])/(24*c^5)
```

Maple [B] time = 0.009, size = 622, normalized size = 2.3

$$\frac{\ln\left(cx^4+bx^2+a\right)abe}{2c^3}-\frac{x^6bf}{6c^2}-\frac{x^4af}{4c^2}-\frac{bx^2d}{2c^2}-\frac{bex^4}{4c^2}+\frac{b^2x^4f}{4c^3}+\frac{b^2ex^2}{2c^3}$$
$$-\frac{b^3fx^2}{2c^4}-\frac{aex^2}{2c^2}+\frac{\ln\left(cx^4+bx^2+a\right)b^2d}{4c^3}+\frac{\ln\left(cx^4+bx^2+a\right)a^2f}{4c^3}$$
$$-\frac{\ln\left(cx^4+bx^2+a\right)ad}{4c^2}+\frac{\ln\left(cx^4+bx^2+a\right)b^4f}{4c^5}-\frac{\ln\left(cx^4+bx^2+a\right)b^3e}{4c^4}$$
$$-\frac{3\ln\left(cx^4+bx^2+a\right)ab^2f}{4c^4}+\frac{fx^8}{8c}+\frac{3abd}{2c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$+\frac{5ab^3f}{2c^4}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-2\frac{ab^2e}{c^3\sqrt{4ac-b^2}}\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)$$
$$+\frac{abfx^2}{c^3}-\frac{b^3d}{2c^3}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$+\frac{a^2e}{c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$-\frac{b^5f}{2c^5}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$+\frac{b^4e}{2c^4}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$-\frac{5a^2bf}{2c^3}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{x^6e}{6c}+\frac{dx^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/c^3*ln(c*x^4+b*x^2+a)*a*b*e-1/6/c^2*x^6*b*f-1/4/c^2*x^4*a*f-1
/2/c^2*x^2*b*d-1/4/c^2*x^4*b*e+1/4/c^3*x^4*b^2*f+1/2/c^3*b^2*e*x^
2-1/2/c^4*b^3*f*x^2-1/2/c^2*x^2*a*e+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2
*d+1/4/c^3*ln(c*x^4+b*x^2+a)*a^2*f-1/4/c^2*ln(c*x^4+b*x^2+a)*a*d+
1/4/c^5*ln(c*x^4+b*x^2+a)*b^4*f-1/4/c^4*ln(c*x^4+b*x^2+a)*b^3*e-3
/4/c^4*ln(c*x^4+b*x^2+a)*a*b^2*f+1/8*f*x^8/c+3/2/c^2/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*d+5/2/c^4/(4*a*c-
b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^3*f-2/c^3/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*e+1/c
^3*a*b*f*x^2-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-
b^2)^(1/2))*b^3*d+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2))*e*a^2-1/2/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*b^5*f+1/2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*b^4*e-5/2/c^3/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b*f+1/6/c*x^6*e+1/4/c*x^4*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.61195, size = 1, normalized size = 0.

$$\left[\frac{6\left(\left(b^{3}c^{2}-3\,abc^{3}\right)d-\left(b^{4}c-4\,ab^{2}c^{2}+2\,a^{2}c^{3}\right)e+\left(b^{5}-5\,ab^{3}c+5\,a^{2}bc^{2}\right)f\right)\log\left(\frac{b^{3}-4\,abc+2\left(b^{2}c-4\,ac^{2}\right)x^{2}+\left(2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,a\right.}{cx^{4}+bx^{2}+a}\right.}{}\right.$$

$$-\frac{12\left(\left(b^{3}c^{2}-3\,abc^{3}\right)d-\left(b^{4}c-4\,ab^{2}c^{2}+2\,a^{2}c^{3}\right)e+\left(b^{5}-5\,ab^{3}c+5\,a^{2}bc^{2}\right)f\right)\arctan\left(-\frac{\left(2\,cx^{2}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-\left(3\,c^{4}fx^{8}+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/24*(6*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*
c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f)*log((b^3 - 4*a*b*c +
2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (3*c^4*f*x^8 + 4*(c^4*e
- b*c^3*f)*x^6 + 6*(c^4*d - b*c^3*e + (b^2*c^2 - a*c^3)*f)*x^4 -
12*(b*c^3*d - (b^2*c^2 - a*c^3)*e + (b^3*c - 2*a*b*c^2)*f)*x^2 +
6*((b^2*c^2 - a*c^3)*d - (b^3*c - 2*a*b*c^2)*e + (b^4 - 3*a*b^2*c
 + a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/(sqrt(b
^2 - 4*a*c)*c^5), -1/24*(12*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4
*a*b^2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f)*ar
ctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (3*c^4*f*
x^8 + 4*(c^4*e - b*c^3*f)*x^6 + 6*(c^4*d - b*c^3*e + (b^2*c^2 - a
*c^3)*f)*x^4 - 12*(b*c^3*d - (b^2*c^2 - a*c^3)*e + (b^3*c - 2*a*b
*c^2)*f)*x^2 + 6*((b^2*c^2 - a*c^3)*d - (b^3*c - 2*a*b*c^2)*e + (
b^4 - 3*a*b^2*c + a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))*sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^5)]
```

Sympy [A] time = 149.096, size = 1392, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5*a*b**3
*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**3*
c**2*d)/(4*c**5*(4*a*c - b**2)) + (a**2*c**2*f - 3*a*b**2*c*f + 2
*a*b*c**2*e - a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)/(4*c**5
))*log(x**2 + (2*a**3*c**2*f - 4*a**2*b**2*c*f + 3*a**2*b*c**2*e
- 2*a**2*c**3*d + a*b**4*f - a*b**3*c*e + a*b**2*c**2*d - 8*a*c**
5*(-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5*a*b*
```

```
*3*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**
3*c**2*d)/(4*c**5*(4*a*c - b**2)) + (a**2*c**2*f - 3*a*b**2*c*f +
 2*a*b*c**2*e - a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)/(4*c*
*5)) + 2*b**2*c**4*(-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**
2*c**3*e - 5*a*b**3*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f
 - b**4*c*e + b**3*c**2*d)/(4*c**5*(4*a*c - b**2)) + (a**2*c**2*f
 - 3*a*b**2*c*f + 2*a*b*c**2*e - a*c**3*d + b**4*f - b**3*c*e + b
**2*c**2*d)/(4*c**5)))/(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5*a*b**
3*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**3
*c**2*d)) + (sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**2*c**3*e
 - 5*a*b**3*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*
c*e + b**3*c**2*d)/(4*c**5*(4*a*c - b**2)) + (a**2*c**2*f - 3*a*b
**2*c*f + 2*a*b*c**2*e - a*c**3*d + b**4*f - b**3*c*e + b**2*c**2
*d)/(4*c**5))*log(x**2 + (2*a**3*c**2*f - 4*a**2*b**2*c*f + 3*a**
2*b*c**2*e - 2*a**2*c**3*d + a*b**4*f - a*b**3*c*e + a*b**2*c**2*
d - 8*a*c**5*(sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**2*c**3*
e - 5*a*b**3*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4
*c*e + b**3*c**2*d)/(4*c**5*(4*a*c - b**2)) + (a**2*c**2*f - 3*a*
b**2*c*f + 2*a*b*c**2*e - a*c**3*d + b**4*f - b**3*c*e + b**2*c**
2*d)/(4*c**5)) + 2*b**2*c**4*(sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*
f - 2*a**2*c**3*e - 5*a*b**3*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d
 + b**5*f - b**4*c*e + b**3*c**2*d)/(4*c**5*(4*a*c - b**2)) + (a*
*2*c**2*f - 3*a*b**2*c*f + 2*a*b*c**2*e - a*c**3*d + b**4*f - b**
3*c*e + b**2*c**2*d)/(4*c**5)))/(5*a**2*b*c**2*f - 2*a**2*c**3*e
- 5*a*b**3*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c
*e + b**3*c**2*d)) + f*x**8/(8*c) - x**6*(b*f - c*e)/(6*c**2) - x
**4*(a*c*f - b**2*f + b*c*e - c**2*d)/(4*c**3) + x**2*(2*a*b*c*f
- a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)/(2*c**4)
```

GIAC/XCAS [A] time = 0.3175, size = 413, normalized size = 1.51

$$\frac{3\,c^3fx^8 - 4\,bc^2fx^6 + 4\,c^3x^6e + 6\,c^3dx^4 + 6\,b^2cfx^4 - 6\,ac^2fx^4 - 6\,bc^2x^4e - 12\,bc^2dx^2 - 12\,b^3fx^2 + 24\,abcfx^2 + 12\,b^2cx^2e}{24\,c^4} + \frac{\left(b^2c^2d - ac^3d + b^4f - 3\,ab^2cf + a^2c^2f - b^3ce + 2\,abc^2e\right)\ln\left(cx^4 + bx^2 + a\right)}{4\,c^5} - \frac{\left(b^3c^2d - 3\,abc^3d + b^5f - 5\,ab^3cf + 5\,a^2bc^2f - b^4ce + 4\,ab^2c^2e - 2\,a^2c^3e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/24*(3*c^3*f*x^8 - 4*b*c^2*f*x^6 + 4*c^3*x^6*e + 6*c^3*d*x^4 + 6
*b^2*c*f*x^4 - 6*a*c^2*f*x^4 - 6*b*c^2*x^4*e - 12*b*c^2*d*x^2 - 1
2*b^3*f*x^2 + 24*a*b*c*f*x^2 + 12*b^2*c*x^2*e - 12*a*c^2*x^2*e)/c
^4 + 1/4*(b^2*c^2*d - a*c^3*d + b^4*f - 3*a*b^2*c*f + a^2*c^2*f -
 b^3*c*e + 2*a*b*c^2*e)*ln(c*x^4 + b*x^2 + a)/c^5 - 1/2*(b^3*c^2*
d - 3*a*b*c^3*d + b^5*f - 5*a*b^3*c*f + 5*a^2*b*c^2*f - b^4*c*e +
 4*a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*
a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

3.48 $\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=203

$$\frac{x^2\left(-c(af+be)+b^2f+c^2d\right)}{2c^3}+\frac{\log\left(a+bx^2+cx^4\right)\left(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{4c^4}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)}{2c^4\sqrt{b^2-4ac}}+\frac{x^4(ce-bf)}{4c^2}+\frac{fx^6}{6c}$$

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x^2)/(2*c^3) + ((c*e - b*f)*x^4)
/(4*c^2) + (f*x^6)/(6*c) + ((b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*
c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(2*c^4*Sqrt[b^2 - 4*a*c]) + ((b^2*c*e - a*c^2*e -
 b^3*f - b*c*(c*d - 2*a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rubi [A] time = 0.850191, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x^2\left(-c(af+be)+b^2f+c^2d\right)}{2c^3}+\frac{\log\left(a+bx^2+cx^4\right)\left(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{4c^4}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)}{2c^4\sqrt{b^2-4ac}}+\frac{x^4(ce-bf)}{4c^2}+\frac{fx^6}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x^2)/(2*c^3) + ((c*e - b*f)*x^4)
/(4*c^2) + (f*x^6)/(6*c) + ((b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*
c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(2*c^4*Sqrt[b^2 - 4*a*c]) + ((b^2*c*e - a*c^2*e -
 b^3*f - b*c*(c*d - 2*a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.262327, size = 193, normalized size = 0.95

$$\frac{6cx^2\left(-c(af+be)+b^2f+c^2d\right)-3\log\left(a+bx^2+cx^4\right)\left(bc(cd-2af)+ac^2e+b^3f-b^2ce\right)+\frac{6\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(b^2c(cd-4af)+3ab\right.}{\sqrt{4ac-b}}}{12c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*c*(c^2*d + b^2*f - c*(b*e + a*f))*x^2 + 3*c^2*(c*e - b*f)*x^4
+ 2*c^3*f*x^6 + (6*(-(b^3*c*e) + 3*a*b*c^2*e + b^4*f + b^2*c*(c*d
 - 4*a*f) + 2*a*c^2*(-(c*d) + a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^
2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(-(b^2*c*e) + a*c^2*e + b^3*f
 + b*c*(c*d - 2*a*f))*Log[a + b*x^2 + c*x^4])/(12*c^4)
```

Maple [B] time = 0.008, size = 474, normalized size = 2.3

$$\frac{fx^6}{6\,c}-\frac{bx^4f}{4\,c^2}+\frac{x^4e}{4\,c}-\frac{x^2af}{2\,c^2}+\frac{b^2fx^2}{2\,c^3}-\frac{x^2be}{2\,c^2}+\frac{dx^2}{2\,c}+\frac{\ln\left(cx^4+bx^2+a\right)abf}{2\,c^3}$$
$$-\frac{\ln\left(cx^4+bx^2+a\right)ae}{4\,c^2}-\frac{\ln\left(cx^4+bx^2+a\right)b^3f}{4\,c^4}+\frac{\ln\left(cx^4+bx^2+a\right)b^2e}{4\,c^3}$$
$$-\frac{\ln\left(cx^4+bx^2+a\right)bd}{4\,c^2}+\frac{a^2f}{c^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-2\,\frac{ab^2f}{c^3\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)+\frac{3\,abe}{2\,c^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-\frac{ad}{c}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$+\frac{b^4f}{2\,c^4}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-\frac{b^3e}{2\,c^3}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$+\frac{b^2d}{2\,c^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/6*f*x^6/c-1/4/c^2*x^4*b*f+1/4/c*x^4*e-1/2/c^2*x^2*a*f+1/2/c^3*b
^2*f*x^2-1/2/c^2*x^2*b*e+1/2/c*d*x^2+1/2/c^3*ln(c*x^4+b*x^2+a)*a*
b*f-1/4/c^2*ln(c*x^4+b*x^2+a)*a*e-1/4/c^4*ln(c*x^4+b*x^2+a)*b^3*f
+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2*e-1/4/c^2*ln(c*x^4+b*x^2+a)*b*d+1/
c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*f
-2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*
b^2*f+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))*a*b*e-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(
1/2))*a*d+1/2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))*b^4*f-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b^3*e+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*b^2*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.457989, size = 1, normalized size = 0.

$$\left[\frac{3\left(\left(b^2c^2-2\,ac^3\right)d-\left(b^3c-3\,abc^2\right)e+\left(b^4-4\,ab^2c+2\,a^2c^2\right)f\right)\log\left(-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2-\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 -
4*a*b^2*c + 2*a^2*c^2)*f)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^
2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))
/(c*x^4 + b*x^2 + a)) + (2*c^3*f*x^6 + 3*(c^3*e - b*c^2*f)*x^4 +
6*(c^3*d - b*c^2*e + (b^2*c - a*c^2)*f)*x^2 - 3*(b*c^2*d - (b^2*c
 - a*c^2)*e + (b^3 - 2*a*b*c)*f)*log(c*x^4 + b*x^2 + a))*sqrt(b^2
 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^4), 1/12*(6*((b^2*c^2 - 2*a*c^3)*
d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*f)*arct
an(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*c^3*f*x^
6 + 3*(c^3*e - b*c^2*f)*x^4 + 6*(c^3*d - b*c^2*e + (b^2*c - a*c^2
)*f)*x^2 - 3*(b*c^2*d - (b^2*c - a*c^2)*e + (b^3 - 2*a*b*c)*f)*lo
g(c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4
)]
```

Sympy [A] time = 110.105, size = 1044, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*
e - 2*a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)/(4*c**4*(4*a*c
- b**2)) + (2*a*b*c*f - a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)/
(4*c**4))*log(x**2 + (-3*a**2*b*c*f + 2*a**2*c**2*e + a*b**3*f -
a*b**2*c*e + a*b*c**2*d + 8*a*c**4*(-sqrt(-4*a*c + b**2)*(2*a**2*
c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e - 2*a*c**3*d + b**4*f - b**3
*c*e + b**2*c**2*d)/(4*c**4*(4*a*c - b**2)) + (2*a*b*c*f - a*c**2
*e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**4)) - 2*b**2*c**3*(-sqrt
(-4*a*c + b**2)*(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e - 2*
a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)/(4*c**4*(4*a*c - b**2
)) + (2*a*b*c*f - a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**
4)))/(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e - 2*a*c**3*d +
b**4*f - b**3*c*e + b**2*c**2*d)) + (sqrt(-4*a*c + b**2)*(2*a**2*
c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e - 2*a*c**3*d + b**4*f - b**3
*c*e + b**2*c**2*d)/(4*c**4*(4*a*c - b**2)) + (2*a*b*c*f - a*c**2
*e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**4))*log(x**2 + (-3*a**2*
b*c*f + 2*a**2*c**2*e + a*b**3*f - a*b**2*c*e + a*b*c**2*d + 8*a*
c**4*(sqrt(-4*a*c + b**2)*(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c
**2*e - 2*a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)/(4*c**4*(4*
a*c - b**2)) + (2*a*b*c*f - a*c**2*e - b**3*f + b**2*c*e - b*c**2
*d)/(4*c**4)) - 2*b**2*c**3*(sqrt(-4*a*c + b**2)*(2*a**2*c**2*f -
 4*a*b**2*c*f + 3*a*b*c**2*e - 2*a*c**3*d + b**4*f - b**3*c*e + b
**2*c**2*d)/(4*c**4*(4*a*c - b**2)) + (2*a*b*c*f - a*c**2*e - b**
3*f + b**2*c*e - b*c**2*d)/(4*c**4)))/(2*a**2*c**2*f - 4*a*b**2*c
*f + 3*a*b*c**2*e - 2*a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)
) + f*x**6/(6*c) - x**4*(b*f - c*e)/(4*c**2) - x**2*(a*c*f - b**2
*f + b*c*e - c**2*d)/(2*c**3)
```

GIAC/XCAS [A] time = 0.30386, size = 289, normalized size = 1.42

$$\frac{2\,c^2fx^6-3\,bcfx^4+3\,c^2x^4e+6\,c^2dx^2+6\,b^2fx^2-6\,acfx^2-6\,bcx^2e}{12\,c^3}-\frac{\left(bc^2d+b^3f-2\,abcf-b^2ce+ac^2e\right)\ln\left(cx^4+bx^2+a\right)}{4\,c^4}+\frac{\left(b^2c^2d-2\,ac^3d+b^4f-4\,ab^2cf+2\,a^2c^2f-b^3ce+3\,abc^2e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/12*(2*c^2*f*x^6 - 3*b*c*f*x^4 + 3*c^2*x^4*e + 6*c^2*d*x^2 + 6*b
^2*f*x^2 - 6*a*c*f*x^2 - 6*b*c*x^2*e)/c^3 - 1/4*(b*c^2*d + b^3*f
- 2*a*b*c*f - b^2*c*e + a*c^2*e)*ln(c*x^4 + b*x^2 + a)/c^4 + 1/2*
(b^2*c^2*d - 2*a*c^3*d + b^4*f - 4*a*b^2*c*f + 2*a^2*c^2*f - b^3*
c*e + 3*a*b*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt
(-b^2 + 4*a*c)*c^4)
```

3.49 $\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=144

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(af+be)+b^2f+c^2d\right)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(ce-bf)}{2c^2}+\frac{fx^4}{4c}$$

```
[Out] ((c*e - b*f)*x^2)/(2*c^2) + (f*x^4)/(4*c) - ((b^2*c*e - 2*a*c^2*e
 - b^3*f - b*c*(c*d - 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*
a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*d + b^2*f - c*(b*e + a*f
))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 0.536187, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(af+be)+b^2f+c^2d\right)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(ce-bf)}{2c^2}+\frac{fx^4}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*e - b*f)*x^2)/(2*c^2) + (f*x^4)/(4*c) - ((b^2*c*e - 2*a*c^2*e
 - b^3*f - b*c*(c*d - 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*
a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*d + b^2*f - c*(b*e + a*f
))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\left(\frac{bf}{2}-\frac{ce}{2}\right)\int^{x^2}\frac{1}{c^2}\,dx+\frac{f\int^{x^2}x\,dx}{2c}+\frac{\left(-acf+b^2f-bce+c^2d\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{\left(-3abcf+2ac^2e+b^3f-b^2ce+bc^2d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^3\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] -(b*f/2 - c*e/2)*Integral(c**(-2), (x, x**2)) + f*Integral(x, (x,
 x**2))/(2*c) + (-a*c*f + b**2*f - b*c*e + c**2*d)*log(a + b*x**2
 + c*x**4)/(4*c**3) + (-3*a*b*c*f + 2*a*c**2*e + b**3*f - b**2*c*
e + b*c**2*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**3*s
qrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.191651, size = 136, normalized size = 0.94

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(af+be)+b^2f+c^2d\right)-\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(bc(cd-3af)+2ac^2e+b^3f-b^2ce\right)}{\sqrt{4ac-b^2}}+2cx^2(ce-bf)+c^2fx^4}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(c*e - b*f)*x^2 + c^2*f*x^4 - (2*(-(b^2*c*e) + 2*a*c^2*e + b
^3*f + b*c*(c*d - 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]
])/Sqrt[-b^2 + 4*a*c] + (c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b
*x^2 + c*x^4])/(4*c^3)
```

Maple [B] time = 0.006, size = 321, normalized size = 2.2

$$\frac{fx^4}{4c} - \frac{bfx^2}{2c^2} + \frac{ex^2}{2c} - \frac{\ln(cx^4+bx^2+a)\,af}{4c^2} + \frac{\ln(cx^4+bx^2+a)\,b^2f}{4c^3} - \frac{\ln(cx^4+bx^2+a)\,be}{4c^2} + \frac{\ln(cx^4+bx^2+a)\,d}{4c} + \frac{3abf}{2c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} - \frac{ae}{c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} - \frac{b^3f}{2c^3}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} + \frac{b^2e}{2c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} - \frac{bd}{2c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*f*x^4/c-1/2/c^2*b*f*x^2+1/2/c*x^2*e-1/4/c^2*ln(c*x^4+b*x^2+a)
*a*f+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2*f-1/4/c^2*ln(c*x^4+b*x^2+a)*b*
e+1/4/c*ln(c*x^4+b*x^2+a)*d+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*f-1/c/(4*a*c-b^2)^(1/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))*a*e-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*f+1/2/c^2/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e-1/2/c/(4*a*c-b^2)^(1/2)
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.323968, size = 1, normalized size = 0.01

$$\left[-\frac{\left(bc^2d-\left(b^2c-2\,ac^2\right)e+\left(b^3-3\,abc\right)f\right)\log\left(-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2-\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(c^2fx^4+2\left(c^2e-\right.\right.}{4\sqrt{b^2-4\,ac}c^3}\right.$$

$$-\frac{2\left(bc^2d-\left(b^2c-2\,ac^2\right)e+\left(b^3-3\,abc\right)f\right)\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(c^2fx^4+2\left(c^2e-bcf\right)x^2+\left(c^2d-bce+\left(b^2-\right.\right.\right.}{4\sqrt{-b^2+4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*log(-(
b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (c^2*f*x
^4 + 2*(c^2*e - b*c*f)*x^2 + (c^2*d - b*c*e + (b^2 - a*c)*f)*log(
c*x^4 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^3), -
1/4*(2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*arctan
(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (c^2*f*x^4 +
2*(c^2*e - b*c*f)*x^2 + (c^2*d - b*c*e + (b^2 - a*c)*f)*log(c*x^4
 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)]
```

Sympy [A] time = 63.4681, size = 721, normalized size = 5.01

$$\left(-\frac{\sqrt{-4ac+b^2}\left(3abcf-2ac^2e-b^3f+b^2ce-bc^2d\right)}{4c^3\left(4ac-b^2\right)}\right.$$

$$\left.-\frac{acf-b^2f+bce-c^2d}{4c^3}\right)\log\left(x^2+\frac{2a^2cf-ab^2f+abce+8ac^3\left(-\frac{\sqrt{-4ac+b^2}(3abcf-2ac^2e-b^3f+b^2ce-bc^2d)}{4c^3(4ac-b^2)}-\frac{acf-b^2f+bce-c^2d}{4c^3}\right)}{3abcf-2ac^2e-b^3f+}\right.$$

$$+\left(\frac{\sqrt{-4ac+b^2}\left(3abcf-2ac^2e-b^3f+b^2ce-bc^2d\right)}{4c^3\left(4ac-b^2\right)}\right.$$

$$\left.-\frac{acf-b^2f+bce-c^2d}{4c^3}\right)\log\left(x^2+\frac{2a^2cf-ab^2f+abce+8ac^3\left(\frac{\sqrt{-4ac+b^2}(3abcf-2ac^2e-b^3f+b^2ce-bc^2d)}{4c^3(4ac-b^2)}-\frac{acf-b^2f+bce-c^2d}{4c^3}\right)}{3abcf-2ac^2e-b^3f+b}\right.$$

$$+\frac{fx^4}{4c}-\frac{x^2\left(bf-ce\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e
 - b*c**2*d)/(4*c**3*(4*a*c - b**2)) - (a*c*f - b**2*f + b*c*e -
c**2*d)/(4*c**3))*log(x**2 + (2*a**2*c*f - a*b**2*f + a*b*c*e + 8
*a*c**3*(-sqrt(-4*a*c + b**2)*(3*a*b*c*f - 2*a*c**2*e - b**3*f +
b**2*c*e - b*c**2*d)/(4*c**3*(4*a*c - b**2)) - (a*c*f - b**2*f +
b*c*e - c**2*d)/(4*c**3)) - 2*a*c**2*d - 2*b**2*c**2*(-sqrt(-4*a*
c + b**2)*(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)
/(4*c**3*(4*a*c - b**2)) - (a*c*f - b**2*f + b*c*e - c**2*d)/(4*c
**3)))/(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)) +
 (sqrt(-4*a*c + b**2)*(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e
 - b*c**2*d)/(4*c**3*(4*a*c - b**2)) - (a*c*f - b**2*f + b*c*e -
c**2*d)/(4*c**3))*log(x**2 + (2*a**2*c*f - a*b**2*f + a*b*c*e + 8
*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*f - 2*a*c**2*e - b**3*f + b
**2*c*e - b*c**2*d)/(4*c**3*(4*a*c - b**2)) - (a*c*f - b**2*f + b
```

```
*c*e - c**2*d)/(4*c**3)) - 2*a*c**2*d - 2*b**2*c**2*(sqrt(-4*a*c
+ b**2)*(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)/(
4*c**3*(4*a*c - b**2)) - (a*c*f - b**2*f + b*c*e - c**2*d)/(4*c**
3)))/(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)) + f
*x**4/(4*c) - x**2*(b*f - c*e)/(2*c**2)
```

GIAC/XCAS [A] time = 0.295227, size = 190, normalized size = 1.32

$$\frac{cfx^4 - 2\,bfx^2 + 2\,cx^2e}{4\,c^2} + \frac{\left(c^2d + b^2f - acf - bce\right)\ln\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(bc^2d + b^3f - 3\,abcf - b^2ce + 2\,ac^2e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4*(c*f*x^4 - 2*b*f*x^2 + 2*c*x^2*e)/c^2 + 1/4*(c^2*d + b^2*f -
a*c*f - b*c*e)*ln(c*x^4 + b*x^2 + a)/c^3 - 1/2*(b*c^2*d + b^3*f -
 3*a*b*c*f - b^2*c*e + 2*a*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2
+ 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.50 $\int \frac{x\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=103

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2acf+b^2f-bce+2c^2d\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{fx^2}{2c}$$

```
[Out] (f*x^2)/(2*c) - ((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*ArcTanh[(b +
 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*e -
 b*f)*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 0.346625, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2acf+b^2f-bce+2c^2d\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{fx^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (f*x^2)/(2*c) - ((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*ArcTanh[(b +
 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*e -
 b*f)*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^2} f\,dx}{2c}-\frac{(bf-ce)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{\left(-2acf+b^2f-bce+2c^2d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(f, (x, x**2))/(2*c) - (b*f - c*e)*log(a + b*x**2 + c*x**
4)/(4*c**2) - (-2*a*c*f + b**2*f - b*c*e + 2*c**2*d)*atanh((b + 2
*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**2*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.118854, size = 100, normalized size = 0.97

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-c(2af+be)+b^2f+2c^2d\right)}{\sqrt{4ac-b^2}}+(ce-bf)\log\left(a+bx^2+cx^4\right)+2cfx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*f*x^2 + (2*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*ArcTan[(b + 2
*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*e - b*f)*Log
[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [B] time = 0.005, size = 211, normalized size = 2.1

$$\frac{f x^2}{2c} - \frac{\ln\left(c x^4 + b x^2 + a\right) b f}{4c^2} + \frac{\ln\left(c x^4 + b x^2 + a\right) e}{4c} - \frac{f a}{c}\arctan\left(\left(2 c x^2 + b\right)\frac{1}{\sqrt{4ac - b^2}}\right)\frac{1}{\sqrt{4ac - b^2}} + d\arctan\left(\left(2 c x^2 + b\right)\frac{1}{\sqrt{4ac - b^2}}\right)\frac{1}{\sqrt{4ac - b^2}} + \frac{b^2 f}{2c^2}\arctan\left(\left(2 c x^2 + b\right)\frac{1}{\sqrt{4ac - b^2}}\right)\frac{1}{\sqrt{4ac - b^2}} - \frac{be}{2c}\arctan\left(\left(2 c x^2 + b\right)\frac{1}{\sqrt{4ac - b^2}}\right)\frac{1}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*f*x^2/c-1/4/c^2*ln(c*x^4+b*x^2+a)*b*f+1/4/c*ln(c*x^4+b*x^2+a)
*e-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*f*
a+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*d+1/2
/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*
f-1/2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b
*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)*x/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28979, size = 1, normalized size = 0.01

$$\left[-\frac{\left(2 c^2 d - bce + \left(b^2 - 2ac\right) f\right) \log\left(\frac{b^3 - 4abc + 2\left(b^2 c - 4ac^2\right) x^2 + \left(2c^2 x^4 + 2bcx^2 + b^2 - 2ac\right)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) - \left(2cfx^2 + (ce - bf)\log\left(cx^4 + b\right.\right.}{4\sqrt{b^2 - 4ac}c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)*x/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*log((b^3 - 4*a*b*c + 2
*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sq
rt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (2*c*f*x^2 + (c*e - b*f)*
log(c*x^4 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^2
), 1/4*(2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*arctan(-(2*c*x^2 +
b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*c*f*x^2 + (c*e - b*f)*l
og(c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^
2)]
```

Sympy [A] time = 36.9497, size = 498, normalized size = 4.83

$$\left(-\frac{\sqrt{-4ac+b^2}\left(2acf-b^2f+bce-2c^2d\right)}{4c^2\left(4ac-b^2\right)} - \frac{bf-ce}{4c^2}\right)\log\left(x^2 + \frac{-abf-8ac^2\left(-\frac{\sqrt{-4ac+b^2}(2acf-b^2f+bce-2c^2d)}{4c^2(4ac-b^2)}-\frac{bf-ce}{4c^2}\right)+2ace+2b^2c\left(-\frac{\sqrt{-4ac+b^2}(2acf-b^2f+bce-2c^2d)}{4c^2(4ac-b^2)}-\cdots\right.}{2acf-b^2f+bce-2c^2d}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\left(2acf-b^2f+bce-2c^2d\right)}{4c^2\left(4ac-b^2\right)} - \frac{bf-ce}{4c^2}\right)\log\left(x^2 + \frac{-abf-8ac^2\left(\frac{\sqrt{-4ac+b^2}(2acf-b^2f+bce-2c^2d)}{4c^2(4ac-b^2)}-\frac{bf-ce}{4c^2}\right)+2ace+2b^2c\left(\frac{\sqrt{-4ac+b^2}(2acf-b^2f+bce-2c^2d)}{4c^2(4ac-b^2)}-\frac{bf}{4}\cdots\right.}{2acf-b^2f+bce-2c^2d}\right.$$
$$+\frac{fx^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*f - b**2*f + b*c*e - 2*c**2*d)/(4*c*
*2*(4*a*c - b**2)) - (b*f - c*e)/(4*c**2))*log(x**2 + (-a*b*f - 8
*a*c**2*(-sqrt(-4*a*c + b**2)*(2*a*c*f - b**2*f + b*c*e - 2*c**2*
d)/(4*c**2*(4*a*c - b**2)) - (b*f - c*e)/(4*c**2)) + 2*a*c*e + 2*
b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*f - b**2*f + b*c*e - 2*c**2*d
)/(4*c**2*(4*a*c - b**2)) - (b*f - c*e)/(4*c**2)) - b*c*d)/(2*a*c
*f - b**2*f + b*c*e - 2*c**2*d)) + (sqrt(-4*a*c + b**2)*(2*a*c*f
- b**2*f + b*c*e - 2*c**2*d)/(4*c**2*(4*a*c - b**2)) - (b*f - c*e
)/(4*c**2))*log(x**2 + (-a*b*f - 8*a*c**2*(sqrt(-4*a*c + b**2)*(2
*a*c*f - b**2*f + b*c*e - 2*c**2*d)/(4*c**2*(4*a*c - b**2)) - (b*
f - c*e)/(4*c**2)) + 2*a*c*e + 2*b**2*c*(sqrt(-4*a*c + b**2)*(2*a
*c*f - b**2*f + b*c*e - 2*c**2*d)/(4*c**2*(4*a*c - b**2)) - (b*f
- c*e)/(4*c**2)) - b*c*d)/(2*a*c*f - b**2*f + b*c*e - 2*c**2*d))
+ f*x**2/(2*c)
```

GIAC/XCAS [A] time = 0.317974, size = 134, normalized size = 1.3

$$\frac{fx^2}{2\,c} - \frac{(bf-ce)\ln\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{\left(2\,c^2d+b^2f-2\,acf-bce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/c - 1/4*(b*f - c*e)*ln(c*x^4 + b*x^2 + a)/c^2 + 1/2*(2*
c^2*d + b^2*f - 2*a*c*f - b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.51 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)} \, dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{2ac\sqrt{b^2-4ac}}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{d\log(x)}{a}$$

```
[Out] ((b*c*d - 2*a*c*e + a*b*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*a*c*Sqrt[b^2 - 4*a*c]) + (d*Log[x])/a - ((c*d - a*f)*Log[a
 + b*x^2 + c*x^4])/(4*a*c)
```

Rubi [A] time = 0.398147, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{2ac\sqrt{b^2-4ac}}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - 2*a*c*e + a*b*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*a*c*Sqrt[b^2 - 4*a*c]) + (d*Log[x])/a - ((c*d - a*f)*Log[a
 + b*x^2 + c*x^4])/(4*a*c)
```

Rubi in Sympy [A] time = 62.573, size = 90, normalized size = 0.93

$$\frac{d\log\left(x^2\right)}{2a}+\frac{(af-cd)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{(abf-2ace+bcd)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2ac\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] d*log(x**2)/(2*a) + (a*f - c*d)*log(a + b*x**2 + c*x**4)/(4*a*c)
+ (a*b*f - 2*a*c*e + b*c*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b*
*2))/(2*a*c*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.265066, size = 178, normalized size = 1.84

$$\frac{-\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}+abf-2ace+bcd\right)+\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(-cd\sqrt{b^2\right.}{4ac\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*c*Sqrt[b^2 - 4*a*c]*d*Log[x] - (b*c*d + c*Sqrt[b^2 - 4*a*c]*d
- 2*a*c*e + a*b*f - a*Sqrt[b^2 - 4*a*c]*f)*Log[b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2] + (b*c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f
 + a*Sqrt[b^2 - 4*a*c]*f)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
```

```
4*a*c*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.01, size = 165, normalized size = 1.7

$$\frac{\ln\left(cx^4+bx^2+a\right)f}{4c}-\frac{\ln\left(cx^4+bx^2+a\right)d}{4a}+e\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$-\frac{bd}{2a}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$-\frac{bf}{2c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{d\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/c*ln(c*x^4+b*x^2+a)*f-1/4/a*ln(c*x^4+b*x^2+a)*d+1/(4*a*c-b^2)
^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*e-1/2/a/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*d-1/2/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b/c*f+d*ln(x)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.447306, size = 1, normalized size = 0.01

$$\left[\frac{(bcd-2ace+abf)\log\left(\frac{b^3-4abc+2\left(b^2c-4ac^2\right)x^2+\left(2c^2x^4+2bcx^2+b^2-2ac\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)+\left(4cd\log(x)-(cd-af)\log\left(cx^4+bx^2+a\right.\right.}{4\sqrt{b^2-4ac}ac}\right.$$
$$\left.-\frac{2(bcd-2ace+abf)\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left(4cd\log(x)-(cd-af)\log\left(cx^4+bx^2+a\right)\right)\sqrt{-b^2+4ac}}{4\sqrt{-b^2+4ac}ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/4*((b*c*d - 2*a*c*e + a*b*f)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4
*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) + (4*c*d*log(x) - (c*d - a*f)*log(c*x^
4 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a*c), -1/4*
(2*(b*c*d - 2*a*c*e + a*b*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) - (4*c*d*log(x) - (c*d - a*f)*log(c*x^4 + b*x
^2 + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.321615, size = 131, normalized size = 1.35

$$\frac{d\ln\left(x^2\right)}{2\,a} - \frac{(cd - af)\ln\left(cx^4 + bx^2 + a\right)}{4\,ac} - \frac{(bcd + abf - 2\,ace)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] 1/2*d*ln(x^2)/a - 1/4*(c*d - a*f)*ln(c*x^4 + b*x^2 + a)/(a*c) - 1
/2*(b*c*d + a*b*f - 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a
*c))/(sqrt(-b^2 + 4*a*c)*a*c)
```

3.52 $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=118

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-abe-2a(cd-af)+b^2d\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{2ax^2}$$

```
[Out] -d/(2*a*x^2) - ((b^2*d - a*b*e - 2*a*(c*d - a*f))*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*
e)*Log[x])/a^2 + ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.55637, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-abe-2a(cd-af)+b^2d\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(2*a*x^2) - ((b^2*d - a*b*e - 2*a*(c*d - a*f))*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*
e)*Log[x])/a^2 + ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi in Sympy [A] time = 85.6312, size = 112, normalized size = 0.95

$$-\frac{d}{2ax^2}+\frac{(ae-bd)\log\left(x^2\right)}{2a^2}-\frac{(ae-bd)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(2a^2f-abe-2acd+b^2d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] -d/(2*a*x**2) + (a*e - b*d)*log(x**2)/(2*a**2) - (a*e - b*d)*log(
a + b*x**2 + c*x**4)/(4*a**2) - (2*a**2*f - a*b*e - 2*a*c*d + b**
2*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a**2*sqrt(-4*a*
c + b**2))
```

Mathematica [A] time = 0.273782, size = 203, normalized size = 1.72

$$\frac{\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(a\left(-e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}-ae\right)+b^2d\right)}{\sqrt{b^2-4ac}}+\frac{\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(-a\left(e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}+ae\right)+b}{\sqrt{b^2-4ac}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*d)/x^2 + 4*(-(b*d) + a*e)*Log[x] + ((b^2*d + b*(Sqrt[b^2 -
 4*a*c]*d - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[
b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-(b^2*d)
```

```
+ b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e
 + 2*a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c
])/(4*a^2)
```

Maple [B] time = 0.013, size = 227, normalized size = 1.9

$$-\frac{\ln\left(cx^4+bx^2+a\right)e}{4a}+\frac{\ln\left(cx^4+bx^2+a\right)bd}{4a^2}+f\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{be}{2a}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{cd}{a}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{b^2d}{2a^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{d}{2ax^2}+\frac{\ln(x)e}{a}-\frac{\ln(x)bd}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/a*ln(c*x^4+b*x^2+a)*e+1/4/a^2*ln(c*x^4+b*x^2+a)*b*d+1/(4*a*c
-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*f-1/2/a/(4*a*c-
b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e-1/a/(4*a*c-b
^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*d+1/2/a^2/(4*a*
c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*d-1/2*d/a/
x^2+1/a*ln(x)*e-1/a^2*ln(x)*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.526774, size = 1, normalized size = 0.01

$$\left[-\frac{\left(abe-2a^2f-\left(b^2-2ac\right)d\right)x^2\log\left(-\frac{b^3-4abc+2\left(b^2c-4ac^2\right)x^2-\left(2c^2x^4+2bcx^2+b^2-2ac\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-\left((bd-ae)x^2\log\left(cx^4+bx^2\right.\right.}{4\sqrt{b^2-4ac}a^2x^2}\right.$$

$$-\frac{2\left(abe-2a^2f-\left(b^2-2ac\right)d\right)x^2\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left((bd-ae)x^2\log\left(cx^4+bx^2+a\right)-4(bd-ae)x^2\log(x)\right.}{4\sqrt{-b^2+4ac}a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*x^2*log(-(b^3 - 4*a*b*
c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*
c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b*d - a*e)*x^2*log
(c*x^4 + b*x^2 + a) - 4*(b*d - a*e)*x^2*log(x) - 2*a*d)*sqrt(b^2
- 4*a*c))/(sqrt(b^2 - 4*a*c)*a^2*x^2), -1/4*(2*(a*b*e - 2*a^2*f -
 (b^2 - 2*a*c)*d)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b
```

```
^2 - 4*a*c)) - ((b*d - a*e)*x^2*log(c*x^4 + b*x^2 + a) - 4*(b*d -
 a*e)*x^2*log(x) - 2*a*d)*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*a^2*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.322527, size = 182, normalized size = 1.54

$$\frac{(bd-ae)\ln\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{(bd-ae)\ln\left(x^2\right)}{2\,a^2}+\frac{\left(b^2d-2\,acd+2\,a^2f-abe\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2}+\frac{bdx^2-ax^2e-ad}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] 1/4*(b*d - a*e)*ln(c*x^4 + b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*ln(x^
2)/a^2 + 1/2*(b^2*d - 2*a*c*d + 2*a^2*f - a*b*e)*arctan((2*c*x^2
+ b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*d*x^2
- a*x^2*e - a*d)/(a^2*x^2)
```

3.53 $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=174

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-abe-a(cd-af)+b^2d\right)}{4a^3}+\frac{\log(x)\left(-abe-a(cd-af)+b^2d\right)}{a^3}+\frac{bd-ae}{2a^2x^2}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)}{2a^3\sqrt{b^2-4ac}}-\frac{d}{4ax^4}$$

```
[Out] -d/(4*a*x^4) + (b*d - a*e)/(2*a^2*x^2) + ((b^3*d - a*b^2*e + 2*a^
2*c*e - a*b*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*a^3*Sqrt[b^2 - 4*a*c]) + ((b^2*d - a*b*e - a*(c*d - a*f))*
Log[x])/a^3 - ((b^2*d - a*b*e - a*(c*d - a*f))*Log[a + b*x^2 + c*
x^4])/(4*a^3)
```

Rubi [A] time = 0.78159, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-abe-a(cd-af)+b^2d\right)}{4a^3}+\frac{\log(x)\left(-abe-a(cd-af)+b^2d\right)}{a^3}+\frac{bd-ae}{2a^2x^2}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)}{2a^3\sqrt{b^2-4ac}}-\frac{d}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(4*a*x^4) + (b*d - a*e)/(2*a^2*x^2) + ((b^3*d - a*b^2*e + 2*a^
2*c*e - a*b*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*a^3*Sqrt[b^2 - 4*a*c]) + ((b^2*d - a*b*e - a*(c*d - a*f))*
Log[x])/a^3 - ((b^2*d - a*b*e - a*(c*d - a*f))*Log[a + b*x^2 + c*
x^4])/(4*a^3)
```

Rubi in Sympy [A] time = 136.097, size = 167, normalized size = 0.96

$$-\frac{d}{4ax^4}-\frac{ae-bd}{2a^2x^2}+\frac{\left(a^2f-abe-acd+b^2d\right)\log\left(x^2\right)}{2a^3}-\frac{\left(a^2f-abe-acd+b^2d\right)\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\left(a^2bf+2a^2ce-ab^2e-3abcd+b^3d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^3\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] -d/(4*a*x**4) - (a*e - b*d)/(2*a**2*x**2) + (a**2*f - a*b*e - a*c
*d + b**2*d)*log(x**2)/(2*a**3) - (a**2*f - a*b*e - a*c*d + b**2*
d)*log(a + b*x**2 + c*x**4)/(4*a**3) + (a**2*b*f + 2*a**2*c*e - a
*b**2*e - 3*a*b*c*d + b**3*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c +
b**2))/(2*a**3*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.680589, size = 314, normalized size = 1.8

$$-\frac{\frac{a^2d}{x^4}-4\log(x)\left(-abe+a(af-cd)+b^2d\right)+\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(ab\left(-e\sqrt{b^2-4ac}+af-3cd\right)+a\left(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+2ace\right)+b^2\left(d\right.\right.}{\sqrt{b^2-4ac}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((a^2*d)/x^4 + (2*a*(-(b*d) + a*e))/x^2 - 4*(b^2*d - a*b*e + a*(
-(c*d) + a*f))*Log[x] + ((b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e)
 + a*b*(-3*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4
*a*c]*d) + 2*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-(b^3*d) + b^2*(Sqrt[b^2
- 4*a*c]*d + a*e) - a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e + a*f) + a*
(-(c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + a*Sqrt[b^2 - 4*a*c]*f))*Log
[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^3)
```

Maple [B] time = 0.016, size = 356, normalized size = 2.1

$$
-\frac{\ln\left(cx^4+bx^2+a\right)f}{4a}+\frac{\ln\left(cx^4+bx^2+a\right)be}{4a^2}+\frac{c\ln\left(cx^4+bx^2+a\right)d}{4a^2}
-\frac{\ln\left(cx^4+bx^2+a\right)b^2d}{4a^3}-\frac{bf}{2a}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}
-\frac{ce}{a}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}
+\frac{b^2e}{2a^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}
+\frac{3bcd}{2a^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}
-\frac{b^3d}{2a^3}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{d}{4ax^4}
-\frac{e}{2ax^2}+\frac{bd}{2a^2x^2}+\frac{\ln(x)f}{a}-\frac{\ln(x)be}{a^2}-\frac{\ln(x)cd}{a^2}+\frac{\ln(x)b^2d}{a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/a*ln(c*x^4+b*x^2+a)*f+1/4/a^2*ln(c*x^4+b*x^2+a)*b*e+1/4/a^2*
c*ln(c*x^4+b*x^2+a)*d-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2*d-1/2/a/(4*a*
c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*f-1/a/(4*a*c
-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*e+1/2/a^2/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e+3/2/a^
2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*d-1
/2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^
3*d-1/4*d/a/x^4-1/2/a/x^2*e+1/2/a^2/x^2*b*d+1/a*ln(x)*f-1/a^2*ln(
x)*b*e-1/a^2*ln(x)*c*d+1/a^3*ln(x)*b^2*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.926894, size = 1, normalized size = 0.01

$$\left[\frac{\left(a^2bf+\left(b^3-3\,abc\right)d-\left(ab^2-2\,a^2c\right)e\right)x^4\log\left(\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2+\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(\left(abe-a^2f-\left(b^2\right.\right.\right.}{4\sqrt{b^2-4\,ac}a^3x}\right.$$

$$-\frac{2\left(a^2bf+\left(b^3-3\,abc\right)d-\left(ab^2-2\,a^2c\right)e\right)x^4\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(\left(abe-a^2f-\left(b^2-ac\right)d\right)x^4\log\left(cx^4+bx^2\right.\right.}{4\sqrt{-b^2+4\,ac}a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b*f + (b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^4*log
((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((a*b*
e - a^2*f - (b^2 - a*c)*d)*x^4*log(c*x^4 + b*x^2 + a) - 4*(a*b*e
- a^2*f - (b^2 - a*c)*d)*x^4*log(x) - a^2*d + 2*(a*b*d - a^2*e)*x
^2)*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^3*x^4), -1/4*(2*(a^2*
b*f + (b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^4*arctan(-(2*c*x
^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((a*b*e - a^2*f - (b^
2 - a*c)*d)*x^4*log(c*x^4 + b*x^2 + a) - 4*(a*b*e - a^2*f - (b^2
- a*c)*d)*x^4*log(x) - a^2*d + 2*(a*b*d - a^2*e)*x^2)*sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.295183, size = 286, normalized size = 1.64

$$-\frac{\left(b^2d-acd+a^2f-abe\right)\ln\left(cx^4+bx^2+a\right)}{4\,a^3}+\frac{\left(b^2d-acd+a^2f-abe\right)\ln\left(x^2\right)}{2\,a^3}-\frac{\left(b^3d-3\,abcd+a^2bf-ab^2e+2\,a^2ce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\sqrt{-b^2+4\,ac}a^3}-\frac{3\,b^2dx^4-3\,acdx^4+3\,a^2fx^4-3\,abx^4e-2\,abdx^2+2\,a^2x^2e+a^2d}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d - a*c*d + a^2*f - a*b*e)*ln(c*x^4 + b*x^2 + a)/a^3 +
1/2*(b^2*d - a*c*d + a^2*f - a*b*e)*ln(x^2)/a^3 - 1/2*(b^3*d - 3*
a*b*c*d + a^2*b*f - a*b^2*e + 2*a^2*c*e)*arctan((2*c*x^2 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3) - 1/4*(3*b^2*d*x^4 - 3*
a*c*d*x^4 + 3*a^2*f*x^4 - 3*a*b*x^4*e - 2*a*b*d*x^2 + 2*a^2*x^2*e
 + a^2*d)/(a^3*x^4)
```

3.54 $\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=244

$$-\frac{-abe-a(cd-af)+b^2d}{2a^3x^2}+\frac{bd-ae}{4a^2x^4}+\frac{\log\left(a+bx^2+cx^4\right)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{4a^4}-\frac{\log(x)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{a^4}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d\right)}{2a^4\sqrt{b^2-4ac}}-\frac{d}{6ax^6}$$

```
[Out] -d/(6*a*x^6) + (b*d - a*e)/(4*a^2*x^4) - (b^2*d - a*b*e - a*(c*d
- a*f))/(2*a^3*x^2) - ((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(
c*d - a*f) - a*b^2*(4*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
- 4*a*c]])/(2*a^4*Sqrt[b^2 - 4*a*c]) - ((b^3*d - a*b^2*e + a^2*c*
e - a*b*(2*c*d - a*f))*Log[x])/a^4 + ((b^3*d - a*b^2*e + a^2*c*e
- a*b*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rubi [A] time = 1.1775, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{-abe-a(cd-af)+b^2d}{2a^3x^2}+\frac{bd-ae}{4a^2x^4}+\frac{\log\left(a+bx^2+cx^4\right)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{4a^4}-\frac{\log(x)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{a^4}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d\right)}{2a^4\sqrt{b^2-4ac}}-\frac{d}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(6*a*x^6) + (b*d - a*e)/(4*a^2*x^4) - (b^2*d - a*b*e - a*(c*d
- a*f))/(2*a^3*x^2) - ((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(
c*d - a*f) - a*b^2*(4*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
- 4*a*c]])/(2*a^4*Sqrt[b^2 - 4*a*c]) - ((b^3*d - a*b^2*e + a^2*c*
e - a*b*(2*c*d - a*f))*Log[x])/a^4 + ((b^3*d - a*b^2*e + a^2*c*e
- a*b*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.789086, size = 416, normalized size = 1.7

$$-\frac{2a^3d}{x^6}-12\log(x)\left(a^2ce-ab^2e+ab(af-2cd)+b^3d\right)+\frac{3\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(a^2c\left(e\sqrt{b^2-4ac}-2af+2cd\right)+ab^2\left(-e\sqrt{b^2-4ac}+af-4cd\right)\cdots}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a^3*d)/x^6 + (3*a^2*(b*d - a*e))/x^4 + (6*a*(-(b^2*d) + a*b*
e + a*(c*d - a*f)))/x^2 - 12*(b^3*d - a*b^2*e + a^2*c*e + a*b*(-2
*c*d + a*f))*Log[x] + (3*(b^4*d + b^3*(Sqrt[b^2 - 4*a*c]*d - a*e)
 + a^2*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f) + a*b^2*(-4*c*d -
Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-2*c*Sqrt[b^2 - 4*a*c]*d + 3*a*
c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/Sqrt[b^2 - 4*a*c] + (3*(-(b^4*d) + b^3*(Sqrt[b^2 - 4*a*c]*d +
a*e) - a*b^2*(-4*c*d + Sqrt[b^2 - 4*a*c]*e + a*f) + a^2*c*(-2*c*d
 + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + a*b*(-2*c*Sqrt[b^2 - 4*a*c]*d -
 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2])/Sqrt[b^2 - 4*a*c])/(12*a^4)
```

Maple [B] time = 0.019, size = 523, normalized size = 2.1

$$\frac{\ln\left(cx^4+bx^2+a\right)bf}{4a^2}+\frac{c\ln\left(cx^4+bx^2+a\right)e}{4a^2}-\frac{\ln\left(cx^4+bx^2+a\right)b^2e}{4a^3}-\frac{c\ln\left(cx^4+bx^2+a\right)bd}{2a^3}+\frac{\ln\left(cx^4+bx^2+a\right)b^3d}{4a^4}-\frac{cf}{a}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{b^2f}{2a^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{3bce}{2a^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{c^2d}{a^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{b^3e}{2a^3}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-2\frac{b^2cd}{a^3\sqrt{4ac-b^2}}\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)+\frac{b^4d}{2a^4}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{d}{6ax^6}-\frac{e}{4ax^4}+\frac{bd}{4a^2x^4}-\frac{f}{2ax^2}+\frac{be}{2a^2x^2}+\frac{cd}{2a^2x^2}-\frac{b^2d}{2a^3x^2}-\frac{\ln(x)bf}{a^2}-\frac{\ln(x)ce}{a^2}+\frac{\ln(x)b^2e}{a^3}+2\frac{\ln(x)bcd}{a^3}-\frac{\ln(x)b^3d}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/a^2*ln(c*x^4+b*x^2+a)*b*f+1/4/a^2*c*ln(c*x^4+b*x^2+a)*e-1/4/a
^3*ln(c*x^4+b*x^2+a)*b^2*e-1/2/a^3*c*ln(c*x^4+b*x^2+a)*b*d+1/4/a^
4*ln(c*x^4+b*x^2+a)*b^3*d-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2))*c*f+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*b^2*f+3/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*e+1/a^2/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2*d-1/2/a^3/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*e-2/a^3/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*c*d+1/2/a^4/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*d-1/6*d/a/x^6-
1/4/a/x^4*e+1/4/a^2/x^4*b*d-1/2/a/x^2*f+1/2/a^2/x^2*b*e+1/2/a^2/x
^2*c*d-1/2/a^3/x^2*b^2*d-1/a^2*ln(x)*b*f-1/a^2*ln(x)*c*e+1/a^3*ln
(x)*b^2*e+2/a^3*ln(x)*b*c*d-1/a^4*ln(x)*b^3*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04713, size = 1, normalized size = 0.

$$\left[-\frac{3\left(\left(b^4-4\,ab^2c+2\,a^2c^2\right)d-\left(ab^3-3\,a^2bc\right)e+\left(a^2b^2-2\,a^3c\right)f\right)x^6\log\left(\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2+\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,}}{cx^4+bx^2+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^7),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*
e + (a^2*b^2 - 2*a^3*c)*f)*x^6*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*
a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a
*c))/(c*x^4 + b*x^2 + a)) - (3*(a^2*b*f + (b^3 - 2*a*b*c)*d - (a*
b^2 - a^2*c)*e)*x^6*log(c*x^4 + b*x^2 + a) - 12*(a^2*b*f + (b^3 -
 2*a*b*c)*d - (a*b^2 - a^2*c)*e)*x^6*log(x) + 6*(a^2*b*e - a^3*f
- (a*b^2 - a^2*c)*d)*x^4 - 2*a^3*d + 3*(a^2*b*d - a^3*e)*x^2)*sqr
t(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^4*x^6), 1/12*(6*((b^4 - 4*a*
b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^2*b^2 - 2*a^3*c
)*f)*x^6*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))
+ (3*(a^2*b*f + (b^3 - 2*a*b*c)*d - (a*b^2 - a^2*c)*e)*x^6*log(c*
x^4 + b*x^2 + a) - 12*(a^2*b*f + (b^3 - 2*a*b*c)*d - (a*b^2 - a^2
*c)*e)*x^6*log(x) + 6*(a^2*b*e - a^3*f - (a*b^2 - a^2*c)*d)*x^4 -
 2*a^3*d + 3*(a^2*b*d - a^3*e)*x^2)*sqrt(-b^2 + 4*a*c))/(sqrt(-b^
2 + 4*a*c)*a^4*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.295154, size = 423, normalized size = 1.73

$$\frac{\left(b^3d-2\,abcd+a^2bf-ab^2e+a^2ce\right)\ln\left(cx^4+bx^2+a\right)}{4\,a^4}-\frac{\left(b^3d-2\,abcd+a^2bf-ab^2e+a^2ce\right)\ln\left(x^2\right)}{2\,a^4}$$
$$+\frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d+a^2b^2f-2\,a^3cf-ab^3e+3\,a^2bce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^4}$$
$$+\frac{11\,b^3dx^6-22\,abcdx^6+11\,a^2bfx^6-11\,ab^2x^6e+11\,a^2cx^6e-6\,ab^2dx^4+6\,a^2cdx^4-6\,a^3fx^4+6\,a^2bx^4e+3\,a^2bdx^2-3\,a^3x}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^7),x, algorithm="giac")
```

```
[Out] 1/4*(b^3*d - 2*a*b*c*d + a^2*b*f - a*b^2*e + a^2*c*e)*ln(c*x^4 +
b*x^2 + a)/a^4 - 1/2*(b^3*d - 2*a*b*c*d + a^2*b*f - a*b^2*e + a^2
```

```
*c*e)*ln(x^2)/a^4 + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d + a^2*
b^2*f - 2*a^3*c*f - a*b^3*e + 3*a^2*b*c*e)*arctan((2*c*x^2 + b)/s
qrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^4) + 1/12*(11*b^3*d*x^6
- 22*a*b*c*d*x^6 + 11*a^2*b*f*x^6 - 11*a*b^2*x^6*e + 11*a^2*c*x^6
*e - 6*a*b^2*d*x^4 + 6*a^2*c*d*x^4 - 6*a^3*f*x^4 + 6*a^2*b*x^4*e
+ 3*a^2*b*d*x^2 - 3*a^3*x^2*e - 2*a^3*d)/(a^4*x^6)
```

3.55 $\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=369

$$\frac{x(-c(af+be)+b^2f+c^2d)}{c^3}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3(ce-bf)}{3c^2}+\frac{fx^5}{5c}$$

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^
2) + (f*x^5)/(5*c) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a
*f) - (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*
c^2*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2 -
4*a*c]]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) + (b^3
*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d -
 a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 10.9386, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x(-c(af+be)+b^2f+c^2d)}{c^3}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3(ce-bf)}{3c^2}+\frac{fx^5}{5c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```


```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^
2) + (f*x^5)/(5*c) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a
*f) - (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*
c^2*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2 -
4*a*c]]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) + (b^3
*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d -
 a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 164.076, size = 405, normalized size = 1.1

$$\frac{fx^5}{5c} - \frac{x^3(bf-ce)}{3c^2} + \frac{x\left(-acf+b^2f-bce+c^2d\right)}{c^3} - \frac{\sqrt{2}\left(-2ac\left(-acf+b^2f-bce+c^2d\right)+b\left(-2abcf+ac^2e+b^3f-b^2ce+bc^2d\right)+\sqrt{-4ac+b^2}\left(-2abcf+ac^2e+b^3f-b^2c\right.\right.}{2c^{\frac{7}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} + \frac{\sqrt{2}\left(-2ac\left(-acf+b^2f-bce+c^2d\right)+b\left(-2abcf+ac^2e+b^3f-b^2ce+bc^2d\right)-\sqrt{-4ac+b^2}\left(-2abcf+ac^2e+b^3f-b^2c\right.\right.}{2c^{\frac{7}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] f*x**5/(5*c) - x**3*(b*f - c*e)/(3*c**2) + x*(-a*c*f + b**2*f - b
*c*e + c**2*d)/c**3 - sqrt(2)*(-2*a*c*(-a*c*f + b**2*f - b*c*e +
c**2*d) + b*(-2*a*b*c*f + a*c**2*e + b**3*f - b**2*c*e + b*c**2*d
) + sqrt(-4*a*c + b**2)*(-2*a*b*c*f + a*c**2*e + b**3*f - b**2*c*
e + b*c**2*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2
)))/(2*c**(7/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)
) + sqrt(2)*(-2*a*c*(-a*c*f + b**2*f - b*c*e + c**2*d) + b*(-2*a*
b*c*f + a*c**2*e + b**3*f - b**2*c*e + b*c**2*d) - sqrt(-4*a*c +
b**2)*(-2*a*b*c*f + a*c**2*e + b**3*f - b**2*c*e + b*c**2*d))*ata
n(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c**(7/2)*sq
rt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 1.16901, size = 456, normalized size = 1.24

$$\frac{x\left(-c(af+be)+b^2f+c^2d\right)}{c^3} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ac^2\left(e\sqrt{b^2-4ac}-2af+2cd\right)-b^2c\left(e\sqrt{b^2-4ac}-4af+cd\right)+bc\left(cd\sqrt{b^2-4ac}-2af\sqrt{b^2-4ac}\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(ac^2\left(e\sqrt{b^2-4ac}+2af-2cd\right)+b^2c\left(-e\sqrt{b^2-4ac}-4af+cd\right)+bc\left(cd\sqrt{b^2-4ac}-2af\sqrt{b^2-4a\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x^3(ce-bf)}{3c^2} + \frac{fx^5}{5c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^
2) + (f*x^5)/(5*c) - ((-(b^4*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*
e - 4*a*f) + a*c^2*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f) + b^3*(c
*e + Sqrt[b^2 - 4*a*c]*f) + b*c*(c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e
- 2*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - ((b^4*f + b^2*c*(c*d - Sqrt[b^2 - 4*a*c]*e -
 4*a*f) + a*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b^3*(-(c
*e) + Sqrt[b^2 - 4*a*c]*f) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e
 - 2*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b +
Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.05, size = 1450, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*f+
1/5*f*x^5/c+1/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*f-1/c/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*f-1/2/c^3/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*f+1/2/c^2/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e-1/2/c/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d-1/c^2*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*a*b*f-1/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*a^2*f-1/2/c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b^4*f+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b^3*e-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^2*d+2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b
^2*f-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*e+
1/3/c*x^3*e+1/c*d*x-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*a*b*e-1/3/c^2*x^3*b*f-1/c^2*a*f*x+1/c^3*b^2*f*x-1/c^2
*b*e*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*
x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e+1/2/c^3*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b^3*f-1/2/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b^2*e+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d-1/2/c*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*a*e-1/2/c^3*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*f+1/2
/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e-1/2/c*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*b*d+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a
*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,c^2 f x^5 + 5\left(c^2 e - bcf\right)x^3 + 15\left(c^2 d - bce + \left(b^2 - ac\right)f\right)x}{15\,c^3} + \frac{-\int \frac{ac^2 d - abce + \left(bc^2 d - \left(b^2 c - ac^2\right)e + \left(b^3 - 2\,abc\right)f\right)x^2 + \left(ab^2 - a^2 c\right)f}{cx^4 + bx^2 + a}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*f*x^5 + 5*(c^2*e - b*c*f)*x^3 + 15*(c^2*d - b*c*e + (
b^2 - a*c)*f)*x)/c^3 + integrate(-(a*c^2*d - a*b*c*e + (b*c^2*d -
 (b^2*c - a*c^2)*e + (b^3 - 2*a*b*c)*f)*x^2 + (a*b^2 - a^2*c)*f)/
```

```
(c*x^4 + b*x^2 + a), x)/c^3
```

Fricas [A] time = 14.7219, size = 20880, normalized size = 56.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*c^2*f*x^5 - 15*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)
*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a
*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d -
(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f + (b^2*c^7
 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5
*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4
*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c
^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5
+ 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^
10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*
b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*
c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a
*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a
^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6
- 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 +
 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2
 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^
6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8
 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*
a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 1
8*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*
b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c
^15)))/(b^2*c^7 - 4*a*c^8))*log(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (
3*a*b^3*c^5 - 5*a^2*b*c^6)*d^3*e + 3*(a*b^4*c^4 - 2*a^2*b^2*c^5)*
d^2*e^2 - (a*b^5*c^3 - a^2*b^3*c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2*b^
4*c^3 - 3*a^3*b^2*c^4 + a^4*c^5)*e^4 + (a^3*b^6 - 5*a^4*b^4*c + 6
*a^5*b^2*c^2 - a^6*c^3)*f^4 + ((a*b^8 - 7*a^2*b^6*c + 18*a^3*b^4*
c^2 - 19*a^4*b^2*c^3 + 4*a^5*c^4)*d - (a^2*b^7 - 3*a^3*b^5*c - 2*
a^4*b^3*c^2 + 5*a^5*b*c^3)*e)*f^3 + 3*((a*b^6*c^2 - 5*a^2*b^4*c^3
 + 7*a^3*b^2*c^4 - 2*a^4*c^5)*d^2 - (a*b^7*c - 5*a^2*b^5*c^2 + 8*
a^3*b^3*c^3 - 5*a^4*b*c^4)*d*e + (a^2*b^6*c - 4*a^3*b^4*c^2 + 3*a
^4*b^2*c^3)*e^2)*f^2 + ((3*a*b^4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c^6)
*d^3 - 3*(2*a*b^5*c^3 - 7*a^2*b^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3*(a
*b^6*c^2 - 3*a^2*b^4*c^3 + a^3*b^2*c^4)*d*e^2 - (3*a^2*b^5*c^2 -
11*a^3*b^3*c^3 + 7*a^4*b*c^4)*e^3)*f)*x + sqrt(1/2)*((b^4*c^6 - 5
*a*b^2*c^7 + 4*a^2*c^8)*d^3 - (3*b^5*c^5 - 17*a*b^3*c^6 + 20*a^2*
b*c^7)*d^2*e + (3*b^6*c^4 - 19*a*b^4*c^5 + 29*a^2*b^2*c^6 - 4*a^3
*c^7)*d*e^2 - (b^7*c^3 - 7*a*b^5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*b*c
^6)*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 +
29*a^4*b^2*c^4 - 4*a^5*c^5)*f^3 + ((3*b^8*c^2 - 25*a*b^6*c^3 + 66
*a^2*b^4*c^4 - 59*a^3*b^2*c^5 + 12*a^4*c^6)*d - (3*b^9*c - 27*a*b
^7*c^2 + 80*a^2*b^5*c^3 - 87*a^3*b^3*c^4 + 28*a^4*b*c^5)*e)*f^2 +
 ((3*b^6*c^4 - 20*a*b^4*c^5 + 35*a^2*b^2*c^6 - 12*a^3*c^7)*d^2 -
2*(3*b^7*c^3 - 22*a*b^5*c^4 + 46*a^2*b^3*c^5 - 24*a^3*b*c^6)*d*e
+ (3*b^8*c^2 - 24*a*b^6*c^3 + 58*a^2*b^4*c^4 - 41*a^3*b^2*c^5 + 4
*a^4*c^6)*e^2)*f - ((b^3*c^9 - 4*a*b*c^10)*d - (b^4*c^8 - 6*a*b^2
*c^9 + 8*a^2*c^10)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*f)*
sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b
^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^
2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b
^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4
*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^
6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3
*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 2
9*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)
*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2
```

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*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*
c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*
c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)
*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*
d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d
^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^
7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13
*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))*sqrt(
-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^
5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b
^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*
c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a
^3*c^4)*e)*f + (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 +
 a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e +
2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 -
 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 +
(b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8
)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 4
6*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*
a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5
*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4
 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^
6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b
^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b
*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*
b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*
b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^
6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c
^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3
- 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^
3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))) + 15*sqrt(1/2
)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4
 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (
b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2
 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^
2*c^3 - 2*a^3*c^4)*e)*f + (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*
a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^
9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9
)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^
8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^
7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*
b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^
10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^
2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*
a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^
4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d
^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^
6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^
4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6
*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 -
15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 -
 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2
- (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^
4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log
(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (3*a*b^3*c^5 - 5*a^2*b*c^6)*d^3*
e + 3*(a*b^4*c^4 - 2*a^2*b^2*c^5)*d^2*e^2 - (a*b^5*c^3 - a^2*b^3*
c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2*b^4*c^3 - 3*a^3*b^2*c^4 + a^4*c^5
)*e^4 + (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*f^4 + (
(a*b^8 - 7*a^2*b^6*c + 18*a^3*b^4*c^2 - 19*a^4*b^2*c^3 + 4*a^5*c^
4)*d - (a^2*b^7 - 3*a^3*b^5*c - 2*a^4*b^3*c^2 + 5*a^5*b*c^3)*e)*f
^3 + 3*((a*b^6*c^2 - 5*a^2*b^4*c^3 + 7*a^3*b^2*c^4 - 2*a^4*c^5)*d
^2 - (a*b^7*c - 5*a^2*b^5*c^2 + 8*a^3*b^3*c^3 - 5*a^4*b*c^4)*d*e
+ (a^2*b^6*c - 4*a^3*b^4*c^2 + 3*a^4*b^2*c^3)*e^2)*f^2 + ((3*a*b^
4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d^3 - 3*(2*a*b^5*c^3 - 7*a^2*b
^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3*(a*b^6*c^2 - 3*a^2*b^4*c^3 + a^3*
b^2*c^4)*d*e^2 - (3*a^2*b^5*c^2 - 11*a^3*b^3*c^3 + 7*a^4*b*c^4)*e
^3)*f)*x - sqrt(1/2)*((b^4*c^6 - 5*a*b^2*c^7 + 4*a^2*c^8)*d^3 - (
3*b^5*c^5 - 17*a*b^3*c^6 + 20*a^2*b*c^7)*d^2*e + (3*b^6*c^4 - 19*
a*b^4*c^5 + 29*a^2*b^2*c^6 - 4*a^3*c^7)*d*e^2 - (b^7*c^3 - 7*a*b^
5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*b*c^6)*e^3 + (b^10 - 10*a*b^8*c +
35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*f^3
 + ((3*b^8*c^2 - 25*a*b^6*c^3 + 66*a^2*b^4*c^4 - 59*a^3*b^2*c^5 +
```

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12*a^4*c^6)*d - (3*b^9*c - 27*a*b^7*c^2 + 80*a^2*b^5*c^3 - 87*a^
3*b^3*c^4 + 28*a^4*b*c^5)*e)*f^2 + ((3*b^6*c^4 - 20*a*b^4*c^5 + 3
5*a^2*b^2*c^6 - 12*a^3*c^7)*d^2 - 2*(3*b^7*c^3 - 22*a*b^5*c^4 + 4
6*a^2*b^3*c^5 - 24*a^3*b*c^6)*d*e + (3*b^8*c^2 - 24*a*b^6*c^3 + 5
8*a^2*b^4*c^4 - 41*a^3*b^2*c^5 + 4*a^4*c^6)*e^2)*f - ((b^3*c^9 -
4*a*b*c^10)*d - (b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*e + (b^5*c^7
 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*f)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 +
a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2
*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 -
4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (
b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)
*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a
*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*
c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4
+ 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6
*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^
9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*
c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b
^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b
^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6
 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^
5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 -
 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3
)*f)/(b^2*c^14 - 4*a*c^15)))*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2
*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3
 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b
*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c -
 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f + (b^2*c^7 - 4*a*c
^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3
*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 1
2*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a
^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2
*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 3
7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24
*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2
 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6
)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3
*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*
b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*
b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*
c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c
^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^
8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^
2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5
 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(
b^2*c^7 - 4*a*c^8))) - 15*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c
^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 -
5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^
2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d
 - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f - (b^2*
c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(
b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*
b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^
5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c
^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a
*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a
^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b
^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c -
9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 -
3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c
^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^
4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*
c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2
*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*
c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 -
 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6
- 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a
^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*
a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-2*((a*b^2*c^6 - a^2*c^7)*d^4
- (3*a*b^3*c^5 - 5*a^2*b*c^6)*d^3*e + 3*(a*b^4*c^4 - 2*a^2*b^2*c^
```

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5)*d^2*e^2 - (a*b^5*c^3 - a^2*b^3*c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2
*b^4*c^3 - 3*a^3*b^2*c^4 + a^4*c^5)*e^4 + (a^3*b^6 - 5*a^4*b^4*c
+ 6*a^5*b^2*c^2 - a^6*c^3)*f^4 + ((a*b^8 - 7*a^2*b^6*c + 18*a^3*b
^4*c^2 - 19*a^4*b^2*c^3 + 4*a^5*c^4)*d - (a^2*b^7 - 3*a^3*b^5*c -
 2*a^4*b^3*c^2 + 5*a^5*b*c^3)*e)*f^3 + 3*((a*b^6*c^2 - 5*a^2*b^4*
c^3 + 7*a^3*b^2*c^4 - 2*a^4*c^5)*d^2 - (a*b^7*c - 5*a^2*b^5*c^2 +
 8*a^3*b^3*c^3 - 5*a^4*b*c^4)*d*e + (a^2*b^6*c - 4*a^3*b^4*c^2 +
3*a^4*b^2*c^3)*e^2)*f^2 + ((3*a*b^4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c
^6)*d^3 - 3*(2*a*b^5*c^3 - 7*a^2*b^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3
*(a*b^6*c^2 - 3*a^2*b^4*c^3 + a^3*b^2*c^4)*d*e^2 - (3*a^2*b^5*c^2
 - 11*a^3*b^3*c^3 + 7*a^4*b*c^4)*e^3)*f)*x + sqrt(1/2)*((b^4*c^6
- 5*a*b^2*c^7 + 4*a^2*c^8)*d^3 - (3*b^5*c^5 - 17*a*b^3*c^6 + 20*a
^2*b*c^7)*d^2*e + (3*b^6*c^4 - 19*a*b^4*c^5 + 29*a^2*b^2*c^6 - 4*
a^3*c^7)*d*e^2 - (b^7*c^3 - 7*a*b^5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*
b*c^6)*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3
 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*f^3 + ((3*b^8*c^2 - 25*a*b^6*c^3 +
 66*a^2*b^4*c^4 - 59*a^3*b^2*c^5 + 12*a^4*c^6)*d - (3*b^9*c - 27*
a*b^7*c^2 + 80*a^2*b^5*c^3 - 87*a^3*b^3*c^4 + 28*a^4*b*c^5)*e)*f^
2 + ((3*b^6*c^4 - 20*a*b^4*c^5 + 35*a^2*b^2*c^6 - 12*a^3*c^7)*d^2
 - 2*(3*b^7*c^3 - 22*a*b^5*c^4 + 46*a^2*b^3*c^5 - 24*a^3*b*c^6)*d
*e + (3*b^8*c^2 - 24*a*b^6*c^3 + 58*a^2*b^4*c^4 - 41*a^3*b^2*c^5
+ 4*a^4*c^6)*e^2)*f + ((b^3*c^9 - 4*a*b*c^10)*d - (b^4*c^8 - 6*a*
b^2*c^9 + 8*a^2*c^10)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*
f)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*
a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12
*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^
2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*
b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37
*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*
a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2
+ 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)
*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*
b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b
^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b
^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c
^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^
9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8
)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2
*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 -
 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))*sq
rt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2
*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*
a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b
^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 -
2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^
9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e
 + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^
2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3
 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*
c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 -
 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 -
a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*
c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a
*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(
3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^
4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a
^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4
*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5
*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^
6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c
^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)
*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))) + 15*sqrt(
1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*
c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2
+ (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*
c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2
*b^2*c^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 -
 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b
*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*
c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b
```

*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2
*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*(
(b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4
*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 -
40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8
*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8
)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3
*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6
*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((
b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5
 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^
4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e
^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3
*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*
log(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (3*a*b^3*c^5 - 5*a^2*b*c^6)*d
^3*e + 3*(a*b^4*c^4 - 2*a^2*b^2*c^5)*d^2*e^2 - (a*b^5*c^3 - a^2*b
^3*c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2*b^4*c^3 - 3*a^3*b^2*c^4 + a^4*
c^5)*e^4 + (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*f^4
+ ((a*b^8 - 7*a^2*b^6*c + 18*a^3*b^4*c^2 - 19*a^4*b^2*c^3 + 4*a^5
*c^4)*d - (a^2*b^7 - 3*a^3*b^5*c - 2*a^4*b^3*c^2 + 5*a^5*b*c^3)*e
)*f^3 + 3*((a*b^6*c^2 - 5*a^2*b^4*c^3 + 7*a^3*b^2*c^4 - 2*a^4*c^5
)*d^2 - (a*b^7*c - 5*a^2*b^5*c^2 + 8*a^3*b^3*c^3 - 5*a^4*b*c^4)*d
*e + (a^2*b^6*c - 4*a^3*b^4*c^2 + 3*a^4*b^2*c^3)*e^2)*f^2 + ((3*a
*b^4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d^3 - 3*(2*a*b^5*c^3 - 7*a^
2*b^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3*(a*b^6*c^2 - 3*a^2*b^4*c^3 + a
^3*b^2*c^4)*d*e^2 - (3*a^2*b^5*c^2 - 11*a^3*b^3*c^3 + 7*a^4*b*c^4
)*e^3)*f)*x - sqrt(1/2)*((b^4*c^6 - 5*a*b^2*c^7 + 4*a^2*c^8)*d^3
- (3*b^5*c^5 - 17*a*b^3*c^6 + 20*a^2*b*c^7)*d^2*e + (3*b^6*c^4 -
19*a*b^4*c^5 + 29*a^2*b^2*c^6 - 4*a^3*c^7)*d*e^2 - (b^7*c^3 - 7*a
*b^5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*b*c^6)*e^3 + (b^10 - 10*a*b^8*c
 + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*
f^3 + ((3*b^8*c^2 - 25*a*b^6*c^3 + 66*a^2*b^4*c^4 - 59*a^3*b^2*c^
5 + 12*a^4*c^6)*d - (3*b^9*c - 27*a*b^7*c^2 + 80*a^2*b^5*c^3 - 87
*a^3*b^3*c^4 + 28*a^4*b*c^5)*e)*f^2 + ((3*b^6*c^4 - 20*a*b^4*c^5
+ 35*a^2*b^2*c^6 - 12*a^3*c^7)*d^2 - 2*(3*b^7*c^3 - 22*a*b^5*c^4
+ 46*a^2*b^3*c^5 - 24*a^3*b*c^6)*d*e + (3*b^8*c^2 - 24*a*b^6*c^3
+ 58*a^2*b^4*c^4 - 41*a^3*b^2*c^5 + 4*a^4*c^6)*e^2)*f + ((b^3*c^9
 - 4*a*b*c^10)*d - (b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*e + (b^5*
c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*f)*sqrt(((b^4*c^8 - 2*a*b^2*c^9
 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e
+ 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2
 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3
+ (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c
^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 -
8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a
^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c
^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*
b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3
*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4
*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^
3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*
a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*
c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6
*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^
3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*
e^3)*f)/(b^2*c^14 - 4*a*c^15)))*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2
- 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*
c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*
c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*
a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7
- 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7
+ 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 +
7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*
a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c
+ 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c
^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 -
 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*
c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*
c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*

```
a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a
^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24
*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a
^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^
3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b
*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3
*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c
^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15))
)/(b^2*c^7 - 4*a*c^8))) + 10*(c^2*e - b*c*f)*x^3 + 30*(c^2*d - b*
c*e + (b^2 - a*c)*f)*x)/c^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.82152, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.56 $\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=282

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(ce-bf)}{c^2}+\frac{fx^3}{3c}$$

```
[Out] ((c*e - b*f)*x)/c^2 + (f*x^3)/(3*c) + ((c^2*d - b*c*e + b^2*f - a
*c*f + (b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c^2*d - b*c*
e + b^2*f - a*c*f - (b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a
*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 7.57338, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(ce-bf)}{c^2}+\frac{fx^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*e - b*f)*x)/c^2 + (f*x^3)/(3*c) + ((c^2*d - b*c*e + b^2*f - a
*c*f + (b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c^2*d - b*c*
e + b^2*f - a*c*f - (b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a
*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 69.8886, size = 294, normalized size = 1.04

$$\frac{fx^3}{3c}-\frac{x(bf-ce)}{c^2}+\frac{\sqrt{2}\left(-2ac(bf-ce)+b\left(-acf+b^2f-bce+c^2d\right)+\sqrt{-4ac+b^2}\left(-acf+b^2f-bce+c^2d\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(-2ac(bf-ce)+b\left(-acf+b^2f-bce+c^2d\right)-\sqrt{-4ac+b^2}\left(-acf+b^2f-bce+c^2d\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] f*x**3/(3*c) - x*(b*f - c*e)/c**2 + sqrt(2)*(-2*a*c*(b*f - c*e) +
 b*(-a*c*f + b**2*f - b*c*e + c**2*d) + sqrt(-4*a*c + b**2)*(-a*c
*f + b**2*f - b*c*e + c**2*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sq
rt(-4*a*c + b**2)))/(2*c**(5/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqr
t(-4*a*c + b**2)) - sqrt(2)*(-2*a*c*(b*f - c*e) + b*(-a*c*f + b**
2*f - b*c*e + c**2*d) - sqrt(-4*a*c + b**2)*(-a*c*f + b**2*f - b*
c*e + c**2*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2
)))/(2*c**(5/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)
)
```

Mathematica [A] time = 1.02542, size = 365, normalized size = 1.29

$$\frac{\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-bc\left(e\sqrt{b^2-4ac}-3af+cd\right)+c\left(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}-2ace\right)+b^2\left(f\sqrt{b^2-4ac}+ce\right)+b^3(-f)\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(bc\left(-e\cdots\right.\right.}{\cdots}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*Sqrt[c]*(c*e - b*f)*x + 2*c^(3/2)*f*x^3 + (3*Sqrt[2]*(-(b^3*f)
 - b*c*(c*d + Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b^2*(c*e + Sqrt[b^2
- 4*a*c]*f) + c*(c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e - a*Sqrt[b^2 - 4
*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(b
^3*f + b*c*(c*d - Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b^2*(-(c*e) + Sq
rt[b^2 - 4*a*c]*f) + c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*Sqrt[
b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5
/2))
```

Maple [B] time = 0.04, size = 1035, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*f*x^3/c-1/c^2*x*b*f+1/c*x*e-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*a*f+1/2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f-1/2/c*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*b*e+1/2*d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-3/2/c/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*f+1/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e+1/2/c^2/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*f-1/2/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*d*b+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*
f-1/2/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f+1/2/c*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b*e-1/2*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-3/
```

```
2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*f+1/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e+1/2/c^2/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*f-1/2/c/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*
x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+1/2/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cfx^3 + 3\,(ce - bf)x}{3\,c^2} - \frac{\int \frac{ace - abf - (c^2d - bce + (b^2 - ac)f)\,x^2}{cx^4 + bx^2 + a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*f*x^3 + 3*(c*e - b*f)*x)/c^2 - integrate((a*c*e - a*b*f -
(c^2*d - b*c*e + (b^2 - a*c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

Fricas [A] time = 3.65615, size = 12641, normalized size = 44.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*f*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 -
2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2
 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*
d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e
^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*
b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a
^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 +
3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e +
(3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 +
4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4
*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*
a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*l
og(2*(c^6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*
b*c^4)*d*e^3 + (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c
 + a^4*c^2)*f^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*
d - (a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*
b^2*c^3 + 2*a^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*
e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d
^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*
e^2 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x + sqrt(1/2)*((b^2*c^5
 - 4*a*c^6)*d^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*
a*b^2*c^4 + 4*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 -
4*a^3*b*c^3)*f^3 - (2*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (
3*b^6*c - 19*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*f^2 - ((b
^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)
*d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c^4)*e^2)*f + (2*(b^2
*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5 - 6*a*b^2*
c^6 + 8*a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6
- a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b
```

```
^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c
^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*
c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3
*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4
*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*
d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 +
a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b
^2*c^10 - 4*a*c^11)))*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*
e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f
^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^
3)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(
3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c
^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4
*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3
- 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*
d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2
- 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6
 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*
b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*
e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/
2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 -
3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^
2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*
c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c
^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^
5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 -
a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*
e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*
c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3
+ 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 -
 (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c
^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(c^6*d^4 - 3*b*c^5*d^
3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*b*c^4)*d*e^3 + (a*b^2*c^3
- a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*f^4 + ((b^6 -
5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3*c - 3
*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 -
 (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c
^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*
c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 - (3*a*b^3*c^2 - 5*a^2
*b*c^3)*e^3)*f)*x - sqrt(1/2)*((b^2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3
*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*e^3
 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^3 - (2*(b^5
*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 + 2
9*a^2*b^2*c^3 - 4*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 -
2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b
^3*c^3 + 20*a^2*b*c^4)*e^2)*f + (2*(b^2*c^7 - 4*a*c^8)*d - (b^3*c
^6 - 4*a*b*c^7)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*f)*sqrt((
c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*
c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 +
4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c -
 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^
4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5
*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^
3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c
^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3
- 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))*sqrt
(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*
e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c
^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c
^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2
 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6
)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d
- (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*
((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b
^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^
2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5
```

```
 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 -
 (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^
11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 -
2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*
a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c
- 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d
^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 -
 a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b
^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*
b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7
*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*
b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5
)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d
^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a
*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5
- 4*a*c^6))*log(2*(c^6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 -
(b^3*c^3 + a*b*c^4)*d*e^3 + (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4
- 3*a^3*b^2*c + a^4*c^2)*f^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2
- 4*a^3*c^3)*d - (a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b
^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*
a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4
 - 4*a*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 -
a*b^2*c^3)*d*e^2 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x + sqrt(1
/2)*((b^2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 +
(b^4*c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a
^2*b^3*c^2 - 4*a^3*b*c^3)*f^3 - (2*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2
*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4*a^3*c^4)
*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4
 + 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c^4)*e^2
)*f - (2*(b^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c
^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e +
2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^
4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^
4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3
+ 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c
^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^
6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c
^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*
c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6
*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^
5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3
- 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5
*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c
^2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^
7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d
*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*
a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7
*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5
+ 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e
+ (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2
+ 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b
^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 +
2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
) - 3*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e
+ (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2
 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)
*e)*f - (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*
b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4
 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a
^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^
2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 -
12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 -
 a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^
2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^
3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(c^6*d^4
 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*b*c^4)*d*e^3
+ (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*f
^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 -
```

```
a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a
^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*e + (a*b^4*c
- 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3*(2*b^3
*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 - (3*a*b^
3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x - sqrt(1/2)*((b^2*c^5 - 4*a*c^6)*d
^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2*c^4 + 4
*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*
f^3 - (2*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19*
a*b^4*c^2 + 29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*
b*c^5)*d^2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*d*e + (3*b^5
*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c^4)*e^2)*f - (2*(b^2*c^7 - 4*a*c^
8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c
^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*
e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*
c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^
4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)
*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 +
 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*
a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2
*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*
c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^
2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a
*c^11)))*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2
- 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*
c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f - (b^
2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a
*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*
c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4
- a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4
)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^
5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^
3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3
 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2
*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 6*(c*e - b*f)*x)/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.69044, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.57 $\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=219

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c}$$

```
[Out] (f*x)/c + ((c*e - b*f + (2*c^2*d + b^2*f - c*(b*e + 2*a*f))/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*e - b*f
 - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.34022, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (f*x)/c + ((c*e - b*f + (2*c^2*d + b^2*f - c*(b*e + 2*a*f))/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*e - b*f
 - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 51.8989, size = 221, normalized size = 1.01

$$\frac{fx}{c}-\frac{\sqrt{2}\left(b\left(bf-ce\right)-2c\left(af-cd\right)+\sqrt{-4ac+b^2}\left(bf-ce\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(b\left(bf-ce\right)-2c\left(af-cd\right)-\sqrt{-4ac+b^2}\left(bf-ce\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] f*x/c - sqrt(2)*(b*(b*f - c*e) - 2*c*(a*f - c*d) + sqrt(-4*a*c +
b**2)*(b*f - c*e))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c +
b**2)))/(2*c**(3/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b
```

```
**2)) + sqrt(2)*(b*(b*f - c*e) - 2*c*(a*f - c*d) - sqrt(-4*a*c +
b**2)*(b*f - c*e))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c +
b**2)))/(2*c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b
**2))
```

Mathematica [A] time = 0.657558, size = 258, normalized size = 1.18

$$\frac{\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(c\left(e\sqrt{b^2-4ac}-2af-be\right)+bf\left(b-\sqrt{b^2-4ac}\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-c\left(e\sqrt{b^2-4ac}+2af+be\right)+bf\left(\sqrt{b^2-4ac}+b\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*f*x + (Sqrt[2]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f
+ c*(-(b*e) + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[2]*(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c]
)*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

Maple [B] time = 0.031, size = 676, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] f*x/c-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+1/2*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
a*f-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f+1
/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-1/(-4*a*c+
b^2)^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+1/2/c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b*f-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*f-1/2/(-4*a*c+b^
2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f+1/2/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-1/(-4*a*c+b^2)^(1/2)*c
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{fx}{c} - \frac{-\int \frac{(ce-bf)x^2+cd-af}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] f*x/c - integrate(-((c*e - b*f)*x^2 + c*d - a*f)/(c*x^4 + b*x^2 +
 a), x)/c
```

Fricas [A] time = 1.89493, size = 7814, normalized size = 35.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 +
(a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)
*f + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^
2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c
^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3
*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^
2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3
*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log
(2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^
2 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3
+ a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^
2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*c^3*d^2*e - 3*a*b^2*c
^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x + sqrt
(1/2)*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 +
(a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c
^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)*f^2 - (3*(a*b
^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (a^2*
b^2*c^2 - 4*a^3*c^3)*e^2)*f - ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a
^2*b^2*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((
c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c
+ a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*
b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2
- (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2
*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7))
)*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b
*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3
- 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2
*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d
- (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^
3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^
5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^
2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt
(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^
2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^
2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 -
 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2
*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*
a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3
- a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6
- 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^4 - b*c^4*d^
3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b
^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*
(a^2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c
^2)*d*e)*f^2 + (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2
*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x - sqrt(1/2)*((b^2*c^4 - 4*a*
c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c
 + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d + 2
*(a^2*b^3*c - 4*a^3*b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)*d^
2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)*e
^2)*f - ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^2*b^2*c^4 - 4*a^3*c^5
)*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*
e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a
```

```
^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a
^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3
*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a
^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^2 - 4
*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d
 - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6
*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a
^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c
^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (
3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e
- a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(
a*b^2*c^3 - 4*a^2*c^4))) + sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3
*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*
b^2*c - 2*a^2*c^2)*e)*f - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 -
 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2
)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)
*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*
b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*
c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*
c^3 - 4*a^2*c^4))*log(2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 -
a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a
^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e^2 + (a*b^
2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*
c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*
c^4)*d^3)*f)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3
 - 4*a^2*c^4)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((
a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*
c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a
^2*b*c^3)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*f + ((a*b^3*c^4 -
4*a^2*b*c^5)*d - 2*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^3 - 4
*a^3*b*c^4)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a
^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*
d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*
c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*
c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*
b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e
^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^
2)*e)*f - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2
 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*
b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*
b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^
3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*
b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4)
)) - sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (
a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*
f - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2
*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^
2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*
d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2
)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*
e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(
2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2
 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 +
 a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2
 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*c^3*d^2*e - 3*a*b^2*c^
2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x - sqrt(
1/2)*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + (
a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c^
2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)*f^2 - (3*(a*b^
2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (a^2*b
^2*c^2 - 4*a^3*c^3)*e^2)*f + ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^
2*b^2*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((c
^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c +
 a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b
*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 -
 (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*
e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))
*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*
c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f - (a*b^2*c^3 -
 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*
b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d -
 (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3
```

```
 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5
*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2
*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - 2*f*x)/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.25712, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.58 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=213

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

```
[Out] -(d/(a*x)) - ((c*d - a*f + (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*a*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((c*d - a*f - (b
*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b + Sq
rt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.75355, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(d/(a*x)) - ((c*d - a*f + (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*a*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((c*d - a*f - (b
*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b + Sq
rt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 74.1687, size = 235, normalized size = 1.1

$$-\frac{f}{cx}+\frac{af-cd}{acx}+\frac{\sqrt{2}\left(b\left(af-cd\right)-2c\left(ae-bd\right)+\sqrt{-4ac+b^2}\left(af-cd\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(-abf+2ace-bcd+\sqrt{-4ac+b^2}\left(af-cd\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] -f/(c*x) + (a*f - c*d)/(a*c*x) + sqrt(2)*(b*(a*f - c*d) - 2*c*(a*
e - b*d) + sqrt(-4*a*c + b**2)*(a*f - c*d))*atan(sqrt(2)*sqrt(c)*
x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*a*sqrt(c)*sqrt(b + sqrt(-4*a*
c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*(-a*b*f + 2*a*c*e - b*c
*d + sqrt(-4*a*c + b**2)*(a*f - c*d))*atan(sqrt(2)*sqrt(c)*x/sqrt
(b - sqrt(-4*a*c + b**2)))/(2*a*sqrt(c)*sqrt(b - sqrt(-4*a*c + b*
*2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.60748, size = 253, normalized size = 1.19

$$\frac{-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}+abf-2ace+bcd\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+abf-2ace+bcd\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2d}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*d)/x - (Sqrt[2]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a
*b*f - a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[2]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e
+ a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt
[b^2 - 4*a*c]]))/(2*a)
```

Maple [B] time = 0.031, size = 563, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-1/2*c/a*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*d+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f-
c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2*c/a/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-1/2*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*f+1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+1/2
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f-c/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2*c/a/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-d/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-\int \frac{(cd-af)x^2+bd-ae}{cx^4+bx^2+a}\,dx}{a}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(-((c*d - a*f)*x^2 + b*d - a*e)/(c*x^4 + b*x^2 + a), x)/
a - d/(a*x)
```

Fricas [A] time = 0.931088, size = 8006, normalized size = 37.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a
*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*
c*e)*f + (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c
^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c
^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^
3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^
4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2
*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c -
4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3
*c^2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*
d^3*e + (a^4*b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e -
(a^2*b^2*c - 2*a^3*c^2)*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^3*b*c*e
^3 + (b^4*c - 3*a*b^2*c^2 + 4*a^2*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c
^2)*d^2*e)*f)*x + sqrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*
d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e + 3*(a^2*b^3
*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e^3 - ((a^3*b^3
 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c - 4*
a^3*b*c^2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f - ((a^3*b^4*c - 6
*a^4*b^2*c^2 + 8*a^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^
5*b^2*c - 4*a^6*c^2)*f)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 +
4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 +
 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^
2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^
2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^
3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a
^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e
+ 2*(a^2*b*c*d - 2*a^3*c*e)*f + (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*
a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^
2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e
- 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*
e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a
^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a
^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))) - sqrt(1/2)*a*x*sqrt(-(a^2*b*
c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*
c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f + (a^3*b^2*c - 4*a^4*c^2)*
sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4
- (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^
3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e
 - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*
d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*
c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d
^2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3*c^2*e^4 - a^5*f^4 + (b^2*c^3 - a
*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d^3*e + (a^4*b*e - (a^3*b^2 - 4*a
^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c - 2*a^3*c^2)*d^2)*f^2
+ (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4*c - 3*a*b^2*c^2 + 4*a^2
*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x - sqrt(1/2)*((b^5
*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2
 + 4*a^3*c^3)*d^2*e + 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^
2*c - 4*a^4*c^2)*e^3 - ((a^3*b^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^
5*c)*e)*f^2 + 2*((a^2*b^3*c - 4*a^3*b*c^2)*d^2 - (a^3*b^2*c - 4*a
^4*c^2)*d*e)*f - ((a^3*b^4*c - 6*a^4*b^2*c^2 + 8*a^5*c^3)*d - (a^
4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*c^2)*f)*sqrt(-(4*
a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^
2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e
- 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*
e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a
^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a
^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^
2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f + (
a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4
*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 +
4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2
```

```
*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2
)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3
*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2)
)) + sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*
b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c
*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^
2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^
4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3
*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4
*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*
c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4
*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3*
c^2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d
^3*e + (a^4*b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (
a^2*b^2*c - 2*a^3*c^2)*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^
3 + (b^4*c - 3*a*b^2*c^2 + 4*a^2*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^
2)*d^2*e)*f)*x + sqrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d
^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e + 3*(a^2*b^3*
c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e^3 - ((a^3*b^3
- 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c - 4*a
^3*b*c^2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f + ((a^3*b^4*c - 6*
a^4*b^2*c^2 + 8*a^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5
*b^2*c - 4*a^6*c^2)*f)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4
*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 +
4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2
*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2
)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3
*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^
3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e +
 2*(a^2*b*c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a
^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2
 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e -
 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e
^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^
4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^
7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))) - sqrt(1/2)*a*x*sqrt(-(a^2*b*c
*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c
^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*s
qrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 -
 (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3
)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e
- a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d
^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c
^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^
2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3*c^2*e^4 - a^5*f^4 + (b^2*c^3 - a*
c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d^3*e + (a^4*b*e - (a^3*b^2 - 4*a^
4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c - 2*a^3*c^2)*d^2)*f^2 +
 (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4*c - 3*a*b^2*c^2 + 4*a^2*
c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x - sqrt(1/2)*((b^5*
c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2
+ 4*a^3*c^3)*d^2*e + 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2
*c - 4*a^4*c^2)*e^3 - ((a^3*b^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5
*c)*e)*f^2 + 2*((a^2*b^3*c - 4*a^3*b*c^2)*d^2 - (a^3*b^2*c - 4*a^
4*c^2)*d*e)*f + ((a^3*b^4*c - 6*a^4*b^2*c^2 + 8*a^5*c^3)*d - (a^4
*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*c^2)*f)*sqrt(-(4*a
^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2
 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e -
 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e
^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^
4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^
7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2
 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f - (a
^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*
a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4
*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*
e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)
*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*
c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))
) + 2*d)/(a*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.26817, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] Done
```

3.59 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=267

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac}-2af+2cd\right)-b\left(d\sqrt{b^2-4ac}+ae\right)+b^2d\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{bd-ae}{a^2x} - \frac{d}{3ax^3}$$

```
[Out] -d/(3*a*x^3) + (b*d - a*e)/(a^2*x) + (Sqrt[c]*(b*d - a*e + (b^2*d
 - a*b*e - 2*a*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e)
 - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.24778, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac}-2af+2cd\right)-b\left(d\sqrt{b^2-4ac}+ae\right)+b^2d\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{bd-ae}{a^2x} - \frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(3*a*x^3) + (b*d - a*e)/(a^2*x) + (Sqrt[c]*(b*d - a*e + (b^2*d
 - a*b*e - 2*a*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e)
 - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 72.4802, size = 258, normalized size = 0.97

$$-\frac{f}{3cx^3} + \frac{af-cd}{3acx^3} - \frac{\sqrt{2}\sqrt{c}\left(2a\left(af-cd\right)-b\left(ae-bd\right)+\sqrt{-4ac+b^2}\left(ae-bd\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a^2\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} + \frac{\sqrt{2}\sqrt{c}\left(2a\left(af-cd\right)-b\left(ae-bd\right)-\sqrt{-4ac+b^2}\left(ae-bd\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a^2\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} - \frac{ae-bd}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] -f/(3*c*x**3) + (a*f - c*d)/(3*a*c*x**3) - sqrt(2)*sqrt(c)*(2*a*(
a*f - c*d) - b*(a*e - b*d) + sqrt(-4*a*c + b**2)*(a*e - b*d))*ata
n(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*a**2*sqrt(b
 + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*sqrt(c)*(2
*a*(a*f - c*d) - b*(a*e - b*d) - sqrt(-4*a*c + b**2)*(a*e - b*d))
*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*a**2*sq
rt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - (a*e - b*d)/(a
**2*x)
```

Mathematica [A] time = 0.688135, size = 284, normalized size = 1.06

$$\frac{\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(a\left(-e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}-ae\right)+b^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}+\right)\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*d)/x^3 + (6*b*d - 6*a*e)/x + (3*Sqrt[2]*Sqrt[c]*(b^2*d + b
*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e +
2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(-2*c*d + Sqrt[
b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))
/(6*a^2)
```

Maple [B] time = 0.036, size = 727, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2*c/a^2*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*b*d-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*f+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+
c^2/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/2*c/a^2
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+1/2*c/a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*e-1/2*c/a^2*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b*d-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*f+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e
+c^2/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/2*c
/a^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d-1/3
*d/a/x^3-1/a/x*e+1/a^2/x*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{abe-a^2f-(bcd-ace)x^2-(b^2-ac)d}{cx^4+bx^2+a}\,dx}{a^2}+\frac{3\,(bd-ae)x^2-ad}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((a*b*e - a^2*f - (b*c*d - a*c*e)*x^2 - (b^2 - a*c)*d)/
(c*x^4 + b*x^2 + a), x)/a^2 + 1/3*(3*(b*d - a*e)*x^2 - a*d)/(a^2*
x^3)
```

Fricas [A] time = 5.1233, size = 13298, normalized size = 49.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5
*a^2*b*c^2)*d^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*
b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*
a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*
a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 1
2*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*
a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2
)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7
*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*
a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2
*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2
*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b
*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(a
^6*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b
^3*c^3 - 3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e
^2 - (3*a^2*b^3*c^2 - 5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3
)*e^4 - (3*a^5*b*c*e - (3*a^4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*
b^2*c*e^2 + (a^2*b^4*c - 3*a^3*b^2*c^2 + 2*a^4*c^3)*d^2 - (2*a^3*
b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 5*a*b^4*c^2 + 9*a^2*b^2
*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2 + 3*a^3*b*c^3)
*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^4*b*c
^2)*e^3)*f)*x + sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17
*a^3*b^2*c^3 + 4*a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*
b^3*c^2 - 20*a^4*b*c^3)*d^2*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^
4*b^2*c^2 - 4*a^5*c^3)*d*e^2 - (a^3*b^5 - 5*a^4*b^3*c + 4*a^5*b*c
^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 + 3*((a^4*b^4 - 5*a^5*b^2*c + 4
*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 + ((3*a^2*b^6 - 19*a^3
*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^5 - 16*a^4
*b^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^
2)*e^2)*f - ((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4
- 6*a^7*b^2*c + 8*a^8*c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8
*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^
3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2
*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 -
 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f
^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3
- 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4
*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^
3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*
e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(
```

```
a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*b^4 - 4*a^
2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*b^
3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)
*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4
*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^
5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 +
(a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a
^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(
3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a
^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5
- 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a
^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)
))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 +
 (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2
*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*b^3 - 3*a^3*b
*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)*sqrt((a^8*
f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3
*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*
e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 -
2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^
3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 -
 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*
a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3
*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e
^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2
 - 4*a^6*c))*log(2*(a^6*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)
*d^4 - (b^5*c^2 - a*b^3*c^3 - 3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 -
 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^2*b^3*c^2 - 5*a^3*b*c^3)*d*e^3 + (
a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e - (3*a^4*b^2*c - 4*a^5*
c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3*b^2*c^2 + 2*a
^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 5*
a*b^4*c^2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b
^3*c^2 + 3*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 -
 (a^3*b^3*c + a^4*b*c^2)*e^3)*f)*x - sqrt(1/2)*((b^8 - 8*a*b^6*c
+ 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*d^3 - (3*a*b^7 - 2
1*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2*e + (3*a^2*b^6 -
 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2 - (a^3*b^5 - 5*
a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 + 3*((a^4*
b^4 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 +
 ((3*a^2*b^6 - 19*a^3*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 -
2*(3*a^3*b^5 - 16*a^4*b^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13
*a^5*b^2*c + 4*a^6*c^2)*e^2)*f - ((a^5*b^5 - 7*a^6*b^3*c + 12*a^7
*b*c^2)*d - (a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*e + (a^7*b^3 - 4*
a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6
*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*
c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b
^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^
2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (
a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2
)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*
f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 -
(3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^
5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2
- 4*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*
d^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*
b*c)*e^2 + 2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f
+ (a^5*b^2 - 4*a^6*c)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c +
7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c
+ 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c +
2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a
^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c +
 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^
7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c
^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*
b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/
(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x
^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*
b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 +
2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f - (a^5*b^2
```

- 4*a^6*c)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c
^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^
2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2
)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a
^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)
d^2 - 2(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f
^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (
3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5
*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 -
 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(a^6*c*f^4 + (b^4*c^3 - 3
*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b^3*c^3 - 3*a^2*b*c^4)*d
^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^2*b^3*c^2 - 5
*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e -
(3*a^4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c
- 3*a^3*b^2*c^2 + 2*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*
e)*f^2 + ((b^6*c - 5*a*b^4*c^2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 -
 3*(a*b^5*c - 3*a^2*b^3*c^2 + 3*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c -
 a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^4*b*c^2)*e^3)*f)*x + sqrt(1/
2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^
4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^4*b*c^3)
*d^2*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*
d*e^2 - (a^3*b^5 - 5*a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*
a^7*c)*f^3 + 3*((a^4*b^4 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3
- 4*a^6*b*c)*e)*f^2 + ((3*a^2*b^6 - 19*a^3*b^4*c + 31*a^4*b^2*c^2
 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^5 - 16*a^4*b^3*c + 16*a^5*b*c^2)*
d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^2)*e^2)*f + ((a^5*b^5 -
 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c
^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a
^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12
*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a
^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)
e^4 - 4(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*
a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a
^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*
c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*
e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*
c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*
b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*
e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3
*b^2 - 2*a^4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*sqrt((a^8*f^4 + (b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a
*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^
2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3
*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c
+ a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^
4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*
d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c +
4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b
*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^
3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))
) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*d^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^
3 - 3*a^3*b*c)*e^2 + 2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^
4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^
2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*
a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^
4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*
e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a
^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^
6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c
^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e
 + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c
)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(a^6
*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b^3
*c^3 - 3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2
 - (3*a^2*b^3*c^2 - 5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*
e^4 - (3*a^5*b*c*e - (3*a^4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^
2*c*e^2 + (a^2*b^4*c - 3*a^3*b^2*c^2 + 2*a^4*c^3)*d^2 - (2*a^3*b^
3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 5*a*b^4*c^2 + 9*a^2*b^2*c

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^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2 + 3*a^3*b*c^3)*d
^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^4*b*c^2
)*e^3)*f)*x - sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a
^3*b^2*c^3 + 4*a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^
3*c^2 - 20*a^4*b*c^3)*d^2*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*
b^2*c^2 - 4*a^5*c^3)*d*e^2 - (a^3*b^5 - 5*a^4*b^3*c + 4*a^5*b*c^2
)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 + 3*((a^4*b^4 - 5*a^5*b^2*c + 4*a
^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 + ((3*a^2*b^6 - 19*a^3*b
^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^5 - 16*a^4*b
^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^2)
*e^2)*f + ((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 -
6*a^7*b^2*c + 8*a^8*c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f
^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*
e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e
^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2
*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3
 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 -
4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a
^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*
c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^
2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(a^
4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*b^4 - 4*a^2*
b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*b^3
- 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*s
qrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b
*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*
c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a
^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7
*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*
a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2
*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 -
9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6
*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))
/(a^5*b^2 - 4*a^6*c))) - 6*(b*d - a*e)*x^2 + 2*a*d)/(a^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.30883, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] Done
```

3.60 $\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=329

$$-\frac{-abe-a(cd-af)+b^2d}{a^3x}+\frac{bd-ae}{3a^2x^3}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{5ax^5}$$

```
[Out] -d/(5*a*x^5) + (b*d - a*e)/(3*a^2*x^3) - (b^2*d - a*b*e - a*(c*d
- a*f))/(a^3*x) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) + (b^3*
d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2*d - a*b*e - a*(c
*d - a*f) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 4.14311, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{-abe-a(cd-af)+b^2d}{a^3x}+\frac{bd-ae}{3a^2x^3}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{5ax^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(5*a*x^5) + (b*d - a*e)/(3*a^2*x^3) - (b^2*d - a*b*e - a*(c*d
- a*f))/(a^3*x) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) + (b^3*
d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2*d - a*b*e - a*(c
*d - a*f) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.11125, size = 394, normalized size = 1.2

$$\frac{-\frac{6a^2d}{x^5}+\frac{30(abe+a(cd-af)+b^2(-d))}{x}-\frac{15\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ab\left(-e\sqrt{b^2-4ac}+af-3cd\right)+a\left(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+2ace\right)+b^2\left(d\sqrt{b^2-4ac}-a\cdots\right.\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{30a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-6*a^2*d)/x^5 + (10*a*(b*d - a*e))/x^3 + (30*(-(b^2*d) + a*b*e
+ a*(c*d - a*f)))/x - (15*Sqrt[2]*Sqrt[c]*(b^3*d + b^2*(Sqrt[b^2
- 4*a*c]*d - a*e) + a*b*(-3*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*
(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (15*Sqrt[2]*Sqrt[c]*(b^3*d
 - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a
*c]*e + a*f) + a*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*Sqrt[b^2 -
4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(30*a^3)
```

Maple [B] time = 0.044, size = 1121, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a^3*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*
d-3/2/a^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d
+1/2/a^3*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-
1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+1
/3/a^2/x^3*b*d+1/a^2/x*b*e+1/a^2/x*c*d-1/a^3/x*b^2*d+1/2/a*c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f-1/5*d/a/x^5-1/
2/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f+1/2/a*c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b*f-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e-3/2/a^2*c^2/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/2/a^3*c*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^2*d+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*e+1/2/a^2*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+1/2/a*c
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-1/2/a^2*c^2*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*d-1/3/a/x^3*e-1/a/x*f+1/2/a^2*c*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*b*e-1/2/a^3*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+1/a*c^
2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e-1/2/a^2*c*2^(
```

```
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{a^2bf-(abce-a^2cf-(b^2c-ac^2)d)x^2+(b^3-2\,abc)d-(ab^2-a^2c)e}{cx^4+bx^2+a}\,dx}{a^3}+\frac{15\left(abe-a^2f-\left(b^2-ac\right)d\right)x^4-3\,a^2d+5\left(abd-a^2e\right)x^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*b*f - (a*b*c*e - a^2*c*f - (b^2*c - a*c^2)*d)*x^2
 + (b^3 - 2*a*b*c)*d - (a*b^2 - a^2*c)*e)/(c*x^4 + b*x^2 + a), x)
/a^3 + 1/15*(15*(a*b*e - a^2*f - (b^2 - a*c)*d)*x^4 - 3*a^2*d + 5
*(a*b*d - a^2*e)*x^2)/(a^3*x^5)
```

Fricas [A] time = 18.4869, size = 21371, normalized size = 64.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*
c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 -
 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^
4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)
*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f + (a^7*b^2 - 4*a^8*
c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^
2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^
6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 -
72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 -
 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e
^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^
8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6
 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^
3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6
*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^
6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7
*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^
3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c
^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3
*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^
4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c
+ 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a
^7*b^2 - 4*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6
 - a^3*c^7)*d^4 - (b^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5*a^3*
b*c^6)*d^3*e + 3*(a*b^6*c^3 - 4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*
e^2 - (3*a^2*b^5*c^3 - 11*a^3*b^3*c^4 + 7*a^4*b*c^5)*d*e^3 + (a^3
*b^4*c^3 - 3*a^4*b^2*c^4 + a^5*c^5)*e^4 + (a^6*b^2*c^2 - a^7*c^3)
*f^4 + ((3*a^4*b^4*c^2 - 9*a^5*b^2*c^3 + 4*a^6*c^4)*d - (3*a^5*b^
3*c^2 - 5*a^6*b*c^3)*e)*f^3 + 3*((a^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7
*a^4*b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b^5*c^2 - 7*a^4*b^3*c^3 +
5*a^5*b*c^4)*d*e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e^2)*f^2 + ((b^8
*c^2 - 7*a*b^6*c^3 + 18*a^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4*c^6)
*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5
)*d^2*e + 3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3 + a^4*b^2*c^4)*d*e^2 - (
```

a^3*b^5*c^2 - a^4*b^3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x + sqrt(1/2)*((
b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*
c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10 - 30*a^2*b^8*c + 105*a^3*b^6*c
^2 - 151*a^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6*c^5)*d^2*e + (3*a^2
*b^9 - 27*a^3*b^7*c + 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 + 32*a^6*b*
c^4)*d*e^2 - (a^3*b^8 - 8*a^4*b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2
*c^3 + 4*a^7*c^4)*e^3 + (a^6*b^5 - 5*a^7*b^3*c + 4*a^8*b*c^2)*f^3
 + ((3*a^4*b^7 - 21*a^5*b^5*c + 40*a^6*b^3*c^2 - 16*a^7*b*c^3)*d
- (3*a^5*b^6 - 18*a^6*b^4*c + 25*a^7*b^2*c^2 - 4*a^8*c^3)*e)*f^2
+ ((3*a^2*b^9 - 27*a^3*b^7*c + 80*a^4*b^5*c^2 - 85*a^5*b^3*c^3 +
20*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^4*b^6*c + 59*a^5*b^4*c^2
- 45*a^6*b^2*c^3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 21*a^5*b^5*c + 4
1*a^6*b^3*c^2 - 20*a^7*b*c^3)*e^2)*f - ((a^7*b^6 - 8*a^8*b^4*c +
18*a^9*b^2*c^2 - 8*a^10*c^3)*d - (a^8*b^5 - 7*a^9*b^3*c + 12*a^10
*b*c^2)*e + (a^9*b^4 - 6*a^10*b^2*c + 8*a^11*c^2)*f)*sqrt(((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4
- 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^
3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e
+ 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3
+ 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c +
16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 -
 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a
^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c
+ 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c
^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a
^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^
7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8
*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^
4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^
3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*
c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*
b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2
 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))*sqrt(-((b^7 - 7*a
*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4
*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*
a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*
b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f
 + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2
 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^
4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 2
2*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c
 + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^
2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^
3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2
- 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^
2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d -
 (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 1
8*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 -
2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e +
 (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 +
 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9
*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*
b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a
^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^
5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*
b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))) - 15*sqrt(1/2)*a^3*x^5*sq
rt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*
b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5
*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a
^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c +
2*a^5*c^2)*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*
a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10
 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^
4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2
- 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c +
11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*
b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2
 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*
((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*
a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a

^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*
c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*
a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*
b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3
*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c
^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)
*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-2*((b^
6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*d^4 - (b^7*c^3 - 3
*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5*a^3*b*c^6)*d^3*e + 3*(a*b^6*c^3 -
4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*e^2 - (3*a^2*b^5*c^3 - 11*a^3*
b^3*c^4 + 7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 - 3*a^4*b^2*c^4 + a^5
*c^5)*e^4 + (a^6*b^2*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c^2 - 9*a^5
*b^2*c^3 + 4*a^6*c^4)*d - (3*a^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 +
3*((a^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7*a^4*b^2*c^4 - 2*a^5*c^5)*d^2
- (2*a^3*b^5*c^2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^4)*d*e + (a^4*b^4*c^
2 - 2*a^5*b^2*c^3)*e^2)*f^2 + ((b^8*c^2 - 7*a*b^6*c^3 + 18*a^2*b^
4*c^4 - 19*a^3*b^2*c^5 + 4*a^4*c^6)*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^
5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2*e + 3*(a^2*b^6*c^2 - 3*a
^3*b^4*c^3 + a^4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a^4*b^3*c^3 - 3*
a^5*b*c^4)*e^3)*f)*x - sqrt(1/2)*((b^11 - 11*a*b^9*c + 44*a^2*b^7
*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*c^4 - 8*a^5*b*c^5)*d^3 - (3*a*
b^10 - 30*a^2*b^8*c + 105*a^3*b^6*c^2 - 151*a^4*b^4*c^3 + 77*a^5*
b^2*c^4 - 4*a^6*c^5)*d^2*e + (3*a^2*b^9 - 27*a^3*b^7*c + 81*a^4*b
^5*c^2 - 92*a^5*b^3*c^3 + 32*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 8*a^4*
b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2*c^3 + 4*a^7*c^4)*e^3 + (a^6*b
^5 - 5*a^7*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4*b^7 - 21*a^5*b^5*c
+ 40*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^6*b^4*c +
25*a^7*b^2*c^2 - 4*a^8*c^3)*e)*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c +
80*a^4*b^5*c^2 - 85*a^5*b^3*c^3 + 20*a^6*b*c^4)*d^2 - 2*(3*a^3*b
^8 - 24*a^4*b^6*c + 59*a^5*b^4*c^2 - 45*a^6*b^2*c^3 + 4*a^7*c^4)*
d*e + (3*a^4*b^7 - 21*a^5*b^5*c + 41*a^6*b^3*c^2 - 20*a^7*b*c^3)*
e^2)*f - ((a^7*b^6 - 8*a^8*b^4*c + 18*a^9*b^2*c^2 - 8*a^10*c^3)*d
- (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*e + (a^9*b^4 - 6*a^10*
b^2*c + 8*a^11*c^2)*f)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2
- 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^
4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 2
2*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c
+ 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^
2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^
3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2
- 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^
2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d -
(a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 1
8*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 -
2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e +
(3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 +
4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9
*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*
b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a
^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^
5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*
b^2 - 4*a^15*c)))*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3
)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*
a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*
b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((
b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4
*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c +
29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d
^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4
*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7
*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*
b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4
+ (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b
^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^
9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 -
19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c +
21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 1
2*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c +
22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 -
(3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a
^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18

```
*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^
3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4
*a^8*c))) + 15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2
*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*
c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2
+ (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b
*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f - (a^7*b^2 - 4
*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c
^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 -
 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 -
 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c
^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*
b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4
)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3
 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^
6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a
^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 3
3*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 -
15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 1
2*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 -
 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 -
a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^
6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a
^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b
^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)
))/(a^7*b^2 - 4*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^
2*c^6 - a^3*c^7)*d^4 - (b^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5
*a^3*b*c^6)*d^3*e + 3*(a*b^6*c^3 - 4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)
*d^2*e^2 - (3*a^2*b^5*c^3 - 11*a^3*b^3*c^4 + 7*a^4*b*c^5)*d*e^3 +
 (a^3*b^4*c^3 - 3*a^4*b^2*c^4 + a^5*c^5)*e^4 + (a^6*b^2*c^2 - a^7
*c^3)*f^4 + ((3*a^4*b^4*c^2 - 9*a^5*b^2*c^3 + 4*a^6*c^4)*d - (3*a
^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 + 3*((a^2*b^6*c^2 - 5*a^3*b^4*c^
3 + 7*a^4*b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b^5*c^2 - 7*a^4*b^3*c
^3 + 5*a^5*b*c^4)*d*e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e^2)*f^2 +
((b^8*c^2 - 7*a*b^6*c^3 + 18*a^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4
*c^6)*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*
b*c^5)*d^2*e + 3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3 + a^4*b^2*c^4)*d*e^
2 - (a^3*b^5*c^2 - a^4*b^3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x + sqrt(1/
2)*((b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4
*b^3*c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10 - 30*a^2*b^8*c + 105*a^3*
b^6*c^2 - 151*a^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6*c^5)*d^2*e + (
3*a^2*b^9 - 27*a^3*b^7*c + 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 + 32*a
^6*b*c^4)*d*e^2 - (a^3*b^8 - 8*a^4*b^6*c + 20*a^5*b^4*c^2 - 17*a^
6*b^2*c^3 + 4*a^7*c^4)*e^3 + (a^6*b^5 - 5*a^7*b^3*c + 4*a^8*b*c^2
)*f^3 + ((3*a^4*b^7 - 21*a^5*b^5*c + 40*a^6*b^3*c^2 - 16*a^7*b*c^
3)*d - (3*a^5*b^6 - 18*a^6*b^4*c + 25*a^7*b^2*c^2 - 4*a^8*c^3)*e)
*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c + 80*a^4*b^5*c^2 - 85*a^5*b^3*c
^3 + 20*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^4*b^6*c + 59*a^5*b^4
*c^2 - 45*a^6*b^2*c^3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 21*a^5*b^5*
c + 41*a^6*b^3*c^2 - 20*a^7*b*c^3)*e^2)*f + ((a^7*b^6 - 8*a^8*b^4
*c + 18*a^9*b^2*c^2 - 8*a^10*c^3)*d - (a^8*b^5 - 7*a^9*b^3*c + 12
*a^10*b*c^2)*e + (a^9*b^4 - 6*a^10*b^2*c + 8*a^11*c^2)*f)*sqrt(((
b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4
*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c +
29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d
^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4
*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7
*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*
b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4
 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b
^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^
9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 -
 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c +
21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 1
2*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c +
22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 -
(3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a
^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18
*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^
3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))*sqrt(-((b^7
- 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^
2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c
```

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 + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5
*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)
*e)*f - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^
3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*
b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^
5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b
^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4
*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^
10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3
)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^
8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d
^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*
d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*
f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^
3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48
*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 -
 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2
- (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(
a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))) - 15*sqrt(1/2)*a^3*x
^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 -
2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^
5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 +
2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2
*c + 2*a^5*c^2)*e)*f - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b
^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2
- 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2
*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b
^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5
*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6
*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2
*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^
2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3
 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3
 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2
- 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 -
 a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2
- 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21
*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e
 + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 +
a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b
*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-2
*((b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*d^4 - (b^7*c^
3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5*a^3*b*c^6)*d^3*e + 3*(a*b^6*c
^3 - 4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*e^2 - (3*a^2*b^5*c^3 - 11
*a^3*b^3*c^4 + 7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 - 3*a^4*b^2*c^4
+ a^5*c^5)*e^4 + (a^6*b^2*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c^2 -
9*a^5*b^2*c^3 + 4*a^6*c^4)*d - (3*a^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f
^3 + 3*((a^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7*a^4*b^2*c^4 - 2*a^5*c^5)
*d^2 - (2*a^3*b^5*c^2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^4)*d*e + (a^4*b
^4*c^2 - 2*a^5*b^2*c^3)*e^2)*f^2 + ((b^8*c^2 - 7*a*b^6*c^3 + 18*a
^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4*c^6)*d^3 - 3*(a*b^7*c^2 - 5*a
^2*b^5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2*e + 3*(a^2*b^6*c^2
- 3*a^3*b^4*c^3 + a^4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a^4*b^3*c^3
 - 3*a^5*b*c^4)*e^3)*f)*x - sqrt(1/2)*((b^11 - 11*a*b^9*c + 44*a^
2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*c^4 - 8*a^5*b*c^5)*d^3 -
(3*a*b^10 - 30*a^2*b^8*c + 105*a^3*b^6*c^2 - 151*a^4*b^4*c^3 + 77
*a^5*b^2*c^4 - 4*a^6*c^5)*d^2*e + (3*a^2*b^9 - 27*a^3*b^7*c + 81*
a^4*b^5*c^2 - 92*a^5*b^3*c^3 + 32*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 8
*a^4*b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2*c^3 + 4*a^7*c^4)*e^3 + (
a^6*b^5 - 5*a^7*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4*b^7 - 21*a^5*b
^5*c + 40*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^6*b^4
*c + 25*a^7*b^2*c^2 - 4*a^8*c^3)*e)*f^2 + ((3*a^2*b^9 - 27*a^3*b^
7*c + 80*a^4*b^5*c^2 - 85*a^5*b^3*c^3 + 20*a^6*b*c^4)*d^2 - 2*(3*
a^3*b^8 - 24*a^4*b^6*c + 59*a^5*b^4*c^2 - 45*a^6*b^2*c^3 + 4*a^7*
c^4)*d*e + (3*a^4*b^7 - 21*a^5*b^5*c + 41*a^6*b^3*c^2 - 20*a^7*b*
c^3)*e^2)*f + ((a^7*b^6 - 8*a^8*b^4*c + 18*a^9*b^2*c^2 - 8*a^10*c
^3)*d - (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*e + (a^9*b^4 - 6*
a^10*b^2*c + 8*a^11*c^2)*f)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
```

```
6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^
3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*
b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^
5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b
^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4
*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^
10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3
)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^
8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d
^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*
d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*
f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^
3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48
*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 -
 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2
- (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(
a^14*b^2 - 4*a^15*c)))*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^
4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3
 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d -
(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f - (a^7*b^2 - 4*a^8*c)*sq
rt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^
4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9
*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c
^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^
5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^
4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 +
(a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4
)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*
a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c +
 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*
c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5
*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*
c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8
*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d
^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3
+ 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2
 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a
^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^
2 - 4*a^8*c))) - 30*(a*b*e - a^2*f - (b^2 - a*c)*d)*x^4 + 6*a^2*d
 - 10*(a*b*d - a^2*e)*x^2)/(a^3*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.76098, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6),x, algorithm="giac")
```

```
[Out] Done
```

3.61 $\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=320

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^3e-b^3c(cd-20af)-12ab^2c^2e+6abc^2(cd-5af)-3b^5f+2b^4ce\right)}{2c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(-2c(4af+be)+3b^2f+4c^2d\right)}{4c^2\left(b^2-4ac\right)}+\frac{x^6\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(a+bx^2+cx^4\right)\left(-2c(af+be)+3b^2f+c^2d\right)}{4c^4}+\frac{x^2\left(-bc(cd-11af)-6ac^2e-3b^3f+2b^2ce\right)}{2c^3\left(b^2-4ac\right)}$$

```
[Out] ((2*b^2*c*e - 6*a*c^2*e - 3*b^3*f - b*c*(c*d - 11*a*f))*x^2)/(2*c
^3*(b^2 - 4*a*c)) + ((4*c^2*d + 3*b^2*f - 2*c*(b*e + 4*a*f))*x^4)
/(4*c^2*(b^2 - 4*a*c)) + (x^6*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d
 - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)) - ((2*b^4*c*e - 12*a*b^2*c^2*e + 12*a^2*c^3*e - 3*b^5*f
- b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(c*d - 5*a*f))*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^2 - 4*a*c)^(3/2)) + ((c^2*d
+ 3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rubi [A] time = 2.46279, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^3e-b^3c(cd-20af)-12ab^2c^2e+6abc^2(cd-5af)-3b^5f+2b^4ce\right)}{2c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(-2c(4af+be)+3b^2f+4c^2d\right)}{4c^2\left(b^2-4ac\right)}+\frac{x^6\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(a+bx^2+cx^4\right)\left(-2c(af+be)+3b^2f+c^2d\right)}{4c^4}+\frac{x^2\left(-bc(cd-11af)-6ac^2e-3b^3f+2b^2ce\right)}{2c^3\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] ((2*b^2*c*e - 6*a*c^2*e - 3*b^3*f - b*c*(c*d - 11*a*f))*x^2)/(2*c
^3*(b^2 - 4*a*c)) + ((4*c^2*d + 3*b^2*f - 2*c*(b*e + 4*a*f))*x^4)
/(4*c^2*(b^2 - 4*a*c)) + (x^6*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d
 - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)) - ((2*b^4*c*e - 12*a*b^2*c^2*e + 12*a^2*c^3*e - 3*b^5*f
- b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(c*d - 5*a*f))*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^2 - 4*a*c)^(3/2)) + ((c^2*d
+ 3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.0414, size = 309, normalized size = 0.97

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-12a^2c^3e+b^3c(cd-20af)+12ab^2c^2e+6abc^2(5af-cd)+3b^5f-2b^4ce\right)}{(4ac-b^2)^{3/2}}+\frac{2\left(2a^3c^2f+a^2c\left(-4b^2f+bc\left(3e+5fx^2\right)-2c^2\left(d+ex^2\right)\right)+ab\left(b^3f-b^2c(\right.\right.}{(b^2-4ac)(a+bx}}{4c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*(c*e - 2*b*f)*x^2 + c^2*f*x^4 + (2*(2*a^3*c^2*f + b^3*(c^2*d
 - b*c*e + b^2*f)*x^2 + a*b*(b^3*f - 3*c^3*d*x^2 + b*c^2*(d + 4*e
*x^2) - b^2*c*(e + 5*f*x^2)) + a^2*c*(-4*b^2*f - 2*c^2*(d + e*x^2
) + b*c*(3*e + 5*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
(2*(-2*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2*c^3*e + 3*b^5*f + b^3*c*
(c*d - 20*a*f) + 6*a*b*c^2*(-(c*d) + 5*a*f))*ArcTan[(b + 2*c*x^2)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c^2*d + 3*b^2*f - 2
*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Maple [B] time = 0.026, size = 1764, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4/c^2*x^4*f+1/2/c^2*x^2*e-1/c^2/(c*x^4+b*x^2+a)*a^3/(4*a*c-b^2)
*f+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*d-1/2/c^4/(c*x^4+b*x^2+a)/
(4*a*c-b^2)*x^2*b^5*f-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^3
*d-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*e+1/2/c^3/(c*x^4+b*x
^2+a)/(4*a*c-b^2)*x^2*b^4*e-1/c^3*b*f*x^2+1/c/(4*a*c-b^2)*ln((4*a
*c-b^2)*(c*x^4+b*x^2+a))*a*d-1/c^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*
a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64
*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^4*e-3/4/c^4/(4*a
*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^4*f+1/2/c^3/(4*a*c-b^2)
*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^3*e-1/4/c^2/(4*a*c-b^2)*ln((4*
a*c-b^2)*(c*x^4+b*x^2+a))*b^2*d-3/c/(64*a^3*c^3-48*a^2*b^2*c^2+12
*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(6
4*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a*b*d-1/2/c^4/(c*
x^4+b*x^2+a)*a/(4*a*c-b^2)*b^4*f-6/c/(64*a^3*c^3-48*a^2*b^2*c^2+1
2*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(
64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*e*a^2+3/2/c^4/(6
4*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b
^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^
6)^(1/2))*b^5*f+1/2/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6
)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48
*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^3*d-2/c^2/(4*a*c-b^2)*ln((4
*a*c-b^2)*(c*x^4+b*x^2+a))*a^2*f+7/2/c^3/(4*a*c-b^2)*ln((4*a*c-b^
2)*(c*x^4+b*x^2+a))*a*b^2*f-2/c^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x
^4+b*x^2+a))*a*b*e-10/c^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b
^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a*b^3*f+2/c^3/(c*x^4+b*x^2+
a)*a^2/(4*a*c-b^2)*b^2*f+1/2/c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^
3*e+15/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arcta
n((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+
12*a*b^4*c-b^6)^(1/2))*a^2*b*f+6/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+1
2*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(
64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a*b^2*e-1/2/c^2/
(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^2*d+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^
```

```
2)*x^2*a^2*e+5/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^3*f-2/c^
2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2*e+3/2/c/(c*x^4+b*x^2+a)/(
4*a*c-b^2)*x^2*a*b*d-5/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*
b*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.657548, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^4*c^2 - 6*a*b^2*c^3 + 6*a
^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*f)*x^4 + ((b^
4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*e
+ (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a
^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^
5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*f)*log(-(b^3 - 4*a*b*c + 2*(b^2*
c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2
 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^2*c^3 - 4*a*c^4)*f*x^8 + (2
*(b^2*c^3 - 4*a*c^4)*e - 3*(b^3*c^2 - 4*a*b*c^3)*f)*x^6 + (2*(b^3
*c^2 - 4*a*b*c^3)*e - (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*f)*x^4
 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 6*a^2*c^3)
*e + (b^5 - 7*a*b^3*c + 13*a^2*b*c^2)*f)*x^2 + 2*(a*b^2*c^2 - 2*a
^2*c^3)*d - 2*(a*b^3*c - 3*a^2*b*c^2)*e + 2*(a*b^4 - 4*a^2*b^2*c
+ 2*a^3*c^2)*f + (((b^2*c^3 - 4*a*c^4)*d - 2*(b^3*c^2 - 4*a*b*c^3
)*e + (3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*f)*x^4 + ((b^3*c^2 - 4
*a*b*c^3)*d - 2*(b^4*c - 4*a*b^2*c^2)*e + (3*b^5 - 14*a*b^3*c + 8
*a^2*b*c^2)*f)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a
^2*b*c^2)*e + (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*f)*log(c*x^4 +
 b*x^2 + a))*sqrt(b^2 - 4*a*c))/((a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^
5 - 4*a*c^6)*x^4 + (b^3*c^4 - 4*a*b*c^5)*x^2)*sqrt(b^2 - 4*a*c)),
 -1/4*(2*(((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^4*c^2 - 6*a*b^2*c^3 + 6
*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*f)*x^4 + ((
b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*
e + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6
*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*
b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(
-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^2*c^3 - 4*a*c^4)*f*x^8 + (2*(b
^2*c^3 - 4*a*c^4)*e - 3*(b^3*c^2 - 4*a*b*c^3)*f)*x^6 + (2*(b^3*c^
2 - 4*a*b*c^3)*e - (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*f)*x^4 +
2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 6*a^2*c^3)*e
+ (b^5 - 7*a*b^3*c + 13*a^2*b*c^2)*f)*x^2 + 2*(a*b^2*c^2 - 2*a^2*
c^3)*d - 2*(a*b^3*c - 3*a^2*b*c^2)*e + 2*(a*b^4 - 4*a^2*b^2*c + 2
*a^3*c^2)*f + (((b^2*c^3 - 4*a*c^4)*d - 2*(b^3*c^2 - 4*a*b*c^3)*e
 + (3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*f)*x^4 + ((b^3*c^2 - 4*a*
b*c^3)*d - 2*(b^4*c - 4*a*b^2*c^2)*e + (3*b^5 - 14*a*b^3*c + 8*a^
2*b*c^2)*f)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*
b*c^2)*e + (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*f)*log(c*x^4 + b*
x^2 + a))*sqrt(-b^2 + 4*a*c))/((a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5
- 4*a*c^6)*x^4 + (b^3*c^4 - 4*a*b*c^5)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.62 $\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=236

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)+b^3(-(ce-2bf))\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(-c(6af+be)+2b^2f+2c^2d\right)}{2c^2\left(b^2-4ac\right)}+\frac{x^4\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(ce-2bf)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

```
[Out] ((2*c^2*d + 2*b^2*f - c*(b*e + 6*a*f))*x^2)/(2*c^2*(b^2 - 4*a*c))
 + (x^4*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a
*c*f)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((12*a^2*c^
2*f - b^3*(c*e - 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*Ar
cTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(3/2
)) + ((c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 0.928076, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)+b^3(-(ce-2bf))\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(-c(6af+be)+2b^2f+2c^2d\right)}{2c^2\left(b^2-4ac\right)}+\frac{x^4\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(ce-2bf)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] ((2*c^2*d + 2*b^2*f - c*(b*e + 6*a*f))*x^2)/(2*c^2*(b^2 - 4*a*c))
 + (x^4*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a
*c*f)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((12*a^2*c^
2*f - b^3*(c*e - 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*Ar
cTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(3/2
)) + ((c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^2} f\,dx}{2c^2}-\frac{\left(bf-\frac{ce}{2}\right)\log\left(a+bx^2+cx^4\right)}{2c^3}-\frac{a\left(-3abcf+2ac^2e+b^3f-b^2ce+bc^2d\right)+x^2\left(2a^2c^2f-4ab^2cf+3abc^2e-2ac^3d+b^4f-b^3ce+b^2c^2d\right)}{2c^3\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-4acf+4b^2f-3bce+2c^2d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^3\sqrt{-4ac+b^2}}+\frac{\left(2a^2c^2f-4ab^2cf+3abc^2e-2ac^3d+b^4f-b^3ce+b^2c^2d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{c^3\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Integral(f, (x, x**2))/(2*c**2) - (b*f - c*e/2)*log(a + b*x**2 +
c*x**4)/(2*c**3) - (a*(-3*a*b*c*f + 2*a*c**2*e + b**3*f - b**2*c*
e + b*c**2*d) + x**2*(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e
 - 2*a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d))/(2*c**3*(-4*a*c
 + b**2)*(a + b*x**2 + c*x**4)) - (-4*a*c*f + 4*b**2*f - 3*b*c*e
+ 2*c**2*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**3*sqr
t(-4*a*c + b**2)) + (2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e
- 2*a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)*atanh((b + 2*c*x*
*2)/sqrt(-4*a*c + b**2))/(c**3*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.682671, size = 236, normalized size = 1.

$$\frac{-\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)+b^3(2bf-ce)\right)}{(4ac-b^2)^{3/2}}-\frac{2\left(a^2c\left(2c\left(e+fx^2\right)-3bf\right)+a\left(b^3f-b^2c\left(e+4fx^2\right)+bc^2\left(d+3ex^2\right)-2c^3dx^2\right)+b^2x^2\left(b^2f-\right.\right.}{(b^2-4ac)(a+bx^2+cx^4)}}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*f*x^2 - (2*(b^2*(c^2*d - b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f
+ 2*c*(e + f*x^2)) + a*(b^3*f - 2*c^3*d*x^2 + b*c^2*(d + 3*e*x^2)
 - b^2*c*(e + 4*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (
2*(12*a^2*c^2*f + b^3*(-(c*e) + 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e
 + 6*b^2*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*
a*c)^(3/2) + (c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [B] time = 0.021, size = 1266, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*x^2*f/c^2+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*f-2/c^2/(c*
x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2*f+3/2/c/(c*x^4+b*x^2+a)/(4*a*c
-b^2)*x^2*a*b*e-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*d+1/2/c^3/(c*
x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^4*f-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c
-b^2)*x^2*b^3*e+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^2*d-3/2/c
^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*f+1/c/(c*x^4+b*x^2+a)*a^2/(4
*a*c-b^2)*e+1/2/c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*f-1/2/c^2/(
c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^2*e+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c
-b^2)*d*b-2/c^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*a*b*f
+1/c/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*a*e+1/2/c^3/(4*a
*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^3*f-1/4/c^2/(4*a*c-b^2)
*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^2*e-6/c/(64*a^3*c^3-48*a^2*b^2
*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2
)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a^2*f+6/c^
2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a
*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*
c-b^6)^(1/2))*a*b^2*f-3/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b
^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a*b*e+2/(64*a^3*c^3-48*a^2*
b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-
b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a*d-1/c
^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*
a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4
*c-b^6)^(1/2))*b^4*f+1/2/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*
c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c
^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^3*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.428719, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d + (b^4*c - 6
*a*b^2*c^2)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^3
*c - 6*a^2*b*c^2)*e - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*log(
(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2
 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (2*(b^2
*c^2 - 4*a*c^3)*f*x^6 + 2*(b^3*c - 4*a*b*c^2)*f*x^4 - 2*a*b*c^2*d
 - 2*((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 5*a*
b^2*c + 6*a^2*c^2)*f)*x^2 + 2*(a*b^2*c - 2*a^2*c^2)*e - 2*(a*b^3
- 3*a^2*b*c)*f + (((b^2*c^2 - 4*a*c^3)*e - 2*(b^3*c - 4*a*b*c^2)*
f)*x^4 + ((b^3*c - 4*a*b*c^2)*e - 2*(b^4 - 4*a*b^2*c)*f)*x^2 + (a
*b^2*c - 4*a^2*c^2)*e - 2*(a*b^3 - 4*a^2*b*c)*f)*log(c*x^4 + b*x^
2 + a))*sqrt(b^2 - 4*a*c))/((a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4
*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2)*sqrt(b^2 - 4*a*c)), -1/4
*(2*(4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2*(b^4*
c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d + (b^4*c - 6*a
*b^2*c^2)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^3*c
 - 6*a^2*b*c^2)*e - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*arctan
(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*(b^2*c^2 -
 4*a*c^3)*f*x^6 + 2*(b^3*c - 4*a*b*c^2)*f*x^4 - 2*a*b*c^2*d - 2*(
(b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 5*a*b^2*c
+ 6*a^2*c^2)*f)*x^2 + 2*(a*b^2*c - 2*a^2*c^2)*e - 2*(a*b^3 - 3*a^
2*b*c)*f + (((b^2*c^2 - 4*a*c^3)*e - 2*(b^3*c - 4*a*b*c^2)*f)*x^4
 + ((b^3*c - 4*a*b*c^2)*e - 2*(b^4 - 4*a*b^2*c)*f)*x^2 + (a*b^2*c
 - 4*a^2*c^2)*e - 2*(a*b^3 - 4*a^2*b*c)*f)*log(c*x^4 + b*x^2 + a)
)*sqrt(-b^2 + 4*a*c))/((a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^
5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.63 $\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=165

$$\frac{x^2\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{f\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] (x^2*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*
f)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((4*a*c^2*e +
b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a
*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (f*Log[a + b*x^2 + c*x^4])/(4
*c^2)
```

Rubi [A] time = 0.561525, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x^2\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{f\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*
f)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((4*a*c^2*e +
b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a
*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (f*Log[a + b*x^2 + c*x^4])/(4
*c^2)
```

Rubi in Sympy [A] time = 59.5452, size = 231, normalized size = 1.4

$$\frac{f\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{a\left(-2acf+b^2f-bce+2c^2d\right)+x^2\left(-3abcf+2ac^2e+b^3f-b^2ce+bc^2d\right)}{2c^2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}+\frac{(3bf-2ce)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\sqrt{-4ac+b^2}}-\frac{\left(-3abcf+2ac^2e+b^3f-b^2ce+bc^2d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{c^2\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] f*log(a + b*x**2 + c*x**4)/(4*c**2) + (a*(-2*a*c*f + b**2*f - b*c
*e + 2*c**2*d) + x**2*(-3*a*b*c*f + 2*a*c**2*e + b**3*f - b**2*c*
e + b*c**2*d))/(2*c**2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)) + (
3*b*f - 2*c*e)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**2*
sqrt(-4*a*c + b**2)) - (-3*a*b*c*f + 2*a*c**2*e + b**3*f - b**2*c
*e + b*c**2*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(c**2*(-
4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.4881, size = 175, normalized size = 1.06

$$\frac{\frac{2\left(-2a^2cf+a\left(b^2f-bc\left(e+3fx^2\right)+2c^2\left(d+ex^2\right)\right)+bx^2\left(b^2f-bce+c^2d\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right)}{(4ac-b^2)^{3/2}}+f\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 + a*(b^2*f + 2*c^
2*(d + e*x^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + (2*(4*a*c^2*e + b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTan[(b
+ 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + f*Log[a +
b*x^2 + c*x^4])/(4*c^2)
```

Maple [B] time = 0.019, size = 671, normalized size = 4.1

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(\frac{\left(3\,abcf-2\,ac^2e-b^3f+b^2ce-c^2db\right)x^2}{(4\,ac-b^2)\,c^2}+\frac{a\left(2\,acf-b^2f+bce-2\,c^2d\right)}{(4\,ac-b^2)\,c^2}\right)$$
$$+\frac{\ln\left(c\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)af}{(4\,ac-b^2)\,c}-\frac{\ln\left(c\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b^2f}{(16\,ac-4\,b^2)\,c^2}$$
$$-3\,\frac{abf}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\arctan\left(\frac{2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)$$
$$+2\,\frac{ace}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\arctan\left(\frac{2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)$$
$$-bcd\arctan\left(\left(2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3}}$$
$$+\frac{b^3f}{2\,c}\arctan\left(\left(2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*((3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)/c^2*
x^2+a*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2
+a)+1/(4*a*c-b^2)/c*ln(c*(4*a*c-b^2)*(c*x^4+b*x^2+a))*a*f-1/4/(4*
a*c-b^2)/c^2*ln(c*(4*a*c-b^2)*(c*x^4+b*x^2+a))*b^2*f-3/(64*a^3*c^
5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*c^2*(4*a*c
-b^2)*x^2+c*(4*a*c-b^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^
3-b^6*c^2)^(1/2))*a*b*f+2/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3
-b^6*c^2)^(1/2)*arctan((2*c^2*(4*a*c-b^2)*x^2+c*(4*a*c-b^2)*b)/(6
4*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2))*a*c*e-1/(64
*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*c^2
*(4*a*c-b^2)*x^2+c*(4*a*c-b^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+12*a
*b^4*c^3-b^6*c^2)^(1/2))*b*c*d+1/2/(64*a^3*c^5-48*a^2*b^2*c^4+12*
a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*c^2*(4*a*c-b^2)*x^2+c*(4*a*c-b
^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2))*b^
3/c*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.328828, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a*b*c^2*d - 4*a^2*c^2*e + (2*b*c^3*d - 4*a*c^3*e - (b^3
*c - 6*a*b*c^2)*f)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2*e - (b^4 - 6*a*
b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*log((b^3 - 4*a*b*c + 2*(b^
2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b
^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (4*a*c^2*d - 2*a*b*c*e + 2*(b
*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*x^2 + 2*(a*b^2
- 2*a^2*c)*f + ((b^2*c - 4*a*c^2)*f*x^4 + (b^3 - 4*a*b*c)*f*x^2 +
 (a*b^2 - 4*a^2*c)*f)*log(c*x^4 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/
((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*
a*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)), 1/4*(2*(2*a*b*c^2*d - 4*a^2*c^2
*e + (2*b*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^2
*c^2*d - 4*a*b*c^2*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*
b*c)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 (4*a*c^2*d - 2*a*b*c*e + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3
 - 3*a*b*c)*f)*x^2 + 2*(a*b^2 - 2*a^2*c)*f + ((b^2*c - 4*a*c^2)*f
*x^4 + (b^3 - 4*a*b*c)*f*x^2 + (a*b^2 - 4*a^2*c)*f)*log(c*x^4 + b
*x^2 + a))*sqrt(-b^2 + 4*a*c))/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3
 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.64 $\int \frac{x\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=123

$$\frac{x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af-be+2cd)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^
2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*c*d - b*e + 2*a*
f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.351927, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$\frac{x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af-be+2cd)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^
2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*c*d - b*e + 2*a*
f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 19.5848, size = 114, normalized size = 0.93

$$\frac{(2af-be+2cd)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{abf-2ace+bcd+x^2\left(-2acf+b^2f-bce+2c^2d\right)}{2c\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (2*a*f - b*e + 2*c*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(
-4*a*c + b**2)**(3/2) - (a*b*f - 2*a*c*e + b*c*d + x**2*(-2*a*c*f
 + b**2*f - b*c*e + 2*c**2*d))/(2*c*(-4*a*c + b**2)*(a + b*x**2 +
 c*x**4))
```

Mathematica [A] time = 0.197209, size = 130, normalized size = 1.06

$$\frac{abf-2ac\left(e+fx^2\right)+b^2fx^2+bc\left(d-ex^2\right)+2c^2dx^2}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)(-2af+be-2cd)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(e + f
*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*c*d + b*e
- 2*a*f)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)
```

```
^(3/2)
```

Maple [A] time = 0.014, size = 205, normalized size = 1.7

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(-\frac{\left(2\,acf-b^2f+bce-2\,c^2d\right)x^2}{\left(4\,ac-b^2\right)c}+\frac{abf-2\,ace+bcd}{\left(4\,ac-b^2\right)c}\right)+2\,\frac{fa}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)-be\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\left(4\,ac-b^2\right)^{-\frac{3}{2}}+2\,\frac{cd}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c*x^2+1/c*(a*b*f-
2*a*c*e+b*c*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)^(3/2)*a
rctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*f*a-1/(4*a*c-b^2)^(3/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e+2/(4*a*c-b^2)^(3/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295108, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(2\,c^3d-bc^2e+2\,ac^2f\right)x^4+2\,ac^2d-abce+2\,a^2cf+\left(2\,bc^2d-b^2ce+2\,abcf\right)x^2\right)\log\left(-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2-\left(2\,c^2x^4+2\right.}{cx^4+bx^2+a}\right.}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,ab\right.\right.}\right.$$

$$-\frac{2\left(\left(2\,c^3d-bc^2e+2\,ac^2f\right)x^4+2\,ac^2d-abce+2\,a^2cf+\left(2\,bc^2d-b^2ce+2\,abcf\right)x^2\right)\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left(bcd\right.}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)\sqrt{-b^2+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d - a*b*c*e
 + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*log(-(b^3 -
 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2
 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b*c*d - 2*a*
c*e + a*b*f + (2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^2)*sqrt(b^2 -
 4*a*c))/(((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c
 - 4*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)), -1/2*(2*((2*c^3*d - b*c^2*
e + 2*a*c^2*f)*x^4 + 2*a*c^2*d - a*b*c*e + 2*a^2*c*f + (2*b*c^2*d
 - b^2*c*e + 2*a*b*c*f)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
```

```
a*c)/(b^2 - 4*a*c)) + (b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e
 + (b^2 - 2*a*c)*f)*x^2)*sqrt(-b^2 + 4*a*c))/(((b^2*c^2 - 4*a*c^3
)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-b^2
+ 4*a*c))]
```

Sympy [A] time = 131.162, size = 474, normalized size = 3.85

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(2af-be+2cd)\log\left(x^2+\frac{-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(2af-be+2cd)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(2af-be+2cd)+2abf-b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(2af-b}{4acf-2bce+4c^2d}\right.}{2}$$

$$+\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(2af-be+2cd)\log\left(x^2+\frac{16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(2af-be+2cd)-8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(2af-be+2cd)+2abf+b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(2af-be}{4acf-2bce+4c^2d}\right.}{2}$$

$$-\frac{-abf+2ace-bcd+x^2\left(2acf-b^2f+bce-2c^2d\right)}{8a^2c^2-2ab^2c+x^4\left(8ac^3-2b^2c^2\right)+x^2\left(8abc^2-2b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d)*log(x**2 + (-16
*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 8*a
*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 2*a*b*
f - b**4*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) - b**2*
e + 2*b*c*d)/(4*a*c*f - 2*b*c*e + 4*c**2*d))/2 + sqrt(-1/(4*a*c -
 b**2)**3)*(2*a*f - b*e + 2*c*d)*log(x**2 + (16*a**2*c**2*sqrt(-1
/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) - 8*a*b**2*c*sqrt(-1/(4
*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 2*a*b*f + b**4*sqrt(-1/(
4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) - b**2*e + 2*b*c*d)/(4*a*
c*f - 2*b*c*e + 4*c**2*d))/2 - (-a*b*f + 2*a*c*e - b*c*d + x**2*(
2*a*c*f - b**2*f + b*c*e - 2*c**2*d))/(8*a**2*c**2 - 2*a*b**2*c +
 x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.65 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=166

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(4a^2ce-2ab(af+3cd)+b^3d)}{2a^2(b^2-4ac)^{3/2}}-\frac{d\log(a+bx^2+cx^4)}{4a^2}+\frac{d\log(x)}{a^2}+\frac{x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

```
[Out] (b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2)
/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*d + 4*a^2*c*e -
2*a*b*(3*c*d + a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2
*a^2*(b^2 - 4*a*c)^(3/2)) + (d*Log[x])/a^2 - (d*Log[a + b*x^2 + c
*x^4])/(4*a^2)
```

Rubi [A] time = 0.825503, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(4a^2ce-2ab(af+3cd)+b^3d)}{2a^2(b^2-4ac)^{3/2}}-\frac{d\log(a+bx^2+cx^4)}{4a^2}+\frac{d\log(x)}{a^2}+\frac{x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2)
/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*d + 4*a^2*c*e -
2*a*b*(3*c*d + a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2
*a^2*(b^2 - 4*a*c)^(3/2)) + (d*Log[x])/a^2 - (d*Log[a + b*x^2 + c
*x^4])/(4*a^2)
```

Rubi in Sympy [A] time = 37.1041, size = 194, normalized size = 1.17

$$\frac{2a^2f-abe-2acd+b^2d+x^2(abf-2ace+bcd)}{2a(-4ac+b^2)(a+bx^2+cx^4)}-\frac{(abf-2ace+bcd)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{a(-4ac+b^2)^{\frac{3}{2}}}+\frac{bd\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2\sqrt{-4ac+b^2}}+\frac{d\log(x^2)}{2a^2}-\frac{d\log(a+bx^2+cx^4)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (2*a**2*f - a*b*e - 2*a*c*d + b**2*d + x**2*(a*b*f - 2*a*c*e + b*
c*d))/(2*a*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)) - (a*b*f - 2*a*
c*e + b*c*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(a*(-4*a*c
 + b**2)**(3/2)) + b*d*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/
(2*a**2*sqrt(-4*a*c + b**2)) + d*log(x**2)/(2*a**2) - d*log(a + b
*x**2 + c*x**4)/(4*a**2)
```

Mathematica [A] time = 0.927853, size = 268, normalized size = 1.61

$$-\frac{-\frac{2a\left(b\left(-ae+afx^2+cdx^2\right)+2a\left(af-c\left(d+ex^2\right)\right)+b^2d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(4ac\left(ae-d\sqrt{b^2-4ac}\right)+b^2d\sqrt{b^2-4ac}-2ab(af+3cd)+b^3d\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\log\left(\sqrt{b^2-4\cdots\right.}{\cdots}}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -((-2*a*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d
 + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - 4*d*Log[x] + (
(b^3*d + b^2*Sqrt[b^2 - 4*a*c]*d + 4*a*c*(-(Sqrt[b^2 - 4*a*c]*d)
+ a*e) - 2*a*b*(3*c*d + a*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/(b^2 - 4*a*c)^(3/2) + ((-(b^3*d) + b^2*Sqrt[b^2 - 4*a*c]*d - 4
*a*c*(Sqrt[b^2 - 4*a*c]*d + a*e) + 2*a*b*(3*c*d + a*f))*Log[b + S
qrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [B] time = 0.024, size = 744, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b*f+c/(c*x^4+b*x^2+a)/(4*a*c
-b^2)*x^2*e-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b*c*d-a/(c*x^4+
b*x^2+a)/(4*a*c-b^2)*f+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*e+1/(c*x
^4+b*x^2+a)/(4*a*c-b^2)*c*d-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2
*d-1/a/(4*a*c-b^2)*c*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*d+1/4/a^2/(4
*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^2*d-1/(64*a^3*c^3-48*
a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*
a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b*f
+2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*
a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4
*c-b^6)^(1/2))*c*e-3/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*
a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b*c*d+1/2/a^2/(64*a^3*c^3-48*a
^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a
*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^3*
d+d*ln(x)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34886, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] [1/4*((4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f + (b^3*
c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b
^2*c)*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*log((b^3 - 4*a*b*c + 2*(b^2
*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^
2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (2*a^2*b*e - 4*a^3*f - 2*(a*b*
c*d - 2*a^2*c*e + a^2*b*f)*x^2 - 2*(a*b^2 - 2*a^2*c)*d + ((b^2*c
- 4*a*c^2)*d*x^4 + (b^3 - 4*a*b*c)*d*x^2 + (a*b^2 - 4*a^2*c)*d)*l
og(c*x^4 + b*x^2 + a) - 4*((b^2*c - 4*a*c^2)*d*x^4 + (b^3 - 4*a*b
*c)*d*x^2 + (a*b^2 - 4*a^2*c)*d)*log(x))*sqrt(b^2 - 4*a*c))/((a^3
*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b
*c)*x^2)*sqrt(b^2 - 4*a*c)), -1/4*(2*(4*a^3*c*e - 2*a^3*b*f + (4*
a^2*c^2*e - 2*a^2*b*c*f + (b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c
*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*c)*d)*x^2 + (a*b^3 - 6*a^2*b*c)
*d)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*
a^2*b*e - 4*a^3*f - 2*(a*b*c*d - 2*a^2*c*e + a^2*b*f)*x^2 - 2*(a*
b^2 - 2*a^2*c)*d + ((b^2*c - 4*a*c^2)*d*x^4 + (b^3 - 4*a*b*c)*d*x
^2 + (a*b^2 - 4*a^2*c)*d)*log(c*x^4 + b*x^2 + a) - 4*((b^2*c - 4*
a*c^2)*d*x^4 + (b^3 - 4*a*b*c)*d*x^2 + (a*b^2 - 4*a^2*c)*d)*log(x
))*sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c
^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.66 $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=234

$$\frac{(2bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2bd-ae)}{a^3}-\frac{2a^2ce+cx^2\left(-abe-2a(cd-af)+b^2d\right)-ab^2e-ab(3cd-af)+b^3d}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{d}{2a^2x^2}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(6a^2bce+4a^2c(3cd-af)-ab^3e-12ab^2cd+2b^4d\right)}{2a^3\left(b^2-4ac\right)^{3/2}}$$

```
[Out] -d/(2*a^2*x^2) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f)
 + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x^2)/(2*a^2*(b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) - ((2*b^4*d - 12*a*b^2*c*d - a*b^3*e + 6*a^2
*b*c*e + 4*a^2*c*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*d - a*e)*Log[x])/a^3
 + ((2*b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi [A] time = 1.4444, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$

$$\frac{(2bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2bd-ae)}{a^3}-\frac{2a^2ce+cx^2\left(-abe-2a(cd-af)+b^2d\right)-ab^2e-ab(3cd-af)+b^3d}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{d}{2a^2x^2}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(6a^2bce+4a^2c(3cd-af)-ab^3e-12ab^2cd+2b^4d\right)}{2a^3\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -d/(2*a^2*x^2) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f)
 + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x^2)/(2*a^2*(b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) - ((2*b^4*d - 12*a*b^2*c*d - a*b^3*e + 6*a^2
*b*c*e + 4*a^2*c*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*d - a*e)*Log[x])/a^3
 + ((2*b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi in Sympy [A] time = 59.2522, size = 265, normalized size = 1.13

$$\frac{c\left(2a^2f-abe-2acd+b^2d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{a^2\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{d}{2a^2x^2}-\frac{a^2bf+2a^2ce-ab^2e-3abcd+b^3d+cx^2\left(2a^2f-abe-2acd+b^2d\right)}{2a^2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}-\frac{\left(\frac{ae}{2}-bd\right)\log\left(a+bx^2+cx^4\right)}{2a^3}+\frac{(ae-2bd)\log\left(x^2\right)}{2a^3}-\frac{\left(-abe-2acd+2b^2d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^3\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] c*(2*a**2*f - a*b*e - 2*a*c*d + b**2*d)*atanh((b + 2*c*x**2)/sqrt
(-4*a*c + b**2))/(a**2*(-4*a*c + b**2)**(3/2)) - d/(2*a**2*x**2)
```

```
- (a**2*b*f + 2*a**2*c*e - a*b**2*e - 3*a*b*c*d + b**3*d + c*x**2
*(2*a**2*f - a*b*e - 2*a*c*d + b**2*d))/(2*a**2*(-4*a*c + b**2)*(
a + b*x**2 + c*x**4)) - (a*e/2 - b*d)*log(a + b*x**2 + c*x**4)/(2
*a**3) + (a*e - 2*b*d)*log(x**2)/(2*a**3) - (-a*b*e - 2*a*c*d + 2
*b**2*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a**3*sqrt(-
4*a*c + b**2))
```

Mathematica [A] time = 1.43176, size = 403, normalized size = 1.72

$$\frac{\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(4a^2c\left(e\sqrt{b^2-4ac}-af+3cd\right)-ab^2\left(e\sqrt{b^2-4ac}+12cd\right)+2abc\left(3ae-4d\sqrt{b^2-4ac}\right)+b^3\left(2d\sqrt{b^2-4ac}-ae\right)+2b^4d\right)}{(b^2-4ac)^{3/2}}+\frac{\log\left(\sqrt{b^2-4ac}+b+2c\cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*d)/x^2 - (2*a*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f -
 c*(3*d + e*x^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 -
4*a*c)*(a + b*x^2 + c*x^4)) + 4*(-2*b*d + a*e)*Log[x] + ((2*b^4*d
 + b^3*(2*Sqrt[b^2 - 4*a*c]*d - a*e) + 2*a*b*c*(-4*Sqrt[b^2 - 4*a
*c]*d + 3*a*e) - a*b^2*(12*c*d + Sqrt[b^2 - 4*a*c]*e) + 4*a^2*c*(
3*c*d + Sqrt[b^2 - 4*a*c]*e - a*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a
*c]*d + a*e) - 2*a*b*c*(4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(1
2*c*d - Sqrt[b^2 - 4*a*c]*e) + 4*a^2*c*(-3*c*d + Sqrt[b^2 - 4*a*c
]*e + a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^3)
```

Maple [B] time = 0.03, size = 1156, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*f-1/2/a/(c*x^4+b*x^2+a)*c/(4*
a*c-b^2)*x^2*b*e-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2*d+1/2/a^
2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b^2*d+1/2/(c*x^4+b*x^2+a)/(4*
a*c-b^2)*b*f+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c*e-1/2/a/(c*x^4+b*x^2
+a)/(4*a*c-b^2)*b^2*e-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*c*d+1/2
/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^3*d-1/a/(4*a*c-b^2)*c*ln((4*a*
c-b^2)*(c*x^4+b*x^2+a))*e+1/4/a^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x
^4+b*x^2+a))*b^2*e+2/a^2/(4*a*c-b^2)*c*ln((4*a*c-b^2)*(c*x^4+b*x^
2+a))*b*d-1/2/a^3/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^3
*d+2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(
4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b
^4*c-b^6)^(1/2))*c*f-3/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^
6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-4
8*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b*c*e-6/a/(64*a^3*c^3-48*a^2
*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c
-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*c^2*d+
1/2/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((
2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*
a*b^4*c-b^6)^(1/2))*b^3*e+6/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b
^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^
3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^2*c*d-1/a^3/(64*a^3
*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c
*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1
/2))*b^4*d-1/2*d/a^2/x^2+1/a^2*ln(x)*e-2/a^3*ln(x)*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.09619, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*(((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d + (a
*b^3*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*c
+ 6*a^2*b*c^2)*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*
(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^
2)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2
*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) -
 (2*(a^2*b*c*e - 2*a^3*c*f - 2*(a*b^2*c - 3*a^2*c^2)*d)*x^4 - 2*(
a^3*b*f + (2*a*b^3 - 7*a^2*b*c)*d - (a^2*b^2 - 2*a^3*c)*e)*x^2 -
2*(a^2*b^2 - 4*a^3*c)*d + ((2*(b^3*c - 4*a*b*c^2)*d - (a*b^2*c -
4*a^2*c^2)*e)*x^6 + (2*(b^4 - 4*a*b^2*c)*d - (a*b^3 - 4*a^2*b*c)*
e)*x^4 + (2*(a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2)*l
og(c*x^4 + b*x^2 + a) - 4*((2*(b^3*c - 4*a*b*c^2)*d - (a*b^2*c -
4*a^2*c^2)*e)*x^6 + (2*(b^4 - 4*a*b^2*c)*d - (a*b^3 - 4*a^2*b*c)*
e)*x^4 + (2*(a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2)*l
og(x))*sqrt(b^2 - 4*a*c))/(((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^
3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*x^2)*sqrt(b^2 - 4*a*c)),
 -1/4*(2*((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d +
(a*b^3*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*
c + 6*a^2*b*c^2)*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f -
2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*
x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2
*(a^2*b*c*e - 2*a^3*c*f - 2*(a*b^2*c - 3*a^2*c^2)*d)*x^4 - 2*(a^3
*b*f + (2*a*b^3 - 7*a^2*b*c)*d - (a^2*b^2 - 2*a^3*c)*e)*x^2 - 2*(
a^2*b^2 - 4*a^3*c)*d + ((2*(b^3*c - 4*a*b*c^2)*d - (a*b^2*c - 4*a
^2*c^2)*e)*x^6 + (2*(b^4 - 4*a*b^2*c)*d - (a*b^3 - 4*a^2*b*c)*e)*
x^4 + (2*(a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2)*log(
c*x^4 + b*x^2 + a) - 4*((2*(b^3*c - 4*a*b*c^2)*d - (a*b^2*c - 4*a
^2*c^2)*e)*x^6 + (2*(b^4 - 4*a*b^2*c)*d - (a*b^3 - 4*a^2*b*c)*e)*
x^4 + (2*(a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2)*log(
x))*sqrt(-b^2 + 4*a*c))/(((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3
- 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.67 $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=329

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-2abe-a(2cd-af)+3b^2d\right)}{4a^4}+\frac{\log(x)\left(-2abe-a(2cd-af)+3b^2d\right)}{a^4}+\frac{2bd-ae}{2a^3x^2}-\frac{d}{4a^2x^4}+\frac{cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)+3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-12a^3c^2e+12a^2b^2ce+6a^2bc(5cd-af)-2ab^4e-ab^3(20cd-af)+3b^5d\right)}{2a^4\left(b^2-4ac\right)^{3/2}}$$

```
[Out] -d/(4*a^2*x^4) + (2*b*d - a*e)/(2*a^3*x^2) + (b^4*d - a*b^3*e + 3
*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f) + c*(b^3*d
 - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*x^2)/(2*a^3*(b^2 - 4*
a*c)*(a + b*x^2 + c*x^4)) + ((3*b^5*d - 2*a*b^4*e + 12*a^2*b^2*c*
e - 12*a^3*c^2*e + 6*a^2*b*c*(5*c*d - a*f) - a*b^3*(20*c*d - a*f)
)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^
(3/2)) + ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[x])/a^4 - ((3
*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^
4)
```

Rubi [A] time = 2.36342, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-2abe-a(2cd-af)+3b^2d\right)}{4a^4}+\frac{\log(x)\left(-2abe-a(2cd-af)+3b^2d\right)}{a^4}+\frac{2bd-ae}{2a^3x^2}-\frac{d}{4a^2x^4}+\frac{cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)+3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-12a^3c^2e+12a^2b^2ce+6a^2bc(5cd-af)-2ab^4e-ab^3(20cd-af)+3b^5d\right)}{2a^4\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -d/(4*a^2*x^4) + (2*b*d - a*e)/(2*a^3*x^2) + (b^4*d - a*b^3*e + 3
*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f) + c*(b^3*d
 - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*x^2)/(2*a^3*(b^2 - 4*
a*c)*(a + b*x^2 + c*x^4)) + ((3*b^5*d - 2*a*b^4*e + 12*a^2*b^2*c*
e - 12*a^3*c^2*e + 6*a^2*b*c*(5*c*d - a*f) - a*b^3*(20*c*d - a*f)
)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^
(3/2)) + ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[x])/a^4 - ((3
*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^
4)
```

Rubi in Sympy [A] time = 119.625, size = 384, normalized size = 1.17

$$-\frac{d}{4a^2x^4}-\frac{c\left(a^2bf+2a^2ce-ab^2e-3abcd+b^3d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{a^3\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{-2a^3cf+a^2b^2f+3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d+cx^2\left(a^2bf+2a^2ce-ab^2e-3abcd+b^3d\right)}{2a^3\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}-\frac{ae-2bd}{2a^3x^2}-\frac{\left(\frac{a^2f}{2}-abe-acd+\frac{3b^2d}{2}\right)\log\left(a+bx^2+cx^4\right)}{2a^4}+\frac{\left(a^2f-2abe-2acd+3b^2d\right)\log\left(x^2\right)}{2a^4}+\frac{\left(a^2bf+2a^2ce-2ab^2e-6abcd+3b^3d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^4\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -d/(4*a**2*x**4) - c*(a**2*b*f + 2*a**2*c*e - a*b**2*e - 3*a*b*c*
d + b**3*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(a**3*(-4*a
*c + b**2)**(3/2)) + (-2*a**3*c*f + a**2*b**2*f + 3*a**2*b*c*e +
2*a**2*c**2*d - a*b**3*e - 4*a*b**2*c*d + b**4*d + c*x**2*(a**2*b
*f + 2*a**2*c*e - a*b**2*e - 3*a*b*c*d + b**3*d))/(2*a**3*(-4*a*c
 + b**2)*(a + b*x**2 + c*x**4)) - (a*e - 2*b*d)/(2*a**3*x**2) - (
a**2*f/2 - a*b*e - a*c*d + 3*b**2*d/2)*log(a + b*x**2 + c*x**4)/(
2*a**4) + (a**2*f - 2*a*b*e - 2*a*c*d + 3*b**2*d)*log(x**2)/(2*a*
*4) + (a**2*b*f + 2*a**2*c*e - 2*a*b**2*e - 6*a*b*c*d + 3*b**3*d)
*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a**4*sqrt(-4*a*c +
b**2))
```

Mathematica [A] time = 3.25721, size = 592, normalized size = 1.8

$$-\frac{\frac{2a\left(2a^2c\left(af-c\left(d+ex^2\right)\right)+b^3\left(ae-cdx^2\right)+ab^2\left(-af+4cd+cex^2\right)-abc\left(3ae+afx^2-3cdx^2\right)+b^4(-d)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(2a^2bc\left(4e\sqrt{b^2-4ac}-3af+1\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -((a^2*d)/x^4 + (2*a*(-2*b*d + a*e))/x^2 + (2*a*(-(b^4*d) + b^3*(
a*e - c*d*x^2) + a*b^2*(4*c*d - a*f + c*e*x^2) - a*b*c*(3*a*e - 3
*c*d*x^2 + a*f*x^2) + 2*a^2*c*(a*f - c*(d + e*x^2))))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) - 4*(3*b^2*d - 2*a*b*e + a*(-2*c*d + a*f
))*Log[x] + ((3*b^5*d + b^4*(3*Sqrt[b^2 - 4*a*c]*d - 2*a*e) + 2*a
^2*b*c*(15*c*d + 4*Sqrt[b^2 - 4*a*c]*e - 3*a*f) + a*b^3*(-20*c*d
- 2*Sqrt[b^2 - 4*a*c]*e + a*f) - 4*a^2*c*(-2*c*Sqrt[b^2 - 4*a*c]*
d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-14*c*Sqrt[b^2 - 4*
a*c]*d + 12*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-3*b^5*d + b^4*(3*Sqrt[b
^2 - 4*a*c]*d + 2*a*e) - a*b^3*(-20*c*d + 2*Sqrt[b^2 - 4*a*c]*e +
 a*f) + 2*a^2*b*c*(-15*c*d + 4*Sqrt[b^2 - 4*a*c]*e + 3*a*f) + 4*a
^2*c*(2*c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e - a*Sqrt[b^2 - 4*a*c]*f)
+ a*b^2*(-2*c*(7*Sqrt[b^2 - 4*a*c]*d + 6*a*e) + a*Sqrt[b^2 - 4*a*
c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))
/(4*a^4)
```

Maple [B] time = 0.038, size = 1675, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c^2*d-1/4*d/a^2/x^4+1/(c*x^4+b*x
^2+a)/(4*a*c-b^2)*c*f-2/a^3*ln(x)*b*e-2/a^3*ln(x)*c*d+3/a^4*ln(x)
*b^2*d+1/a^3/x^2*b*d-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^4*d-1/
2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2*f-1/2/a/(c*x^4+b*x^2+a)*c/(4*
a*c-b^2)*x^2*b*f+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b^2*e+
3/2/a^2/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2*b*d-1/2/a^3/(c*x^4+b*
x^2+a)*c/(4*a*c-b^2)*x^2*b^3*d-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2
)*x^2*e-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*c*e+2/a^2/(c*x^4+b*x^
2+a)/(4*a*c-b^2)*b^2*c*d-3/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*
c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c
^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b*c*f+6/a^2/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+
(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*
b^2*c*e+15/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*a
rctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*
c^2+12*a*b^4*c-b^6)^(1/2))*b*c^2*d-10/a^3/(64*a^3*c^3-48*a^2*b^2*
c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)
*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^3*c*d+2/a
^2/(4*a*c-b^2)*c*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b*e-7/2/a^3/(4*a
*c-b^2)*c*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^2*d+1/2/a^2/(c*x^4+b*
x^2+a)/(4*a*c-b^2)*b^3*e-6/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*
c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c
^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*c^2*e+1/2/a^2/(64*a^3*c^
3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^
2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)
)*b^3*f-1/a^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*ar
ctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)^(1/2))*b^4*e-1/2/a^3/(4*a*c-b^2)*ln((4*a*c-b^2
)*(c*x^4+b*x^2+a))*b^3*e+3/2/a^4/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*
b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a
^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^5*d-1/a/(4*a*c-b^2
)*c*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*f+2/a^2/(4*a*c-b^2)*c^2*ln((4
*a*c-b^2)*(c*x^4+b*x^2+a))*d+3/4/a^4/(4*a*c-b^2)*ln((4*a*c-b^2)*(
c*x^4+b*x^2+a))*b^4*d+1/4/a^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b
*x^2+a))*b^2*f-1/2/a^2/x^2*e+1/a^2*ln(x)*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.93827, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^5),x, algorithm="fricas")
```

```
[Out] [1/4*((((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c -
6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 +
 ((3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*d - 2*(a*b^5 - 6*a^2*b^3*
c + 6*a^3*b*c^2)*e + (a^2*b^4 - 6*a^3*b^2*c)*f)*x^6 + ((3*a*b^5 -
```

```
20*a^2*b^3*c + 30*a^3*b*c^2)*d - 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^
4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)*f)*x^4)*log((b^3 - 4*a*b*c + 2*(
b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt
(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (2*(a^3*b*c*f + (3*a*b^3*c
- 11*a^2*b*c^2)*d - 2*(a^2*b^2*c - 3*a^3*c^2)*e)*x^6 + ((6*a*b^4
- 25*a^2*b^2*c + 8*a^3*c^2)*d - 2*(2*a^2*b^3 - 7*a^3*b*c)*e + 2*(
a^3*b^2 - 2*a^4*c)*f)*x^4 + (3*(a^2*b^3 - 4*a^3*b*c)*d - 2*(a^3*b
^2 - 4*a^4*c)*e)*x^2 - (a^3*b^2 - 4*a^4*c)*d - (((3*b^4*c - 14*a*
b^2*c^2 + 8*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*e + (a^2*b^2*c
 - 4*a^3*c^2)*f)*x^8 + ((3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*d - 2*
(a*b^4 - 4*a^2*b^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6 + ((3*a*b^
4 - 14*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*c)*e + (a^
3*b^2 - 4*a^4*c)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^4*c -
14*a*b^2*c^2 + 8*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*e + (a^2*
b^2*c - 4*a^3*c^2)*f)*x^8 + ((3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*d
 - 2*(a*b^4 - 4*a^2*b^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6 + ((3
*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*c)*e
+ (a^3*b^2 - 4*a^4*c)*f)*x^4)*log(x))*sqrt(b^2 - 4*a*c))/(((a^4*b
^2*c - 4*a^5*c^2)*x^8 + (a^4*b^3 - 4*a^5*b*c)*x^6 + (a^5*b^2 - 4*
a^6*c)*x^4)*sqrt(b^2 - 4*a*c)), -1/4*(2*(((3*b^5*c - 20*a*b^3*c^2
 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e +
(a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*b^6 - 20*a*b^4*c + 30*a^2*
b^2*c^2)*d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^2)*e + (a^2*b^4 -
 6*a^3*b^2*c)*f)*x^6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*d
 - 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c
)*f)*x^4)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))
 - (2*(a^3*b*c*f + (3*a*b^3*c - 11*a^2*b*c^2)*d - 2*(a^2*b^2*c -
3*a^3*c^2)*e)*x^6 + ((6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*d - 2*(
2*a^2*b^3 - 7*a^3*b*c)*e + 2*(a^3*b^2 - 2*a^4*c)*f)*x^4 + (3*(a^2
*b^3 - 4*a^3*b*c)*d - 2*(a^3*b^2 - 4*a^4*c)*e)*x^2 - (a^3*b^2 - 4
*a^4*c)*d - (((3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*d - 2*(a*b^3*c
 - 4*a^2*b*c^2)*e + (a^2*b^2*c - 4*a^3*c^2)*f)*x^8 + ((3*b^5 - 14
*a*b^3*c + 8*a^2*b*c^2)*d - 2*(a*b^4 - 4*a^2*b^2*c)*e + (a^2*b^3
- 4*a^3*b*c)*f)*x^6 + ((3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*d - 2
*(a^2*b^3 - 4*a^3*b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4)*log(c*x^4
+ b*x^2 + a) + 4*(((3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*d - 2*(a*
b^3*c - 4*a^2*b*c^2)*e + (a^2*b^2*c - 4*a^3*c^2)*f)*x^8 + ((3*b^5
 - 14*a*b^3*c + 8*a^2*b*c^2)*d - 2*(a*b^4 - 4*a^2*b^2*c)*e + (a^2
*b^3 - 4*a^3*b*c)*f)*x^6 + ((3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*
d - 2*(a^2*b^3 - 4*a^3*b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4)*log(x
))*sqrt(-b^2 + 4*a*c))/(((a^4*b^2*c - 4*a^5*c^2)*x^8 + (a^4*b^3 -
 4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^5),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.68 $\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=550

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(3cd-7\ldots\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(3cd-7a\ldots\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x\left(a\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)+x^2\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)\right)}{2c^3(b^2-4ac)(a+bx^2+cx^4)}$$
$$+\frac{x(ce-2bf)}{c^3}+\frac{fx^3}{3c^2}$$

```
[Out] ((c*e - 2*b*f)*x)/c^3 + (f*x^3)/(3*c^2) + (x*(a*(b^2*c*e - 2*a*c^
2*e - b^3*f - b*c*(c*d - 3*a*f)) + (b^3*c*e - 3*a*b*c^2*e - b^4*f
 - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))*x^2))/(2*c^3*(b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4
*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) - (3*b^4*c*e
- 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f)
+ 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*c*e - 13*a*b*c^2*
e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) + (3
*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d -
 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7
/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 25.0298, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(3cd-7\ldots\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(3cd-7a\ldots\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x\left(a\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)+x^2\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)\right)}{2c^3(b^2-4ac)(a+bx^2+cx^4)}$$
$$+\frac{x(ce-2bf)}{c^3}+\frac{fx^3}{3c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*e - 2*b*f)*x)/c^3 + (f*x^3)/(3*c^2) + (x*(a*(b^2*c*e - 2*a*c^
2*e - b^3*f - b*c*(c*d - 3*a*f)) + (b^3*c*e - 3*a*b*c^2*e - b^4*f
 - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))*x^2))/(2*c^3*(b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4
*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) - (3*b^4*c*e
- 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f)
+ 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*c*e - 13*a*b*c^2*
e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) + (3
*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d -
```

```
 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7
/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.35173, size = 648, normalized size = 1.18

$$\frac{-\frac{6\sqrt{c}x\left(a^2c\left(2c\left(e+fx^2\right)-3bf\right)+a\left(b^3f-b^2c\left(e+4fx^2\right)+bc^2\left(d+3ex^2\right)-2c^3dx^2\right)+b^2x^2\left(b^2f-bce+c^2d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(abc^2\left(13e\sqrt{b^2-4ac}-52af\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (12*Sqrt[c]*(c*e - 2*b*f)*x + 4*c^(3/2)*f*x^3 - (6*Sqrt[c]*x*(b^2
*(c^2*d - b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e + f*x^2)) +
 a*(b^3*f - 2*c^3*d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^
2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*f
+ a*b*c^2*(8*c*d + 13*Sqrt[b^2 - 4*a*c]*e - 52*a*f) - b^3*c*(c*d
+ 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + b^4*(3*c*e + 5*Sqrt[b^2 - 4*a
*c]*f) + b^2*c*(c*Sqrt[b^2 - 4*a*c]*d - 19*a*c*e - 24*a*Sqrt[b^2
- 4*a*c]*f) + 2*a*c^2*(-3*c*Sqrt[b^2 - 4*a*c]*d + 10*a*c*e + 7*a*
Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
 (3*Sqrt[2]*(5*b^5*f + b^3*c*(c*d - 3*Sqrt[b^2 - 4*a*c]*e - 34*a*
f) + a*b*c^2*(-8*c*d + 13*Sqrt[b^2 - 4*a*c]*e + 52*a*f) + b^4*(-3
*c*e + 5*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt[b^2 - 4*a*c]*d + 19
*a*c*e - 24*a*Sqrt[b^2 - 4*a*c]*f) - 2*a*c^2*(3*c*Sqrt[b^2 - 4*a*
c]*d + 10*a*c*e - 7*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]))/(12*c^(7/2))
```

Maple [B] time = 0.101, size = 6868, normalized size = 12.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(\left(b^2c^2-2\,ac^3\right)d-\left(b^3c-3\,abc^2\right)e+\left(b^4-4\,ab^2c+2\,a^2c^2\right)f\right)x^3+\left(abc^2d-\left(ab^2c-2\,a^2c^2\right)e+\left(ab^3-3\,a^2bc\right)f\right)x}{2\left(ab^2c^3-4\,a^2c^4+\left(b^2c^4-4\,ac^5\right)x^4+\left(b^3c^3-4\,abc^4\right)x^2\right)}$$
$$+\frac{\int\frac{abc^2d+\left(\left(b^2c^2-6\,ac^3\right)d-\left(3\,b^3c-13\,abc^2\right)e+\left(5\,b^4-24\,ab^2c+14\,a^2c^2\right)f\right)x^2-\left(3\,ab^2c-10\,a^2c^2\right)e+\left(5\,ab^3-19\,a^2bc\right)f}{cx^4+bx^2+a}\,dx}{2\left(b^2c^3-4\,ac^4\right)}$$
$$+\frac{cfx^3+3\left(ce-2\,bf\right)x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^6/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a
*b^2*c + 2*a^2*c^2)*f)*x^3 + (a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e
 + (a*b^3 - 3*a^2*b*c)*f)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 -
4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*integrate((a*b*c^
2*d + ((b^2*c^2 - 6*a*c^3)*d - (3*b^3*c - 13*a*b*c^2)*e + (5*b^4
- 24*a*b^2*c + 14*a^2*c^2)*f)*x^2 - (3*a*b^2*c - 10*a^2*c^2)*e +
(5*a*b^3 - 19*a^2*b*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 - 4*a*
c^4) + 1/3*(c*f*x^3 + 3*(c*e - 2*b*f)*x)/c^3
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^6/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^6/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.69 $\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=436

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{b^2c(19af+cd)-8abc^2e+4ac^2(cd-5af)-3b^4f+b^3ce}{\sqrt{b^2-4ac}}+bc(13af+cd)-6ac^2e-3b^3f+b^2ce\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{b^2c(19af+cd)-8abc^2e+4ac^2(cd-5af)-3b^4f+b^3ce}{\sqrt{b^2-4ac}}+bc(13af+cd)-6ac^2e-3b^3f+b^2ce\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c^2}$$

```
[Out] (f*x)/c^2 + (x*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) - (b^2*c*e
- 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*x^2))/(2*c^2*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + ((b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(
c*d + 13*a*f) - (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d -
 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^
2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - 6*a*c^2*e -
 3*b^3*f + b*c*(c*d + 13*a*f) + (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f
+ 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqr
t[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 12.6618, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x\left(a\left(-c(2af+be)+b^2f+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{b^2c(19af+cd)-8abc^2e+4ac^2(cd-5af)-3b^4f+b^3ce}{\sqrt{b^2-4ac}}+bc(13af+cd)-6ac^2e-3b^3f+b^2ce\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{b^2c(19af+cd)-8abc^2e+4ac^2(cd-5af)-3b^4f+b^3ce}{\sqrt{b^2-4ac}}+bc(13af+cd)-6ac^2e-3b^3f+b^2ce\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (f*x)/c^2 + (x*(a*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - (b^2*c*e
- 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*x^2))/(2*c^2*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + ((b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(
c*d + 13*a*f) - (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d -
 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^
2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - 6*a*c^2*e -
 3*b^3*f + b*c*(c*d + 13*a*f) + (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f
+ 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqr
t[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 127.085, size = 478, normalized size = 1.1

$$\frac{fx}{c^2} + \frac{x\left(a\left(-2acf + b^2f - bce + 2c^2d\right) + x^2\left(-3abcf + 2ac^2e + b^3f - b^2ce + bc^2d\right)\right)}{2c^2\left(-4ac + b^2\right)\left(a + bx^2 + cx^4\right)}$$

$$- \frac{\sqrt{2}\left(-2ac\left(-10acf + 3b^2f - bce + 2c^2d\right) + b\left(-13abcf + 6ac^2e + 3b^3f - b^2ce - bc^2d\right) + \sqrt{-4ac + b^2}\left(-13abcf + 6ac^2e \cdots\right.\right.}{4c^{\frac{5}{2}}\sqrt{b + \sqrt{-4ac + b^2}}\left(-4ac + b^2\right)^{\frac{3}{2}}}$$

$$+ \frac{\sqrt{2}\left(-2ac\left(-10acf + 3b^2f - bce + 2c^2d\right) + b\left(-13abcf + 6ac^2e + 3b^3f - b^2ce - bc^2d\right) - \sqrt{-4ac + b^2}\left(-13abcf + 6ac^2e + \cdots\right.\right.}{4c^{\frac{5}{2}}\sqrt{b - \sqrt{-4ac + b^2}}\left(-4ac + b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] f*x/c**2 + x*(a*(-2*a*c*f + b**2*f - b*c*e + 2*c**2*d) + x**2*(-3
*a*b*c*f + 2*a*c**2*e + b**3*f - b**2*c*e + b*c**2*d))/(2*c**2*(-
4*a*c + b**2)*(a + b*x**2 + c*x**4)) - sqrt(2)*(-2*a*c*(-10*a*c*f
 + 3*b**2*f - b*c*e + 2*c**2*d) + b*(-13*a*b*c*f + 6*a*c**2*e + 3
*b**3*f - b**2*c*e - b*c**2*d) + sqrt(-4*a*c + b**2)*(-13*a*b*c*f
 + 6*a*c**2*e + 3*b**3*f - b**2*c*e - b*c**2*d))*atan(sqrt(2)*sqr
t(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*c**(5/2)*sqrt(b + sqrt(-
4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*(-2*a*c*(-10*a*c
*f + 3*b**2*f - b*c*e + 2*c**2*d) + b*(-13*a*b*c*f + 6*a*c**2*e +
 3*b**3*f - b**2*c*e - b*c**2*d) - sqrt(-4*a*c + b**2)*(-13*a*b*c
*f + 6*a*c**2*e + 3*b**3*f - b**2*c*e - b*c**2*d))*atan(sqrt(2)*s
qrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(4*c**(5/2)*sqrt(b - sqrt
(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 3.49949, size = 511, normalized size = 1.17

$$\frac{\frac{2\sqrt{c}x\left(-2a^2cf+a\left(b^2f-bc\left(e+3fx^2\right)+2c^2\left(d+ex^2\right)\right)+bx^2\left(b^2f-bce+c^2d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2ac^2\left(3e\sqrt{b^2-4ac}-10af+2cd\right)+b^2c\left(-e\sqrt{b^2-4ac}+19af \cdots\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\cdots}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*f*x + (2*Sqrt[c]*x*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^
2*f)*x^2 + a*(b^2*f + 2*c^2*(d + e*x^2) - b*c*(e + 3*f*x^2))))/((
b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*f + 2*a*c^2*
(2*c*d + 3*Sqrt[b^2 - 4*a*c]*e - 10*a*f) + b^2*c*(c*d - Sqrt[b^2
- 4*a*c]*e + 19*a*f) + b^3*(c*e + 3*Sqrt[b^2 - 4*a*c]*f) - b*c*(c
*Sqrt[b^2 - 4*a*c]*d + 8*a*c*e + 13*a*Sqrt[b^2 - 4*a*c]*f))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*f + 2*a*c^
2*(-2*c*d + 3*Sqrt[b^2 - 4*a*c]*e + 10*a*f) - b^2*c*(c*d + Sqrt[b
^2 - 4*a*c]*e + 19*a*f) + b^3*(-(c*e) + 3*Sqrt[b^2 - 4*a*c]*f) -
b*c*(c*Sqrt[b^2 - 4*a*c]*d - 8*a*c*e + 13*a*Sqrt[b^2 - 4*a*c]*f))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

Maple [B] time = 0.086, size = 5928, normalized size = 13.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bc^2d - \left(b^2c - 2\,ac^2\right)e + \left(b^3 - 3\,abc\right)f\right)x^3 + \left(2\,ac^2d - abce + \left(ab^2 - 2\,a^2c\right)f\right)x}{2\,(ab^2c^2 - 4\,a^2c^3 + (b^2c^3 - 4\,ac^4)x^4 + (b^3c^2 - 4\,abc^3)x^2)} + \frac{fx}{c^2} + \frac{-\int \frac{2\,ac^2d - abce - \left(bc^2d + \left(b^2c - 6\,ac^2\right)e - \left(3\,b^3 - 13\,abc\right)f\right)x^2 + \left(3\,ab^2 - 10\,a^2c\right)f}{cx^4 + bx^2 + a}\,dx}{2\,(b^2c^2 - 4\,ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*x^3 + (2
*a*c^2*d - a*b*c*e + (a*b^2 - 2*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c
^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + f*x/c
^2 + 1/2*integrate(-(2*a*c^2*d - a*b*c*e - (b*c^2*d + (b^2*c - 6*
a*c^2)*e - (3*b^3 - 13*a*b*c)*f)*x^2 + (3*a*b^2 - 10*a^2*c)*f)/(c
*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Fricas [A] time = 9.91134, size = 17006, normalized size = 39.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*f*x^5 + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e
 + (3*b^3 - 11*a*b*c)*f)*x^3 + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 +
 (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-((b^3
*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d
*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 - 1
3*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2
*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b
^2*c^7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 -
 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18
*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b
^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 1
08*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351
*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4
*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^
4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*
c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*
b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2
*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/
(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6
*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(((3*b^2*c
^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^4*c^4
 - 28*a*b^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*
c^5)*d*e^3 - (5*a*b^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (
189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*f
^4 - ((81*b^8 - 945*a*b^6*c + 3213*a^2*b^4*c^2 - 3000*a^3*b^2*c^3
 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2*b^5*c + 2727*a^3*b^3*c
^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a*b^4*c^3 - 15
0*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^2 + 14
61*a^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c
^2 + 1672*a^3*b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3
```

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+ 3*(18*b^5*c^3 - 123*a*b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6
*c^2 - 165*a*b^4*c^3 + 692*a^2*b^2*c^4)*d*e^2 - (45*a*b^5*c^2 - 6
47*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)*f)*x + 1/2*sqrt(1/2)*(2*(b^
4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*c^5 - 8*a*b^3*c^6
+ 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 16*a^3*c^
7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^3*b*c
^6)*e^3 + (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^
4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^
3 - 121*a^2*b^4*c^4 + 392*a^3*b^2*c^5 - 400*a^4*c^6)*d + (9*b^9*c
- 153*a*b^7*c^2 + 947*a^2*b^5*c^3 - 2536*a^3*b^3*c^4 + 2480*a^4*
b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c^5 - 32*a^2*b^2*c^6 + 1
60*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*d
*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^3*b^2*c^5
+ 160*a^4*c^6)*e^2)*f + (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*
c^9 - 64*a^3*b*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^
8 - 640*a^3*b^2*c^9 + 768*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6
+ 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*f)*sqrt((c
^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c
^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4
+ (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c
^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^
3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*
c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b
^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 -
4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e +
3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 4
9*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^
2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*
b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*
c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*
b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*a^3*c^
4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*
sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4
*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c
^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2
*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^
2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2
*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 +
25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e
+ (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2
)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*
d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*
c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 +
48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^
3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-((
b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5
)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 - 10
5*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2
- 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*
a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^
2*b^2*c^7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^
6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 -
18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^
2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2
- 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c -
351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*
b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3
*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b
^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*
(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*
a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*
f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(
b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(((3*b^
2*c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^4*
c^4 - 28*a*b^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2
*b*c^5)*d*e^3 - (5*a*b^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4
- (189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3
)*f^4 - ((81*b^8 - 945*a*b^6*c + 3213*a^2*b^4*c^2 - 3000*a^3*b^2*
c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2*b^5*c + 2727*a^3*b^
```

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3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a*b^4*c^3 -
 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^2 +
 1461*a^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^
4*c^2 + 1672*a^3*b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d
^3 + 3*(18*b^5*c^3 - 123*a*b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*
b^6*c^2 - 165*a*b^4*c^3 + 692*a^2*b^2*c^4)*d*e^2 - (45*a*b^5*c^2
- 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)*f)*x - 1/2*sqrt(1/2)*(2*
(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*c^5 - 8*a*b^3*c
^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 16*a^3
*c^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^3*
b*c^6)*e^3 + (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3
*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6
*c^3 - 121*a^2*b^4*c^4 + 392*a^3*b^2*c^5 - 400*a^4*c^6)*d + (9*b^
9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c^3 - 2536*a^3*b^3*c^4 + 2480*a
^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c^5 - 32*a^2*b^2*c^6
+ 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6
)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^3*b^2*
c^5 + 160*a^4*c^6)*e^2)*f + (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b
^3*c^9 - 64*a^3*b*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4
*c^8 - 640*a^3*b^2*c^9 + 768*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*f)*sqrt
((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^
3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*
e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^
2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3
*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a
^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (
9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^
2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*
e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3
- 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)
*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15
*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b
^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a
^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*a^3
*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^
8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2
+ 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^
2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180
*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*
a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5
 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*
d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*
e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^
6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b
^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(a*b^2*c^2 - 4*a^2
*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(
-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*
c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 -
 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c
^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 1
50*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48
*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2
*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^
4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051
*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c
^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c
 - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*(
(9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*
b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^
2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 +
 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 -
15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^
3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(((3
*b^2*c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b
^4*c^4 - 28*a*b^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*
```

```
a^2*b*c^5)*d*e^3 - (5*a*b^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e
^4 - (189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*
c^3)*f^4 - ((81*b^8 - 945*a*b^6*c + 3213*a^2*b^4*c^2 - 3000*a^3*b
^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2*b^5*c + 2727*a^3
*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a*b^4*c^
3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^
2 + 1461*a^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2
*b^4*c^2 + 1672*a^3*b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6
)*d^3 + 3*(18*b^5*c^3 - 123*a*b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*
(9*b^6*c^2 - 165*a*b^4*c^3 + 692*a^2*b^2*c^4)*d*e^2 - (45*a*b^5*c
^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)*f)*x + 1/2*sqrt(1/2)*
(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*c^5 - 8*a*b^
3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 16*
a^3*c^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a
^3*b*c^6)*e^3 + (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*
a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*
b^6*c^3 - 121*a^2*b^4*c^4 + 392*a^3*b^2*c^5 - 400*a^4*c^6)*d + (9
*b^9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c^3 - 2536*a^3*b^3*c^4 + 248
0*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c^5 - 32*a^2*b^2*c
^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*
c^6)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^3*b
^2*c^5 + 160*a^4*c^6)*e^2)*f - (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^
2*b^3*c^9 - 64*a^3*b*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*
b^4*c^8 - 640*a^3*b^2*c^9 + 768*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*f)*s
qrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*
(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^
6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2
*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*
b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 2
5*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e
+ (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)
*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d
^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c
^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c
^5)*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 -
 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 - 13*a*b^3*c^3 - 1
2*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*
a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3
*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e
^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81
*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a
^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 -
180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 11
97*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*
c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^
5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^
5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b
*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (
3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4
*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b^2*c^2 - 4*
a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sq
rt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a
^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^
7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^
5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2
+ 150*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 +
 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(
b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4
*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^
6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^
7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 +
6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51
*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484
*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^
3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5
 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)
```

```
*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^1
3)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(
((3*b^2*c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(
3*b^4*c^4 - 28*a*b^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 3
24*a^2*b*c^5)*d*e^3 - (5*a*b^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5
)*e^4 - (189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a
^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6*c + 3213*a^2*b^4*c^2 - 3000*a^
3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2*b^5*c + 2727*
a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a*b^4
*c^3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5
*c^2 + 1461*a^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*
a^2*b^4*c^2 + 1672*a^3*b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*
c^6)*d^3 + 3*(18*b^5*c^3 - 123*a*b^3*c^4 - 100*a^2*b*c^5)*d^2*e +
 3*(9*b^6*c^2 - 165*a*b^4*c^3 + 692*a^2*b^2*c^4)*d*e^2 - (45*a*b^
5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)*f)*x - 1/2*sqrt(1/
2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*c^5 - 8*a
*b^3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 +
16*a^3*c^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 14
4*a^3*b*c^6)*e^3 + (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 88
18*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12
*a*b^6*c^3 - 121*a^2*b^4*c^4 + 392*a^3*b^2*c^5 - 400*a^4*c^6)*d +
 (9*b^9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c^3 - 2536*a^3*b^3*c^4 +
2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c^5 - 32*a^2*b^
2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3
*b*c^6)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^
3*b^2*c^5 + 160*a^4*c^6)*e^2)*f - (4*(b^7*c^7 - 12*a*b^5*c^8 + 48
*a^2*b^3*c^9 - 64*a^3*b*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a
^2*b^4*c^8 - 640*a^3*b^2*c^9 + 768*a^4*c^10)*e - (3*b^9*c^5 - 52*
a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*f
)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 +
 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2
*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b
^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*
a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a
^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5
+ 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d
*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e
^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6
)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^
5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*
b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^
2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385
*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 - 13*a*b^3*c^3
- 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 1
20*a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*
a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^
2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 +
 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 255
0*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3
 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 +
 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b
^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b
*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3
*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*
a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2
+ (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*
b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(2*a*c^2*d - a*b*c*e
 + (3*a*b^2 - 10*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 -
 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.70 $\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=362

$$-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{c\sqrt{b^2-4ac}}+6af-\frac{b^2f}{c}-be+2cd\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{c\sqrt{b^2-4ac}}+6af-\frac{b^2f}{c}-be+2cd\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(x*(b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e + b^2*f - 2*a*c*f
)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c*d - b*e +
 6*a*f - (b^2*f)/c + (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d +
2*a*f))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - ((2*c*d - b*e + 6*a*f - (b^2*f)/c - (b^2*c*
e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 - 4*a*c]
))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 5.50739, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{c\sqrt{b^2-4ac}}+6af-\frac{b^2f}{c}-be+2cd\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{c\sqrt{b^2-4ac}}+6af-\frac{b^2f}{c}-be+2cd\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] -(x*(b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e + b^2*f - 2*a*c*f
)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c*d - b*e +
 6*a*f - (b^2*f)/c + (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d +
2*a*f))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - ((2*c*d - b*e + 6*a*f - (b^2*f)/c - (b^2*c*
e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 - 4*a*c]
))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 75.0333, size = 374, normalized size = 1.03

$$-\frac{x\left(abf-2ace+bcd+x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{2c\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b\left(-6acf+b^2f+bce-2c^2d\right)-2c\left(abf-2ace+bcd\right)+\sqrt{-4ac+b^2}\left(-6acf+b^2f+bce-2c^2d\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(b\left(-6acf+b^2f+bce-2c^2d\right)-2c\left(abf-2ace+bcd\right)-\sqrt{-4ac+b^2}\left(-6acf+b^2f+bce-2c^2d\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -x*(a*b*f - 2*a*c*e + b*c*d + x**2*(-2*a*c*f + b**2*f - b*c*e + 2
*c**2*d))/(2*c*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)) + sqrt(2)*(
b*(-6*a*c*f + b**2*f + b*c*e - 2*c**2*d) - 2*c*(a*b*f - 2*a*c*e +
 b*c*d) + sqrt(-4*a*c + b**2)*(-6*a*c*f + b**2*f + b*c*e - 2*c**2
*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*c**
(3/2)*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) - sqr
t(2)*(b*(-6*a*c*f + b**2*f + b*c*e - 2*c**2*d) - 2*c*(a*b*f - 2*a
*c*e + b*c*d) - sqrt(-4*a*c + b**2)*(-6*a*c*f + b**2*f + b*c*e -
2*c**2*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/
(4*c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 2.36313, size = 414, normalized size = 1.14

$$\frac{-\frac{2\sqrt{c}x\left(abf-2ac\left(e+fx^2\right)+b^2fx^2+bc\left(d-ex^2\right)+2c^2dx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(bc\left(e\sqrt{b^2-4ac}+8af+4cd\right)-2c\left(cd\sqrt{b^2-4ac}+3af\sqrt{b^2-4ac}+2ace\right)+b^2\left(f\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2)
 - 2*a*c*(e + f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqr
t[2]*(-(b^3*f) + b*c*(4*c*d + Sqrt[b^2 - 4*a*c]*e + 8*a*f) + b^2*
(-(c*e) + Sqrt[b^2 - 4*a*c]*f) - 2*c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a
*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[2]*(b^3*f + b*c*(-4*c*d + Sqrt[b^2 - 4*a*c]*e -
 8*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) - 2*c*(c*Sqrt[b^2 - 4*a
*c]*d - 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

Maple [B] time = 0.079, size = 5528, normalized size = 15.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,c^2d - bce + \left(b^2 - 2\,ac\right)f\right)x^3 + (bcd - 2\,ace + abf)x}{2\left((b^2c^2 - 4\,ac^3)x^4 + ab^2c - 4\,a^2c^2 + (b^3c - 4\,abc^2)x^2\right)} - \frac{-\int \frac{bcd-2\,ace+abf-\left(2\,c^2d-bce-\left(b^2-6\,ac\right)f\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(b^2c - 4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + (b*c*d - 2*a*c*e
+ a*b*f)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3
*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-(b*c*d - 2*a*c*e + a*b*f -
(2*c^2*d - b*c*e - (b^2 - 6*a*c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/
(b^2*c - 4*a*c^2)
```

Fricas [A] time = 5.47541, size = 12084, normalized size = 33.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + sqrt(1/2)*((b^2
*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x
^2)*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^
4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c +
 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c
 - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*c^3 - 12*a^2*b^4*c^4
 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 +
 a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(
a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2
*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^
2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*
c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*
a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*
b^2*c^5 - 64*a^4*c^6))*log(((3*b^2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4
+ 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b
^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^2
)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65
*a^3*b^3*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*
c^3 - 24*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)
*d*e + (3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a
*b^2*c^4 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e -
 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*a^3*b
*c^3)*e^3)*f)*x + 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*
b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (
a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3
- 8*a^3*b^2*c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*
a^4*b^3*c^2 - 144*a^5*b*c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 1
36*a^3*b^3*c^3 - 240*a^4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*e)*f^2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*
b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a
^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*e^2)
*f - ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^8)
*d - 4*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*
c^7)*e - (a^2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^
5*b^2*c^6 + 768*a^6*c^7)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2
*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*
b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12
*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 -
3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*
d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*
```

```
b^2*c^8 - 64*a^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*
a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*
b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2
*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*
c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4
 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a
^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a
^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)
*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*
b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 -
12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a
^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))) - sqrt(1/2)*((b^2*c^2
 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*
sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d
*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*
a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6
*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*c^3 - 12*a^2*b^4*c^4 + 4
8*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2
*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*
b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12
*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 -
3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*
d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*
b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*
c^5 - 64*a^4*c^6))*log(((3*b^2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12
*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b^2*c
^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^2)*f^
4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3
*b^3*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3
- 24*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e
 + (3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2
*c^4 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(
a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*a^3*b*c^3
)*e^3)*f)*x - 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^
6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^
5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8*
a^3*b^2*c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*
b^3*c^2 - 144*a^5*b*c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a
^3*b^3*c^3 - 240*a^4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
 16*a^5*c^4)*e)*f^2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^
5)*d^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a^4*c
^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*e^2)*f -
 ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^8)*d -
 4*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)
*e - (a^2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^
2*c^6 + 768*a^6*c^7)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4
*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*
c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2
*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^
3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^
2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*
c^8 - 64*a^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^
2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5
- 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c
^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*c^3
- 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2
*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c
^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b
*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2
 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^
4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a
^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b
^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))) + sqrt(1/2)*((b^2*c^2 - 4
*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt
(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e +
 (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*
b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2
*b^2*c^2 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^
3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4
*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*
c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2
*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^
```

```
3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^
2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*
c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5
- 64*a^4*c^6))*log(((3*b^2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12*a*b
*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b^2*c^3 +
 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^2)*f^4 +
((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3*b^3
*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24
*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (
3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4
 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^
4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*e^
3)*f)*x + 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d
^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^5*c^
3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8*a^3*
b^2*c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*b^3*
c^2 - 144*a^5*b*c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b
^3*c^3 - 240*a^4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*
a^5*c^4)*e)*f^2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d
^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a^4*c^5)*
d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*e^2)*f + ((a
*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^8)*d - 4*(
a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*e -
 (a^2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^
6 + 768*a^6*c^7)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4
 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2
- 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c
^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^
3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 +
a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8
- 64*a^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^
3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15
*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*
d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12
*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c
^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*
f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2
)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3
*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^
2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b
^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c
^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c
^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-((
b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*
b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^
2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2
*c^2 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^
2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4
 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2
- 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c
^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^
3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 +
a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8
- 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64
*a^4*c^6))*log(((3*b^2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5
)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b^2*c^3 + 4*a
^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^2)*f^4 + ((a*
b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3*b^3*c +
 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24*a^3
*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^
2*b^4*c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 4
8*a^2*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^
2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*e^3)*f
)*x - 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^3 -
 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^5*c^3 -
8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8*a^3*b^2*
c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*b^3*c^2
- 144*a^5*b*c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c
^3 - 240*a^4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*e)*f^2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^2 -
 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a^4*c^5)*d*e
+ 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*e^2)*f + ((a*b^8
```

```
*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^8)*d - 4*(a^2*
b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*e - (a^
2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^6 +
768*a^6*c^7)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (
a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*
a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d
*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e
^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*
b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64
*a^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 +
4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2
*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d -
(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2
*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d
^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4
- 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)
*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^
2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e
- 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c
^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 +
 48*a^3*b^2*c^5 - 64*a^4*c^6))) + 2*(b*c*d - 2*a*c*e + a*b*f)*x)/
((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c
^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.71 $\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=346

$$\frac{x\left(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x
^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*e
+ a*b*f + (4*a*b*c*e + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) + ((b*c*d - 2*a*c*e + a*b*f - (4*a*b*c*e + b^2*(c*d - a*f)
 - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*
a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 3.99895, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{x\left(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x
^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*e
+ a*b*f + (4*a*b*c*e + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) + ((b*c*d - 2*a*c*e + a*b*f - (4*a*b*c*e + b^2*(c*d - a*f)
 - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*
a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 72.7995, size = 364, normalized size = 1.05

$$\frac{x\left(2a^2f-abe-2acd+b^2d+x^2\left(abf-2ace+bcd\right)\right)}{2a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b\left(abf-2ace+bcd\right)+2c\left(2a^2f-abe+6acd-b^2d\right)+\sqrt{-4ac+b^2}\left(abf-2ace+bcd\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(b\left(abf-2ace+bcd\right)+2c\left(2a^2f-abe+6acd-b^2d\right)-\sqrt{-4ac+b^2}\left(abf-2ace+bcd\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] x*(2*a**2*f - a*b*e - 2*a*c*d + b**2*d + x**2*(a*b*f - 2*a*c*e +
b*c*d))/(2*a*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)) + sqrt(2)*(b*
(a*b*f - 2*a*c*e + b*c*d) + 2*c*(2*a**2*f - a*b*e + 6*a*c*d - b**
2*d) + sqrt(-4*a*c + b**2)*(a*b*f - 2*a*c*e + b*c*d))*atan(sqrt(2
)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a*sqrt(c)*sqrt(b +
sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) - sqrt(2)*(b*(a*b*f
- 2*a*c*e + b*c*d) + 2*c*(2*a**2*f - a*b*e + 6*a*c*d - b**2*d) -
sqrt(-4*a*c + b**2)*(a*b*f - 2*a*c*e + b*c*d))*atan(sqrt(2)*sqrt(
c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(4*a*sqrt(c)*sqrt(b - sqrt(-4
*a*c + b**2))*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 2.18308, size = 382, normalized size = 1.1

$$\frac{\frac{2x\left(b\left(-ae+afx^2+cdx^2\right)+2a\left(af-c\left(d+ex^2\right)\right)+b^2d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(b\left(cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+4ace\right)-2ac\left(e\sqrt{b^2-4ac}+2af+6cd\right)+b^2(cd-af)\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d +
 e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c
*d - a*f) - 2*a*c*(6*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sq
rt[b^2 - 4*a*c]*d + 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a
*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d) +
a*f) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sqrt[b^
2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.12, size = 5350, normalized size = 15.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bcd - 2\,ace + abf)x^3 - \left(abe - 2\,a^2f - \left(b^2 - 2\,ac\right)d\right)x}{2\left((ab^2c - 4\,a^2c^2)x^4 + a^2b^2 - 4\,a^3c + (ab^3 - 4\,a^2bc)x^2\right)} + \frac{\int \frac{abe - 2\,a^2f + (bcd - 2\,ace + abf)x^2 + (b^2 - 6\,ac)d}{cx^4 + bx^2 + a}\,dx}{2\,(ab^2 - 4\,a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*e + a*b*f)*x^3 - (a*b*e - 2*a^2*f - (b^2 - 2*
a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2) + 1/2*integrate((a*b*e - 2*a^2*f + (b*c*d - 2
*a*c*e + a*b*f)*x^2 + (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a
*b^2 - 4*a^2*c)
```

Fricas [A] time = 5.47374, size = 12138, normalized size = 35.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c*d - 2*a*c*e + a*b*f)*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^
2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(
(b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^
2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b
^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a
^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5
*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12
*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^
4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^
3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c
^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c
^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^
2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^
3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^
4 - (3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4
*c^2 - 28*a^2*b^2*c^3)*d^2*e^2 - (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d
*e^3 - (3*a^3*b^2*c^2 + 4*a^4*c^3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^
4 - ((a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d + (a^4*b^3 + 12*a^5*
b*c)*e)*f^3 - 9*((a^2*b^4*c - 6*a^3*b^2*c^2 - 24*a^4*c^3)*d^2 + (
a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 15*a*b^4*c^2 + 432
*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d^2*e
 + 3*(a^2*b^4*c + 12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^4*b*c
^2)*e^3)*f)*x + 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^
4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*
b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^2*e + 3*(a^2*b^6*c -
10*a^3*b^4*c^2 + 32*a^4*b^2*c^3 - 32*a^5*c^4)*d*e^2 + (a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3 + 2*(a^5*b^4 - 8*a^6*b^2*c +
16*a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c + 160*a^5*b^2*c^2 - 28
8*a^6*c^3)*d + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e)*f^2 - 2*
((4*a^2*b^6*c - 59*a^3*b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d
^2 + 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e + (a^4*b^4*
c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^2)*f - ((a^3*b^9*c - 20*a^4*b^7
*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d + (a^
4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*e - 4*(a
^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*f)*sqr
t((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (
b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b
*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*
d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b
*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a
```

```
^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt
(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2
*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^
3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(
3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*
a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 +
 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)
*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3
*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^
4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^
2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8
*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2
*c^3 - 64*a^6*c^4))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^
3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c
^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)
*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4
*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^
6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a
^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2
 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(
2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 1
2*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*
d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c
^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))
*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^4 - (3*b^5*c^2 -
 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4*c^2 - 28*a^2*b^
2*c^3)*d^2*e^2 - (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*a^3*b^
2*c^2 + 4*a^4*c^3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 -
24*a^4*b^2*c - 48*a^5*c^2)*d + (a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*
((a^2*b^4*c - 6*a^3*b^2*c^2 - 24*a^4*c^3)*d^2 + (a^3*b^3*c + 12*a
^4*b*c^2)*d*e)*f^2 + ((b^6*c - 15*a*b^4*c^2 + 432*a^3*c^4)*d^3 +
3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c
+ 12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^4*b*c^2)*e^3)*f)*x -
1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*
b^2*c^4 + 864*a^4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3
*b^3*c^3 - 112*a^4*b*c^4)*d^2*e + 3*(a^2*b^6*c - 10*a^3*b^4*c^2 +
 32*a^4*b^2*c^3 - 32*a^5*c^4)*d*e^2 + (a^3*b^5*c - 8*a^4*b^3*c^2
+ 16*a^5*b*c^3)*e^3 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*f^3
- ((a^3*b^6 - 26*a^4*b^4*c + 160*a^5*b^2*c^2 - 288*a^6*c^3)*d + (
a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e)*f^2 - 2*((4*a^2*b^6*c -
59*a^3*b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d^2 + 5*(a^3*b^5*
c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*b^2*c^
2 + 16*a^6*c^3)*e^2)*f - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b
^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d + (a^4*b^8*c - 8*a^5*
b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*e - 4*(a^5*b^7*c - 12*a^
6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*f)*sqrt((4*a^3*b*c^2*d
*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b
^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*
(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2
+ (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4
*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*
a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a
*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^
3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b
*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*
a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64
*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3
+ a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*
c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 -
2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2
- 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^
3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^
9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^
4))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b
*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b
^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2
*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f - (a^
3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a
^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^
2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*
```

```
d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e +
a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d
^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6
*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c
 - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3
 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^4 - (3*b^5*c^2 - 65*a*b^3*c^3 +
324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4*c^2 - 28*a^2*b^2*c^3)*d^2*e^2 -
 (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*a^3*b^2*c^2 + 4*a^4*c^
3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a^4*b^2*c - 4
8*a^5*c^2)*d + (a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c - 6*
a^3*b^2*c^2 - 24*a^4*c^3)*d^2 + (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f
^2 + ((b^6*c - 15*a*b^4*c^2 + 432*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a
^2*b^3*c^2 - 108*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c + 12*a^3*b^2*c^2
)*d*e^2 + (a^3*b^3*c + 12*a^4*b*c^2)*e^3)*f)*x + 1/2*sqrt(1/2)*((
b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^
4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a
^4*b*c^4)*d^2*e + 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b^2*c^3
 - 32*a^5*c^4)*d*e^2 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*
e^3 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*f^3 - ((a^3*b^6 - 26
*a^4*b^4*c + 160*a^5*b^2*c^2 - 288*a^6*c^3)*d + (a^4*b^5 - 8*a^5*
b^3*c + 16*a^6*b*c^2)*e)*f^2 - 2*((4*a^2*b^6*c - 59*a^3*b^4*c^2 +
 280*a^4*b^2*c^3 - 432*a^5*c^4)*d^2 + 5*(a^3*b^5*c - 8*a^4*b^3*c^
2 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*
e^2)*f + ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6
*b^3*c^4 + 512*a^7*b*c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^
7*b^2*c^4 - 256*a^8*c^5)*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a
^7*b^3*c^3 - 64*a^8*b*c^4)*f)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e
^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*
c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3
*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 2
7*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^
2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48
*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^
2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^
2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*
a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f -
(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((
4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4
*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^
3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e
 + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^
2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*
b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^
6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) - sqrt(1/2)
*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*
b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a
*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*
c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*
b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f - (a^3*b^6*c - 12*a^4
*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 +
 a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3
 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b
^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3
*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d
*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^
4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^
2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4
+ 324*a^2*c^5)*d^4 - (3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d
^3*e - 3*(3*a*b^4*c^2 - 28*a^2*b^2*c^3)*d^2*e^2 - (9*a^2*b^3*c^2
- 20*a^3*b*c^3)*d*e^3 - (3*a^3*b^2*c^2 + 4*a^4*c^3)*e^4 + (3*a^5*
b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d + (
a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c - 6*a^3*b^2*c^2 - 24
*a^4*c^3)*d^2 + (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 1
5*a*b^4*c^2 + 432*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108
*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c + 12*a^3*b^2*c^2)*d*e^2 + (a^3*b
^3*c + 12*a^4*b*c^2)*e^3)*f)*x - 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6
*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^3 + 3*(
a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^2*e
+ 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b^2*c^3 - 32*a^5*c^4)*d*
e^2 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3 + 2*(a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c + 160
*a^5*b^2*c^2 - 288*a^6*c^3)*d + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b
```

```
*c^2)*e)*f^2 - 2*((4*a^2*b^6*c - 59*a^3*b^4*c^2 + 280*a^4*b^2*c^3
 - 432*a^5*c^4)*d^2 + 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3
)*d*e + (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^2)*f + ((a^3*b
^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a
^7*b*c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*
a^8*c^5)*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*
a^8*b*c^4)*f)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*
f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*
b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^
2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*
f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^
3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 6
4*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2
*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3
*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a
^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f - (a^3*b^6*c - 12*
a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^
3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*
c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^
2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (
a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^
2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7
*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4
*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) - 2*(a*b*e - 2*a^2*f - (b^2
 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
(a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.72 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=399

$$-\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2\left(-abe-2a(cd-af)+b^2d\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2e-4ab(af+4cd)+3b^3d}{\sqrt{b^2-4ac}}-abe-2a(5cd-af)+3b^2d\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12a^2ce-ab^2e-4ab(af+4cd)+3b^3d}{\sqrt{b^2-4ac}}-abe-2a(5cd-af)+3b^2d\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{a^2x}$$

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*
f)) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x^2))/(2*a^2*(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d
 - a*f) + (3*b^3*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) - (3*b^3*d -
a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 6.04412, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{x\left(a^2(bf+2ce)+cx^2\left(-abe-2a(cd-af)+b^2d\right)-ab(be+3cd)+b^3d\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2e-4ab(af+4cd)+3b^3d}{\sqrt{b^2-4ac}}-abe-2a(5cd-af)+3b^2d\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12a^2ce-ab^2e-4ab(af+4cd)+3b^3d}{\sqrt{b^2-4ac}}-abe-2a(5cd-af)+3b^2d\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{a^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(d/(a^2*x)) - (x*(b^3*d - a*b*(3*c*d + b*e) + a^2*(2*c*e + b*f)
+ c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x^2))/(2*a^2*(b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a
*f) + (3*b^3*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
- (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) - (3*b^3*d - a*b^
2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2
*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.64542, size = 444, normalized size = 1.11

$$\frac{-\frac{2x\left(b^2\left(cdx^2-ae\right)+ab\left(af-c\left(3d+ex^2\right)\right)+2ac\left(a\left(e+fx^2\right)-cdx^2\right)+b^3d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ab\left(e\sqrt{b^2-4ac}+4af+16cd\right)-2a\left(-5cd\sqrt{b^2-4ac}+af\sqrt{b\ldots}\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*d)/x - (2*x*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(
3*d + e*x^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3*d + b^2*(-3*Sq
rt[b^2 - 4*a*c]*d + a*e) + a*b*(16*c*d + Sqrt[b^2 - 4*a*c]*e + 4*
a*f) - 2*a*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e + a*Sqrt[b^2 - 4*a
*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt
[c]*(3*b^3*d - b^2*(3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-16*c*d +
 Sqrt[b^2 - 4*a*c]*e - 4*a*f) + 2*a*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*
a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]))/(4*a^2)
```

Maple [B] time = 0.086, size = 5142, normalized size = 12.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(abce-2\,a^2cf-\left(3\,b^2c-10\,ac^2\right)d\right)x^4-\left(a^2bf+\left(3\,b^3-11\,abc\right)d-\left(ab^2-2\,a^2c\right)e\right)x^2-2\left(ab^2-4\,a^2c\right)d}{2\left(\left(a^2b^2c-4\,a^3c^2\right)x^5+\left(a^2b^3-4\,a^3bc\right)x^3+\left(a^3b^2-4\,a^4c\right)x\right)}-\frac{-\int\frac{a^2bf+\left(abce-2\,a^2cf-\left(3\,b^2c-10\,ac^2\right)d\right)x^2-\left(3\,b^3-13\,abc\right)d+\left(ab^2-6\,a^2c\right)e}{cx^4+bx^2+a}\,dx}{2\left(a^2b^2-4\,a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b*
f + (3*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2 -
4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*
x^3 + (a^3*b^2 - 4*a^4*c)*x) - 1/2*integrate(-(a^2*b*f + (a*b*c*e
 - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^2 - (3*b^3 - 13*a*b*c)*d
 + (a*b^2 - 6*a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*
c)
```

Fricas [A] time = 11.9325, size = 17700, normalized size = 44.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - 2*(a^
2*b*f + (3*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 + sqrt(1/
2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^
3*b^2 - 4*a^4*c)*x)*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2
 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c
^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e
^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c -
12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^
5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4
 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2
 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*
b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198
*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4
*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*
b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^
2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*
b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5
*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 -
 (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3))*log(-((189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*
b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^7*c^2 - 1323*a*b^5*c^3 + 272
7*a^2*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a*b^6*c^2 - 558*a^2
*b^4*c^3 + 1672*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 - 647*a^3*
b^3*c^3 + 2268*a^4*b*c^4)*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^3
 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4*a^7*c^2)*f^4 + ((27*a^4*b^
4*c + 80*a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^2)*e)*f^3 - 3*((2
7*a^2*b^6*c - 117*a^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*c^4)*d^
2 - (18*a^3*b^5*c - 123*a^4*b^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a^4
*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b^8*c - 945*a*b^6*c^2 +
3213*a^2*b^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*d^3 - 3*(27*a
*b^7*c - 405*a^2*b^5*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d^2*
e + 3*(9*a^2*b^6*c - 165*a^3*b^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 - (
3*a^3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^3)*e^3)*f)*x + 1/2*sqr
t(1/2)*((27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5
*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 - 17
7*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138*a^4*b^4*c^3 + 5216*a^5*b^2*
c^4 - 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^3*b^7*c + 495*a^4*
b^5*c^2 - 1656*a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 2
3*a^4*b^6*c + 190*a^5*b^4*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e^
3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*f^3 + 3*((3*a^4*b^7 -
25*a^5*b^5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (a^5*b^6 - 10*a
^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2*b^9 -
105*a^3*b^7*c + 373*a^4*b^5*c^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c^4
)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*a^5*b^4*c^2 - 176*a^6*b
^2*c^3 - 160*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c + 72*a^6*b^3*
c^2 - 112*a^7*b*c^3)*e^2)*f - ((3*a^5*b^10 - 55*a^6*b^8*c + 392*a
^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)
*d - (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*c^2 - 448*a^9*b^3*c^3
+ 512*a^10*b*c^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^10*b^2*c^3 -
 256*a^11*c^4)*f)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^
2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 3
51*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2
*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*
(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*
a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*
d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5
*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2
*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*
```

a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-((189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^7*c^2 - 1323*a*b^5*c^3 + 2727*a^2*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a*b^6*c^2 - 558*a^2*b^4*c^3 + 1672*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 - 647*a^3*b^3*c^3 + 2268*a^4*b*c^4)*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a^4*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b^8*c - 945*a*b^6*c^2 + 3213*a^2*b^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*d^3 - 3*(27*a*b^7*c - 405*a^2*b^5*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d^2*e + 3*(9*a^2*b^6*c - 165*a^3*b^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^3)*e^3)*f)*x - 1/2*sqrt(1/2)*((27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 - 177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 - 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^3*b^7*c + 495*a^4*b^5*c^2 - 1656*a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 23*a^4*b^6*c + 190*a^5*b^4*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 160*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c + 72*a^6*b^3*c^2 - 112*a^7*b*c^3)*e^2)*f - ((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*d - (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^10*b^2*c^3 - 256*a^11*c^4)*f)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 -

```
351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^
2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4
*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18
*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)
*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^
5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^
2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9
*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22
*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-
((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*
(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a
^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c
)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4
 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6 - 12*a^6*b^4*c + 48*
a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c +
 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a
*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e +
6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*
e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b
^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5
*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 -
2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*
((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3
 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b
^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3
)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(
1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (
a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c
^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2
*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)
*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c
- 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f - (
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f
^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
+ 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c
^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^
4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 1
98*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 +
 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^
5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*
b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^
4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a
^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2
 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))*log(-((189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^
2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^7*c^2 - 1323*a*b^5*c^3 + 2
727*a^2*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a*b^6*c^2 - 558*a
^2*b^4*c^3 + 1672*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 - 647*a^
3*b^3*c^3 + 2268*a^4*b*c^4)*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c
^3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4*a^7*c^2)*f^4 + ((27*a^4*
b^4*c + 80*a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^2)*e)*f^3 - 3*(
(27*a^2*b^6*c - 117*a^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*c^4)*
d^2 - (18*a^3*b^5*c - 123*a^4*b^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a
^4*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b^8*c - 945*a*b^6*c^2
+ 3213*a^2*b^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*d^3 - 3*(27
*a*b^7*c - 405*a^2*b^5*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d^
2*e + 3*(9*a^2*b^6*c - 165*a^3*b^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 -
 (3*a^3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^3)*e^3)*f)*x + 1/2*s
qrt(1/2)*((27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b
^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 -
177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138*a^4*b^4*c^3 + 5216*a^5*b^
2*c^4 - 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^3*b^7*c + 495*a^
4*b^5*c^2 - 1656*a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3*b^8 -
 23*a^4*b^6*c + 190*a^5*b^4*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*
e^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*f^3 + 3*((3*a^4*b^7
- 25*a^5*b^5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (a^5*b^6 - 10
*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2*b^9
- 105*a^3*b^7*c + 373*a^4*b^5*c^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c
^4)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*a^5*b^4*c^2 - 176*a^6
```

*b^2*c^3 - 160*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c + 72*a^6*b^
3*c^2 - 112*a^7*b*c^3)*e^2)*f + ((3*a^5*b^10 - 55*a^6*b^8*c + 392
*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^
5)*d - (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*c^2 - 448*a^9*b^3*c^
3 + 512*a^10*b*c^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^10*b^2*c^3
 - 256*a^11*c^4)*f)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*
a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 -
 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a
^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 -
4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 1
8*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c
)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a
^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a
^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(
9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 2
2*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(
-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2
*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (
a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*
c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^
4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f - (a^5*b^6 - 12*a^6*b^4*c + 48
*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*
a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e +
 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2
e^2 - 4(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*
b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 +
5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 -
 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4
*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^
3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*
b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^
3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))
)/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt
(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 +
(a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*
c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^
2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2
)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c
 - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f -
(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*
f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*
c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a
^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c +
198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4
+ 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a
^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6
*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a
^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*
a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^
2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^
2*c^2 - 64*a^8*c^3))*log(-((189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a
^2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^7*c^2 - 1323*a*b^5*c^3 +
2727*a^2*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a*b^6*c^2 - 558*
a^2*b^4*c^3 + 1672*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 - 647*a
^3*b^3*c^3 + 2268*a^4*b*c^4)*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*
c^3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4*a^7*c^2)*f^4 + ((27*a^4
*b^4*c + 80*a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^2)*e)*f^3 - 3*
((27*a^2*b^6*c - 117*a^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*c^4)
*d^2 - (18*a^3*b^5*c - 123*a^4*b^3*c^2 - 100*a^5*b*c^3)*d*e + (3*
a^4*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b^8*c - 945*a*b^6*c^2
 + 3213*a^2*b^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*d^3 - 3*(2
7*a*b^7*c - 405*a^2*b^5*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d
^2*e + 3*(9*a^2*b^6*c - 165*a^3*b^4*c^2 + 692*a^4*b^2*c^3)*d*e^2
- (3*a^3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^3)*e^3)*f)*x - 1/2*
sqrt(1/2)*((27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*
b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 -
 177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138*a^4*b^4*c^3 + 5216*a^5*b
^2*c^4 - 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^3*b^7*c + 495*a

```
^4*b^5*c^2 - 1656*a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3*b^8
- 23*a^4*b^6*c + 190*a^5*b^4*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)
*e^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*f^3 + 3*((3*a^4*b^7
 - 25*a^5*b^5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (a^5*b^6 - 1
0*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2*b^9
 - 105*a^3*b^7*c + 373*a^4*b^5*c^2 - 248*a^5*b^3*c^3 - 560*a^6*b*
c^4)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*a^5*b^4*c^2 - 176*a^
6*b^2*c^3 - 160*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c + 72*a^6*b
^3*c^2 - 112*a^7*b*c^3)*e^2)*f + ((3*a^5*b^10 - 55*a^6*b^8*c + 39
2*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c
^5)*d - (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*c^2 - 448*a^9*b^3*c
^3 + 512*a^10*b*c^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^10*b^2*c^
3 - 256*a^11*c^4)*f)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051
*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7
- 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*
a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 -
 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 -
18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*
c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*
a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*
a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*
(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 -
22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt
(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 -
2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e +
(a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b
*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b
^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f - (a^5*b^6 - 12*a^6*b^4*c + 4
8*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c
 + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27
*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e
+ 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^
2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4
*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 +
 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2
- 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 -
4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d
^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4
*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e
^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)
))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - 4*(
a*b^2 - 4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a
^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.73 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=575

$$\frac{2bd-ae}{a^3x}-\frac{d}{3a^2x^3}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2a^2c\left(5e\sqrt{b^2-4ac}-6af+14cd\right)-ab^2\left(3e\sqrt{b^2-4ac}-af+29cd\right)-ab\left(19cd\sqrt{b^2-4ac}-af\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(2a^2c\left(-5e\sqrt{b^2-4ac}-6af+14cd\right)-ab^2\left(-3e\sqrt{b^2-4ac}-af+29cd\right)+ab\left(19cd\sqrt{b^2-4ac}-\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2
*c^2*d + 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - 2*a*c*f) + c*(
b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*x^2))/(2*a^3*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt
[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*c*d + 5*Sqrt[b^2 - 4*a*c]*
e - 6*a*f) - a*b^2*(29*c*d + 3*Sqrt[b^2 - 4*a*c]*e - a*f) - a*b*(
19*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(
5*b^4*d - b^3*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + 2*a^2*c*(14*c*d -
 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d - 3*Sqrt[b^2 - 4*
a*c]*e - a*f) + a*b*(19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt
[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Rubi [A] time = 18.2041, antiderivative size = 575, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2bd-ae}{a^3x}-\frac{d}{3a^2x^3}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2a^2c\left(5e\sqrt{b^2-4ac}-6af+14cd\right)-ab^2\left(3e\sqrt{b^2-4ac}-af+29cd\right)-ab\left(19cd\sqrt{b^2-4ac}-af\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(2a^2c\left(-5e\sqrt{b^2-4ac}-6af+14cd\right)-ab^2\left(-3e\sqrt{b^2-4ac}-af+29cd\right)+ab\left(19cd\sqrt{b^2-4ac}-\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2
*c^2*d + 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - 2*a*c*f) + c*(
b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*x^2))/(2*a^3*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt
[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*c*d + 5*Sqrt[b^2 - 4*a*c]*
e - 6*a*f) - a*b^2*(29*c*d + 3*Sqrt[b^2 - 4*a*c]*e - a*f) - a*b*(
19*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(
```

```
5*b^4*d - b^3*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + 2*a^2*c*(14*c*d -
 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d - 3*Sqrt[b^2 - 4*
a*c]*e - a*f) + a*b*(19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt
[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Rubi in Sympy [A] time = 76.1512, size = 216, normalized size = 0.38

$$-\frac{f}{3acx^3}+\frac{bf}{a^2cx}-\frac{\sqrt{2}f\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a^2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}f\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a^2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -f/(3*a*c*x**3) + b*f/(a**2*c*x) - sqrt(2)*f*(-2*a*c + b**2 - b*s
qrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c +
 b**2)))/(2*a**2*sqrt(c)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*
c + b**2)) + sqrt(2)*f*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*at
an(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*a**2*sqrt(
c)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 3.60531, size = 548, normalized size = 0.95

$$\frac{\frac{6x\left(2a^2c\left(c\left(d+ex^2\right)-af\right)+b^3\left(cdx^2-ae\right)+ab^2\left(af-c\left(4d+ex^2\right)\right)+abc\left(3ae+afx^2-3cdx^2\right)+b^4d\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2a^2c\left(5e\sqrt{b^2-4ac}-6af+14cd\right)\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) +
 c*d*x^2) + a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x^2) + 2*a^2*c*(-(a*f)
 + c*(d + e*x^2)) + a*b^2*(a*f - c*(4*d + e*x^2))))/((b^2 - 4*a*c
)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqr
t[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*c*d + 5*Sqrt[b^2 - 4*a*c]
*e - 6*a*f) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b
*(-19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-
5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(-29*c*d +
3*Sqrt[b^2 - 4*a*c]*e + a*f) + 2*a^2*c*(-14*c*d + 5*Sqrt[b^2 - 4*
a*c]*e + 6*a*f) + a*b*(-19*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e + a*S
qrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
12*a^3)
```

Maple [B] time = 0.095, size = 6122, normalized size = 10.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,(a^2bcf + (5\,b^3c - 19\,abc^2)\,d - (3\,ab^2c - 10\,a^2c^2)\,e)\,x^6 + ((15\,b^4 - 62\,ab^2c + 14\,a^2c^2)\,d - 3\,(3\,ab^3 - 11\,a^2bc)\,e + 3\,(a^2b^2 - \cdots}{6\,((a^3b^2c - 4\,a^4c^2)x^7 + (a^3b^3 - 4\,a^4bc)x^5 + (a^4b^2 - 4 \cdots} + \frac{\int \frac{(a^2bcf+(5\,b^3c-19\,abc^2)\,d-(3\,ab^2c-10\,a^2c^2)\,e)\,x^2+(5\,b^4-24\,ab^2c+14\,a^2c^2)\,d-(3\,ab^3-13\,a^2bc)\,e+(a^2b^2-6\,a^3c)\,f}{cx^4+bx^2+a}\,dx}{2\,(a^3b^2 - 4\,a^4c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] 1/6*(3*(a^2*b*c*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^
2*c^2)*e)*x^6 + ((15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d - 3*(3*a*b^
3 - 11*a^2*b*c)*e + 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 -
4*a^2*b*c)*d - 3*(a^2*b^2 - 4*a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*
c)*d)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 +
(a^4*b^2 - 4*a^5*c)*x^3) + 1/2*integrate(((a^2*b*c*f + (5*b^3*c -
 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 - 24*a*
b^2*c + 14*a^2*c^2)*d - (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a
^3*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^4),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.74 $\int \frac{x^9\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=68

$$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+2\log\left(x^2+1\right)+392\log\left(x^2+2\right)+\frac{415x^2+414}{2\left(x^4+3x^2+2\right)}$$

```
[Out] (-293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8 + (414 + 415*x^
2)/(2*(2 + 3*x^2 + x^4)) + 2*Log[1 + x^2] + 392*Log[2 + x^2]
```

Rubi [A] time = 0.198708, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+2\log\left(x^2+1\right)+392\log\left(x^2+2\right)+\frac{415x^2+414}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8 + (414 + 415*x^
2)/(2*(2 + 3*x^2 + x^4)) + 2*Log[1 + x^2] + 392*Log[2 + x^2]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{5x^{12}}{8\left(x^4+3x^2+2\right)}-\frac{21x^6}{8}+2\log\left(x^2+1\right)+392\log\left(x^2+2\right)+\frac{\int^{x^2}\left(-\frac{1097}{4}\right)dx}{2}+\frac{161\int^{x^2}x\,dx}{4}+\frac{248}{x^2+2}-\frac{9}{8\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**12/(8*(x**4 + 3*x**2 + 2)) - 21*x**6/8 + 2*log(x**2 + 1) + 3
92*log(x**2 + 2) + Integral(-1097/4, (x, x**2))/2 + 161*Integral(
x, (x, x**2))/4 + 248/(x**2 + 2) - 9/(8*(x**2 + 1))
```

Mathematica [A] time = 0.0478874, size = 62, normalized size = 0.91

$$\frac{1}{8}\left(5x^8-36x^6+196x^4-1172x^2+16\log\left(x^2+1\right)+3136\log\left(x^2+2\right)+\frac{4\left(415x^2+414\right)}{x^4+3x^2+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-1172*x^2 + 196*x^4 - 36*x^6 + 5*x^8 + (4*(414 + 415*x^2))/(2 +
3*x^2 + x^4) + 16*Log[1 + x^2] + 3136*Log[2 + x^2])/8
```

Maple [A] time = 0.024, size = 56, normalized size = 0.8

$$\frac{5\,x^{8}}{8}-\frac{9\,x^{6}}{2}+\frac{49\,x^{4}}{2}-\frac{293\,x^{2}}{2}+392\,\ln\left(x^{2}+2\right)+208\,\left(x^{2}+2\right)^{-1}-\frac{1}{2\,x^{2}+2}+2\,\ln\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/8*x^8-9/2*x^6+49/2*x^4-293/2*x^2+392*ln(x^2+2)+208/(x^2+2)-1/2/
(x^2+1)+2*ln(x^2+1)
```

Maxima [A] time = 0.731391, size = 78, normalized size = 1.15

$$\frac{5}{8}\,x^{8}-\frac{9}{2}\,x^{6}+\frac{49}{2}\,x^{4}-\frac{293}{2}\,x^{2}+\frac{415\,x^{2}+414}{2\,(x^{4}+3\,x^{2}+2)}+392\,\log\left(x^{2}+2\right)+2\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^9/(x^4 + 3*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] 5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 + 1/2*(415*x^2 + 414)/(x
^4 + 3*x^2 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

Fricas [A] time = 0.259687, size = 111, normalized size = 1.63

$$\frac{5\,x^{12}-21\,x^{10}+98\,x^{8}-656\,x^{6}-3124\,x^{4}-684\,x^{2}+3136\,\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+2\right)+16\,\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+1\right)+165}{8\,(x^{4}+3\,x^{2}+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^9/(x^4 + 3*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(5*x^12 - 21*x^10 + 98*x^8 - 656*x^6 - 3124*x^4 - 684*x^2 + 3
136*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 16*(x^4 + 3*x^2 + 2)*log(x^2
 + 1) + 1656)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.411799, size = 61, normalized size = 0.9

$$\frac{5x^{8}}{8}-\frac{9x^{6}}{2}+\frac{49x^{4}}{2}-\frac{293x^{2}}{2}+\frac{415x^{2}+414}{2x^{4}+6x^{2}+4}+2\log\left(x^{2}+1\right)+392\log\left(x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**8/8 - 9*x**6/2 + 49*x**4/2 - 293*x**2/2 + (415*x**2 + 414)/(
2*x**4 + 6*x**2 + 4) + 2*log(x**2 + 1) + 392*log(x**2 + 2)
```

GIAC/XCAS [A] time = 0.285976, size = 85, normalized size = 1.25

$$\frac{5}{8}\,x^{8}-\frac{9}{2}\,x^{6}+\frac{49}{2}\,x^{4}-\frac{293}{2}\,x^{2}-\frac{394\,x^{4}+767\,x^{2}+374}{2\,(x^{4}+3\,x^{2}+2)}+392\ln\left(x^{2}+2\right)+2\ln\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^9/(x^4 + 3*x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] 5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 - 1/2*(394*x^4 + 767*x^2
 + 374)/(x^4 + 3*x^2 + 2) + 392*ln(x^2 + 2) + 2*ln(x^2 + 1)
```

3.75 $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=61

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{5}{2}\log\left(x^2+1\right)-144\log\left(x^2+2\right)-\frac{207x^2+206}{2\left(x^4+3x^2+2\right)}$$

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 - (206 + 207*x^2)/(2*(2 + 3*x^2 + 
 x^4)) - (5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Rubi [A] time = 0.180364, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{5}{2}\log\left(x^2+1\right)-144\log\left(x^2+2\right)-\frac{207x^2+206}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 - (206 + 207*x^2)/(2*(2 + 3*x^2 + 
 x^4)) - (5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{5x^{10}}{6\left(x^4+3x^2+2\right)}-\frac{5\log\left(x^2+1\right)}{2}-144\log\left(x^2+2\right)+\frac{\int^{x^2}\frac{259}{3}\,dx}{2}-\frac{17\int^{x^2}x\,dx}{2}-\frac{392}{3\left(x^2+2\right)}+\frac{4}{3\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**10/(6*(x**4 + 3*x**2 + 2)) - 5*log(x**2 + 1)/2 - 144*log(x**
2 + 2) + Integral(259/3, (x, x**2))/2 - 17*Integral(x, (x, x**2))
/2 - 392/(3*(x**2 + 2)) + 4/(3*(x**2 + 1))
```

Mathematica [A] time = 0.0503608, size = 61, normalized size = 1.

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{5}{2}\log\left(x^2+1\right)-144\log\left(x^2+2\right)+\frac{-207x^2-206}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 + (-206 - 207*x^2)/(2*(2 + 3*x^2
+ x^4)) - (5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Maple [A] time = 0.022, size = 51, normalized size = 0.8

$$\frac{5\,x^6}{6}-\frac{27\,x^4}{4}+49\,x^2-144\,\ln\left(x^2+2\right)-104\,\left(x^2+2\right)^{-1}+\frac{1}{2\,x^2+2}-\frac{5\,\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/6*x^6-27/4*x^4+49*x^2-144*ln(x^2+2)-104/(x^2+2)+1/2/(x^2+1)-5/2
*ln(x^2+1)
```

Maxima [A] time = 0.736648, size = 72, normalized size = 1.18

$$\frac{5}{6}\,x^{6} - \frac{27}{4}\,x^{4} + 49\,x^{2} - \frac{207\,x^{2} + 206}{2\,(x^{4} + 3\,x^{2} + 2)} - 144\,\log\left(x^{2} + 2\right) - \frac{5}{2}\,\log\left(x^{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^7/(x^4 + 3*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 27/4*x^4 + 49*x^2 - 1/2*(207*x^2 + 206)/(x^4 + 3*x^2 +
2) - 144*log(x^2 + 2) - 5/2*log(x^2 + 1)
```

Fricas [A] time = 0.274135, size = 104, normalized size = 1.7

$$\frac{10\,x^{10} - 51\,x^{8} + 365\,x^{6} + 1602\,x^{4} - 66\,x^{2} - 1728\,\left(x^{4} + 3\,x^{2} + 2\right)\log\left(x^{2} + 2\right) - 30\,\left(x^{4} + 3\,x^{2} + 2\right)\log\left(x^{2} + 1\right) - 1236}{12\,(x^{4} + 3\,x^{2} + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^7/(x^4 + 3*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(10*x^10 - 51*x^8 + 365*x^6 + 1602*x^4 - 66*x^2 - 1728*(x^4
+ 3*x^2 + 2)*log(x^2 + 2) - 30*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 1
236)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.416023, size = 54, normalized size = 0.89

$$\frac{5x^{6}}{6} - \frac{27x^{4}}{4} + 49x^{2} - \frac{207x^{2} + 206}{2x^{4} + 6x^{2} + 4} - \frac{5\log\left(x^{2} + 1\right)}{2} - 144\log\left(x^{2} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**6/6 - 27*x**4/4 + 49*x**2 - (207*x**2 + 206)/(2*x**4 + 6*x**
2 + 4) - 5*log(x**2 + 1)/2 - 144*log(x**2 + 2)
```

GIAC/XCAS [A] time = 0.27494, size = 78, normalized size = 1.28

$$\frac{5}{6}\,x^{6} - \frac{27}{4}\,x^{4} + 49\,x^{2} + \frac{293\,x^{4} + 465\,x^{2} + 174}{4\,(x^{4} + 3\,x^{2} + 2)} - 144\,\ln\left(x^{2} + 2\right) - \frac{5}{2}\,\ln\left(x^{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^7/(x^4 + 3*x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 27/4*x^4 + 49*x^2 + 1/4*(293*x^4 + 465*x^2 + 174)/(x^4
+ 3*x^2 + 2) - 144*ln(x^2 + 2) - 5/2*ln(x^2 + 1)
```

3.76 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=54

$$\frac{5x^4}{4}-\frac{27x^2}{2}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)+\frac{103x^2+102}{2\left(x^4+3x^2+2\right)}$$

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) +
 3*Log[1 + x^2] + 46*Log[2 + x^2]
```

Rubi [A] time = 0.173221, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{5x^4}{4}-\frac{27x^2}{2}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)+\frac{103x^2+102}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) +
 3*Log[1 + x^2] + 46*Log[2 + x^2]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{5x^8}{4\left(x^4+3x^2+2\right)}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)+\frac{\int^{x^2}\left(-\frac{39}{2}\right)dx}{2}+\frac{72}{x^2+2}-\frac{7}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**8/(4*(x**4 + 3*x**2 + 2)) + 3*log(x**2 + 1) + 46*log(x**2 +
2) + Integral(-39/2, (x, x**2))/2 + 72/(x**2 + 2) - 7/(4*(x**2 +
1))
```

Mathematica [A] time = 0.0428921, size = 54, normalized size = 1.

$$\frac{5x^4}{4}-\frac{27x^2}{2}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)+\frac{103x^2+102}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) +
 3*Log[1 + x^2] + 46*Log[2 + x^2]
```

Maple [A] time = 0.022, size = 46, normalized size = 0.9

$$\frac{5\,x^4}{4}-\frac{27\,x^2}{2}+46\,\ln\left(x^2+2\right)+52\,\left(x^2+2\right)^{-1}-\frac{1}{2\,x^2+2}+3\,\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/4*x^4-27/2*x^2+46*ln(x^2+2)+52/(x^2+2)-1/2/(x^2+1)+3*ln(x^2+1)
```

Maxima [A] time = 0.722565, size = 65, normalized size = 1.2

$$\frac{5}{4}\,x^{4} - \frac{27}{2}\,x^{2} + \frac{103\,x^{2} + 102}{2\,(x^{4} + 3\,x^{2} + 2)} + 46\,\log\left(x^{2} + 2\right) + 3\,\log\left(x^{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^5/(x^4 + 3*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] 5/4*x^4 - 27/2*x^2 + 1/2*(103*x^2 + 102)/(x^4 + 3*x^2 + 2) + 46*l
og(x^2 + 2) + 3*log(x^2 + 1)
```

Fricas [A] time = 0.255956, size = 97, normalized size = 1.8

$$\frac{5\,x^{8} - 39\,x^{6} - 152\,x^{4} + 98\,x^{2} + 184\,\left(x^{4} + 3\,x^{2} + 2\right)\log\left(x^{2} + 2\right) + 12\,\left(x^{4} + 3\,x^{2} + 2\right)\log\left(x^{2} + 1\right) + 204}{4\,(x^{4} + 3\,x^{2} + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^5/(x^4 + 3*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(5*x^8 - 39*x^6 - 152*x^4 + 98*x^2 + 184*(x^4 + 3*x^2 + 2)*lo
g(x^2 + 2) + 12*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 204)/(x^4 + 3*x^
2 + 2)
```

Sympy [A] time = 0.407439, size = 48, normalized size = 0.89

$$\frac{5x^{4}}{4} - \frac{27x^{2}}{2} + \frac{103x^{2} + 102}{2x^{4} + 6x^{2} + 4} + 3\log\left(x^{2} + 1\right) + 46\log\left(x^{2} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**4/4 - 27*x**2/2 + (103*x**2 + 102)/(2*x**4 + 6*x**2 + 4) + 3
*log(x**2 + 1) + 46*log(x**2 + 2)
```

GIAC/XCAS [A] time = 0.271441, size = 72, normalized size = 1.33

$$\frac{5}{4}\,x^{4} - \frac{27}{2}\,x^{2} - \frac{49\,x^{4} + 44\,x^{2} - 4}{2\,(x^{4} + 3\,x^{2} + 2)} + 46\ln\left(x^{2} + 2\right) + 3\ln\left(x^{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^5/(x^4 + 3*x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] 5/4*x^4 - 27/2*x^2 - 1/2*(49*x^4 + 44*x^2 - 4)/(x^4 + 3*x^2 + 2)
+ 46*ln(x^2 + 2) + 3*ln(x^2 + 1)
```

3.77 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=49

$$\frac{5x^2}{2}-\frac{7}{2}\log\left(x^2+1\right)-10\log\left(x^2+2\right)-\frac{51x^2+50}{2\left(x^4+3x^2+2\right)}$$

```
[Out] (5*x^2)/2 - (50 + 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2]
)/2 - 10*Log[2 + x^2]
```

Rubi [A] time = 0.144343, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{5x^2}{2}-\frac{7}{2}\log\left(x^2+1\right)-10\log\left(x^2+2\right)-\frac{51x^2+50}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 - (50 + 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2]
)/2 - 10*Log[2 + x^2]
```

Rubi in Sympy [A] time = 22.557, size = 48, normalized size = 0.98

$$\frac{5x^6}{2\left(x^4+3x^2+2\right)}-\frac{7\log\left(x^2+1\right)}{2}-10\log\left(x^2+2\right)-\frac{46}{x^2+2}+\frac{3}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**6/(2*(x**4 + 3*x**2 + 2)) - 7*log(x**2 + 1)/2 - 10*log(x**2
+ 2) - 46/(x**2 + 2) + 3/(x**2 + 1)
```

Mathematica [A] time = 0.0404558, size = 49, normalized size = 1.

$$\frac{5x^2}{2}-\frac{7}{2}\log\left(x^2+1\right)-10\log\left(x^2+2\right)+\frac{-51x^2-50}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 + (-50 - 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2
])/2 - 10*Log[2 + x^2]
```

Maple [A] time = 0.025, size = 41, normalized size = 0.8

$$\frac{5\,x^2}{2}-10\,\ln\left(x^2+2\right)-26\,\left(x^2+2\right)^{-1}+\frac{1}{2\,x^2+2}-\frac{7\,\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/2*x^2-10*ln(x^2+2)-26/(x^2+2)+1/2/(x^2+1)-7/2*ln(x^2+1)
```

Maxima [A] time = 0.738623, size = 58, normalized size = 1.18

$$\frac{5}{2}x^2 - \frac{51x^2+50}{2(x^4+3x^2+2)} - 10\log\left(x^2+2\right) - \frac{7}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^3/(x^4 + 3*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] 5/2*x^2 - 1/2*(51*x^2 + 50)/(x^4 + 3*x^2 + 2) - 10*log(x^2 + 2) -
 7/2*log(x^2 + 1)
```

Fricas [A] time = 0.282631, size = 90, normalized size = 1.84

$$\frac{5x^6+15x^4-41x^2-20\left(x^4+3x^2+2\right)\log\left(x^2+2\right)-7\left(x^4+3x^2+2\right)\log\left(x^2+1\right)-50}{2(x^4+3x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^3/(x^4 + 3*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(5*x^6 + 15*x^4 - 41*x^2 - 20*(x^4 + 3*x^2 + 2)*log(x^2 + 2)
- 7*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 50)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.413091, size = 42, normalized size = 0.86

$$\frac{5x^2}{2} - \frac{51x^2+50}{2x^4+6x^2+4} - \frac{7\log\left(x^2+1\right)}{2} - 10\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**2/2 - (51*x**2 + 50)/(2*x**4 + 6*x**2 + 4) - 7*log(x**2 + 1)
/2 - 10*log(x**2 + 2)
```

GIAC/XCAS [A] time = 0.273641, size = 61, normalized size = 1.24

$$\frac{5}{2}x^2 - \frac{51x^2+50}{2(x^2+2)(x^2+1)} - 10\ln\left(x^2+2\right) - \frac{7}{2}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^3/(x^4 + 3*x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] 5/2*x^2 - 1/2*(51*x^2 + 50)/((x^2 + 2)*(x^2 + 1)) - 10*ln(x^2 + 2
) - 7/2*ln(x^2 + 1)
```

3.78 $\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=42

$$4\log\left(x^2+1\right)-\frac{3}{2}\log\left(x^2+2\right)+\frac{25x^2+24}{2\left(x^4+3x^2+2\right)}$$

```
[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 +
 x^2])/2
```

Rubi [A] time = 0.084288, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$4\log\left(x^2+1\right)-\frac{3}{2}\log\left(x^2+2\right)+\frac{25x^2+24}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 +
 x^2])/2
```

Rubi in Sympy [A] time = 17.3863, size = 48, normalized size = 1.14

$$\frac{0.25\left(50x^2+48\right)}{x^4+3x^2+2}+2.75\log\left(x^2+1\right)-2.75\log\left(x^2+2\right)+\frac{5\log\left(x^4+3x^2+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 0.25*(50*x**2 + 48)/(x**4 + 3*x**2 + 2) + 2.75*log(x**2 + 1) - 2.
75*log(x**2 + 2) + 5*log(x**4 + 3*x**2 + 2)/4
```

Mathematica [A] time = 0.0312979, size = 42, normalized size = 1.

$$4\log\left(x^2+1\right)-\frac{3}{2}\log\left(x^2+2\right)+\frac{25x^2+24}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 +
 x^2])/2
```

Maple [A] time = 0.022, size = 36, normalized size = 0.9

$$-\frac{3\ln\left(x^2+2\right)}{2}+13\left(x^2+2\right)^{-1}-\frac{1}{2x^2+2}+4\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] -3/2*ln(x^2+2)+13/(x^2+2)-1/2/(x^2+1)+4*ln(x^2+1)
```

Maxima [A] time = 0.725112, size = 51, normalized size = 1.21

$$\frac{25\,x^2+24}{2\,(x^4+3\,x^2+2)}-\frac{3}{2}\,\log\left(x^2+2\right)+4\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x/(x^4 + 3*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(25*x^2 + 24)/(x^4 + 3*x^2 + 2) - 3/2*log(x^2 + 2) + 4*log(x^
2 + 1)
```

Fricas [A] time = 0.252317, size = 77, normalized size = 1.83

$$\frac{25\,x^2-3\,\left(x^4+3\,x^2+2\right)\,\log\left(x^2+2\right)+8\,\left(x^4+3\,x^2+2\right)\,\log\left(x^2+1\right)+24}{2\,(x^4+3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x/(x^4 + 3*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(25*x^2 - 3*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 8*(x^4 + 3*x^2 +
 2)*log(x^2 + 1) + 24)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.394701, size = 36, normalized size = 0.86

$$\frac{25x^2+24}{2x^4+6x^2+4}+4\log\left(x^2+1\right)-\frac{3\log\left(x^2+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] (25*x**2 + 24)/(2*x**4 + 6*x**2 + 4) + 4*log(x**2 + 1) - 3*log(x*
*2 + 2)/2
```

GIAC/XCAS [A] time = 0.274234, size = 54, normalized size = 1.29

$$\frac{25\,x^2+24}{2\,(x^2+2)(x^2+1)}-\frac{3}{2}\ln\left(x^2+2\right)+4\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x/(x^4 + 3*x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(25*x^2 + 24)/((x^2 + 2)*(x^2 + 1)) - 3/2*ln(x^2 + 2) + 4*ln(
x^2 + 1)
```

3.79 $\int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{9}{2}\log\left(x^2+1\right)+4\log\left(x^2+2\right)-\frac{12x^2+11}{2\left(x^4+3x^2+2\right)}+\log(x)$$

```
[Out] -(11 + 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/
2 + 4*Log[2 + x^2]
```

Rubi [A] time = 0.140799, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$-\frac{9}{2}\log\left(x^2+1\right)+4\log\left(x^2+2\right)-\frac{12x^2+11}{2\left(x^4+3x^2+2\right)}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -(11 + 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/
2 + 4*Log[2 + x^2]
```

Rubi in Sympy [A] time = 22.1618, size = 56, normalized size = 1.27

$$-\frac{5x^2}{2\left(x^4+3x^2+2\right)}+\frac{\log\left(x^2\right)}{2}-\frac{9\log\left(x^2+1\right)}{2}+4\log\left(x^2+2\right)-\frac{3}{2\left(x^2+2\right)}-\frac{2}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+3*x**2+2)**2,x)
```

```
[Out] -5*x**2/(2*(x**4 + 3*x**2 + 2)) + log(x**2)/2 - 9*log(x**2 + 1)/2
 + 4*log(x**2 + 2) - 3/(2*(x**2 + 2)) - 2/(x**2 + 1)
```

Mathematica [A] time = 0.0351933, size = 44, normalized size = 1.

$$-\frac{9}{2}\log\left(x^2+1\right)+4\log\left(x^2+2\right)+\frac{-12x^2-11}{2\left(x^4+3x^2+2\right)}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-11 - 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/
2 + 4*Log[2 + x^2]
```

Maple [A] time = 0.027, size = 38, normalized size = 0.9

$$\ln(x)+4\,\ln\left(x^2+2\right)-\frac{13}{2\,x^2+4}+\frac{1}{2\,x^2+2}-\frac{9\,\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x)
```

```
[Out] ln(x)+4*ln(x^2+2)-13/2/(x^2+2)+1/2/(x^2+1)-9/2*ln(x^2+1)
```

Maxima [A] time = 0.721556, size = 59, normalized size = 1.34

$$-\frac{12\,x^2+11}{2\,(x^4+3\,x^2+2)}+4\,\log\left(x^2+2\right)-\frac{9}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x),x, algorithm="maxima")
```

```
[Out] -1/2*(12*x^2 + 11)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x
^2 + 1) + 1/2*log(x^2)
```

Fricas [A] time = 0.260401, size = 96, normalized size = 2.18

$$-\frac{12\,x^2-8\,\left(x^4+3\,x^2+2\right)\,\log\left(x^2+2\right)+9\,\left(x^4+3\,x^2+2\right)\,\log\left(x^2+1\right)-2\,\left(x^4+3\,x^2+2\right)\,\log\left(x\right)+11}{2\,(x^4+3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x),x, algorithm="fricas")
```

```
[Out] -1/2*(12*x^2 - 8*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 9*(x^4 + 3*x^2
+ 2)*log(x^2 + 1) - 2*(x^4 + 3*x^2 + 2)*log(x) + 11)/(x^4 + 3*x^2
 + 2)
```

Sympy [A] time = 0.453258, size = 39, normalized size = 0.89

$$-\frac{12x^2+11}{2x^4+6x^2+4}+\log\left(x\right)-\frac{9\log\left(x^2+1\right)}{2}+4\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+3*x**2+2)**2,x)
```

```
[Out] -(12*x**2 + 11)/(2*x**4 + 6*x**2 + 4) + log(x) - 9*log(x**2 + 1)/
2 + 4*log(x**2 + 2)
```

GIAC/XCAS [A] time = 0.272368, size = 63, normalized size = 1.43

$$\frac{x^4-21\,x^2-20}{4\,(x^4+3\,x^2+2)}+4\ln\left(x^2+2\right)-\frac{9}{2}\ln\left(x^2+1\right)+\frac{1}{2}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x),x, algorithm="giac")
```

```
[Out] 1/4*(x^4 - 21*x^2 - 20)/(x^4 + 3*x^2 + 2) + 4*ln(x^2 + 2) - 9/2*l
n(x^2 + 1) + 1/2*ln(x^2)
```

3.80 $\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{1}{2x^2}+5\log\left(x^2+1\right)-\frac{29}{8}\log\left(x^2+2\right)+\frac{11x^2+9}{4\left(x^4+3x^2+2\right)}-\frac{11\log(x)}{4}$$

```
[Out] -1/(2*x^2) + (9 + 11*x^2)/(4*(2 + 3*x^2 + x^4)) - (11*Log[x])/4 +
 5*Log[1 + x^2] - (29*Log[2 + x^2])/8
```

Rubi [A] time = 0.167633, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$-\frac{1}{2x^2}+5\log\left(x^2+1\right)-\frac{29}{8}\log\left(x^2+2\right)+\frac{11x^2+9}{4\left(x^4+3x^2+2\right)}-\frac{11\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(2*x^2) + (9 + 11*x^2)/(4*(2 + 3*x^2 + x^4)) - (11*Log[x])/4 +
 5*Log[1 + x^2] - (29*Log[2 + x^2])/8
```

Rubi in Sympy [A] time = 22.6491, size = 61, normalized size = 1.11

$$-\frac{11\log\left(x^2\right)}{8}+5\log\left(x^2+1\right)-\frac{29\log\left(x^2+2\right)}{8}-\frac{5}{4\left(x^4+3x^2+2\right)}+\frac{2}{x^2+2}+\frac{3}{4\left(x^2+1\right)}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+3*x**2+2)**2,x)
```

```
[Out] -11*log(x**2)/8 + 5*log(x**2 + 1) - 29*log(x**2 + 2)/8 - 5/(4*(x*
*4 + 3*x**2 + 2)) + 2/(x**2 + 2) + 3/(4*(x**2 + 1)) - 1/(2*x**2)
```

Mathematica [A] time = 0.0425769, size = 50, normalized size = 0.91

$$\frac{1}{8}\left(-\frac{4}{x^2}+40\log\left(x^2+1\right)-29\log\left(x^2+2\right)+\frac{22x^2+18}{x^4+3x^2+2}-22\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-4/x^2 + (18 + 22*x^2)/(2 + 3*x^2 + x^4) - 22*Log[x] + 40*Log[1
+ x^2] - 29*Log[2 + x^2])/8
```

Maple [A] time = 0.028, size = 45, normalized size = 0.8

$$-\frac{1}{2\,x^2}-\frac{11\,\ln(x)}{4}-\frac{29\,\ln\left(x^2+2\right)}{8}+\frac{13}{4\,x^2+8}-\frac{1}{2\,x^2+2}+5\,\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/2/x^2-11/4*ln(x)-29/8*ln(x^2+2)+13/4/(x^2+2)-1/2/(x^2+1)+5*ln(
x^2+1)
```

Maxima [A] time = 0.720326, size = 72, normalized size = 1.31

$$\frac{9\,x^4+3\,x^2-4}{4\,(x^6+3\,x^4+2\,x^2)}-\frac{29}{8}\,\log\left(x^2+2\right)+5\,\log\left(x^2+1\right)-\frac{11}{8}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^3),x, algorithm="maxima
```

```
[Out] 1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2)
 + 5*log(x^2 + 1) - 11/8*log(x^2)
```

Fricas [A] time = 0.278775, size = 124, normalized size = 2.25

$$\frac{18\,x^4+6\,x^2-29\,\left(x^6+3\,x^4+2\,x^2\right)\log\left(x^2+2\right)+40\,\left(x^6+3\,x^4+2\,x^2\right)\log\left(x^2+1\right)-22\,\left(x^6+3\,x^4+2\,x^2\right)\log\left(x\right)-8}{8\,(x^6+3\,x^4+2\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^3),x, algorithm="fricas
```

```
[Out] 1/8*(18*x^4 + 6*x^2 - 29*(x^6 + 3*x^4 + 2*x^2)*log(x^2 + 2) + 40*
(x^6 + 3*x^4 + 2*x^2)*log(x^2 + 1) - 22*(x^6 + 3*x^4 + 2*x^2)*log
(x) - 8)/(x^6 + 3*x^4 + 2*x^2)
```

Sympy [A] time = 0.545322, size = 51, normalized size = 0.93

$$\frac{9x^4+3x^2-4}{4x^6+12x^4+8x^2}-\frac{11\log\left(x\right)}{4}+5\log\left(x^2+1\right)-\frac{29\log\left(x^2+2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+3*x**2+2)**2,x)
```

```
[Out] (9*x**4 + 3*x**2 - 4)/(4*x**6 + 12*x**4 + 8*x**2) - 11*log(x)/4 +
 5*log(x**2 + 1) - 29*log(x**2 + 2)/8
```

GIAC/XCAS [A] time = 0.271813, size = 72, normalized size = 1.31

$$\frac{9\,x^4+3\,x^2-4}{4\,(x^6+3\,x^4+2\,x^2)}-\frac{29}{8}\ln\left(x^2+2\right)+5\ln\left(x^2+1\right)-\frac{11}{8}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^3),x, algorithm="giac")
```

```
[Out] 1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*ln(x^2 + 2)
+ 5*ln(x^2 + 1) - 11/8*ln(x^2)
```

3.81 $\int \frac{4+x^2+3x^4+5x^6}{x^5(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{11}{2}\log\left(x^2+1\right)+\frac{21}{8}\log\left(x^2+2\right)-\frac{9x^2+5}{8\left(x^4+3x^2+2\right)}+\frac{23\log(x)}{4}$$

```
[Out] -1/(4*x^4) + 11/(8*x^2) - (5 + 9*x^2)/(8*(2 + 3*x^2 + x^4)) + (23
*Log[x])/4 - (11*Log[1 + x^2])/2 + (21*Log[2 + x^2])/8
```

Rubi [A] time = 0.172182, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$-\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{11}{2}\log\left(x^2+1\right)+\frac{21}{8}\log\left(x^2+2\right)-\frac{9x^2+5}{8\left(x^4+3x^2+2\right)}+\frac{23\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(4*x^4) + 11/(8*x^2) - (5 + 9*x^2)/(8*(2 + 3*x^2 + x^4)) + (23
*Log[x])/4 - (11*Log[1 + x^2])/2 + (21*Log[2 + x^2])/8
```

Rubi in Sympy [A] time = 22.8318, size = 75, normalized size = 1.17

$$\frac{23\log\left(x^2\right)}{8}-\frac{11\log\left(x^2+1\right)}{2}+\frac{21\log\left(x^2+2\right)}{8}-\frac{29}{24\left(x^2+2\right)}-\frac{1}{3\left(x^2+1\right)}+\frac{43}{24x^2}-\frac{5}{6x^2\left(x^4+3x^2+2\right)}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+3*x**2+2)**2,x)
```

```
[Out] 23*log(x**2)/8 - 11*log(x**2 + 1)/2 + 21*log(x**2 + 2)/8 - 29/(24
*(x**2 + 2)) - 1/(3*(x**2 + 1)) + 43/(24*x**2) - 5/(6*x**2*(x**4
+ 3*x**2 + 2)) - 1/(4*x**4)
```

Mathematica [A] time = 0.0514542, size = 56, normalized size = 0.88

$$\frac{1}{8}\left(-\frac{2}{x^4}+\frac{11}{x^2}-44\log\left(x^2+1\right)+21\log\left(x^2+2\right)-\frac{9x^2+5}{x^4+3x^2+2}+46\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-2/x^4 + 11/x^2 - (5 + 9*x^2)/(2 + 3*x^2 + x^4) + 46*Log[x] - 44
*Log[1 + x^2] + 21*Log[2 + x^2])/8
```

Maple [A] time = 0.029, size = 50, normalized size = 0.8

$$-\frac{1}{4\,x^4}+\frac{11}{8\,x^2}+\frac{23\,\ln(x)}{4}+\frac{21\,\ln\left(x^2+2\right)}{8}-\frac{13}{8\,x^2+16}+\frac{1}{2\,x^2+2}-\frac{11\,\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/4/x^4+11/8/x^2+23/4*ln(x)+21/8*ln(x^2+2)-13/8/(x^2+2)+1/2/(x^2
+1)-11/2*ln(x^2+1)
```

Maxima [A] time = 0.739014, size = 76, normalized size = 1.19

$$\frac{x^{6}+13\,x^{4}+8\,x^{2}-2}{4\,(x^{8}+3\,x^{6}+2\,x^{4})}+\frac{21}{8}\,\log\left(x^{2}+2\right)-\frac{11}{2}\,\log\left(x^{2}+1\right)+\frac{23}{8}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^5),x, algorithm="maxima
```

```
[Out] 1/4*(x^6 + 13*x^4 + 8*x^2 - 2)/(x^8 + 3*x^6 + 2*x^4) + 21/8*log(x
^2 + 2) - 11/2*log(x^2 + 1) + 23/8*log(x^2)
```

Fricas [A] time = 0.26491, size = 131, normalized size = 2.05

$$\frac{2\,x^{6}+26\,x^{4}+16\,x^{2}+21\,\left(x^{8}+3\,x^{6}+2\,x^{4}\right)\log\left(x^{2}+2\right)-44\,\left(x^{8}+3\,x^{6}+2\,x^{4}\right)\log\left(x^{2}+1\right)+46\,\left(x^{8}+3\,x^{6}+2\,x^{4}\right)\log\left(x\right)-}{8\,(x^{8}+3\,x^{6}+2\,x^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^5),x, algorithm="fricas
```

```
[Out] 1/8*(2*x^6 + 26*x^4 + 16*x^2 + 21*(x^8 + 3*x^6 + 2*x^4)*log(x^2 +
 2) - 44*(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 1) + 46*(x^8 + 3*x^6 + 2
*x^4)*log(x) - 4)/(x^8 + 3*x^6 + 2*x^4)
```

Sympy [A] time = 0.613668, size = 56, normalized size = 0.88

$$\frac{23\log\left(x\right)}{4}-\frac{11\log\left(x^{2}+1\right)}{2}+\frac{21\log\left(x^{2}+2\right)}{8}+\frac{x^{6}+13x^{4}+8x^{2}-2}{4x^{8}+12x^{6}+8x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+3*x**2+2)**2,x)
```

```
[Out] 23*log(x)/4 - 11*log(x**2 + 1)/2 + 21*log(x**2 + 2)/8 + (x**6 + 1
3*x**4 + 8*x**2 - 2)/(4*x**8 + 12*x**6 + 8*x**4)
```

GIAC/XCAS [A] time = 0.277373, size = 89, normalized size = 1.39

$$\frac{23\,x^{4}+51\,x^{2}+36}{16\,(x^{4}+3\,x^{2}+2)}-\frac{69\,x^{4}-22\,x^{2}+4}{16\,x^{4}}+\frac{21}{8}\ln\left(x^{2}+2\right)-\frac{11}{2}\ln\left(x^{2}+1\right)+\frac{23}{8}\ln\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^5),x, algorithm="giac")
```

```
[Out] 1/16*(23*x^4 + 51*x^2 + 36)/(x^4 + 3*x^2 + 2) - 1/16*(69*x^4 - 22
*x^2 + 4)/x^4 + 21/8*ln(x^2 + 2) - 11/2*ln(x^2 + 1) + 23/8*ln(x^2
)
```

3.82 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=70

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-\frac{(207x^2+206)\,x}{2\,(x^4+3x^2+2)}-293x+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 - (x*(206 + 207*x^2)
)/(2*(2 + 3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/
Sqrt[2]]
```

Rubi [A] time = 0.137117, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-\frac{(207x^2+206)\,x}{2\,(x^4+3x^2+2)}-293x+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 - (x*(206 + 207*x^2)
)/(2*(2 + 3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/
Sqrt[2]]
```

Rubi in Sympy [A] time = 25.1349, size = 66, normalized size = 0.94

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-\frac{x\,(905418x^2+901044)}{8748\,(x^4+3x^2+2)}-293x+\frac{9\,\mathrm{atan}\,(x)}{2}+340\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**7/7 - 27*x**5/5 + 98*x**3/3 - x*(905418*x**2 + 901044)/(8748
*(x**4 + 3*x**2 + 2)) - 293*x + 9*atan(x)/2 + 340*sqrt(2)*atan(sq
rt(2)*x/2)
```

Mathematica [A] time = 0.0921298, size = 71, normalized size = 1.01

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}+\frac{-207x^3-206x}{2\,(x^4+3x^2+2)}-293x+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 + (-206*x - 207*x^3)
/(2*(2 + 3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/S
qrt[2]]
```

Maple [A] time = 0.029, size = 56, normalized size = 0.8

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x-104\frac{x}{x^2+2}+340\arctan\left(1/2\sqrt{2}x\right)\sqrt{2}+\frac{x}{2x^2+2}+\frac{9\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/7*x^7-27/5*x^5+98/3*x^3-293*x-104*x/(x^2+2)+340*arctan(1/2*2^(1
/2)*x)*2^(1/2)+1/2*x/(x^2+1)+9/2*arctan(x)
```

Maxima [A] time = 0.801077, size = 78, normalized size = 1.11

$$\frac{5}{7}x^7-\frac{27}{5}x^5+\frac{98}{3}x^3+340\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-293x-\frac{207x^3+206x}{2(x^4+3x^2+2)}+\frac{9}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 3*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] 5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x)
 - 293*x - 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x
)
```

Fricas [A] time = 0.276454, size = 107, normalized size = 1.53

$$\frac{150x^{11}-684x^9+3758x^7-43218x^5-192605x^3+71400\sqrt{2}\left(x^4+3x^2+2\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+945\left(x^4+3x^2+2\right)\arctan(\cdots}{210(x^4+3x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 3*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/210*(150*x^11 - 684*x^9 + 3758*x^7 - 43218*x^5 - 192605*x^3 + 7
1400*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) + 945*(x^4 +
 3*x^2 + 2)*arctan(x) - 144690*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.603544, size = 66, normalized size = 0.94

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x-\frac{207x^3+206x}{2x^4+6x^2+4}+\frac{9\operatorname{atan}(x)}{2}+340\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**7/7 - 27*x**5/5 + 98*x**3/3 - 293*x - (207*x**3 + 206*x)/(2*
x**4 + 6*x**2 + 4) + 9*atan(x)/2 + 340*sqrt(2)*atan(sqrt(2)*x/2)
```

GIAC/XCAS [A] time = 0.271722, size = 78, normalized size = 1.11

$$\frac{5}{7}x^7-\frac{27}{5}x^5+\frac{98}{3}x^3+340\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-293x-\frac{207x^3+206x}{2(x^4+3x^2+2)}+\frac{9}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 3*x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] 5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x)
 - 293*x - 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x
)
```

3.83 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=57

$$x^5-9x^3+\frac{(103x^2+102)\,x}{2\,(x^4+3x^2+2)}+98x-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 98*x - 9*x^3 + x^5 + (x*(102 + 103*x^2))/(2*(2 + 3*x^2 + x^4)) -
(11*ArcTan[x])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] time = 0.129946, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$x^5-9x^3+\frac{(103x^2+102)\,x}{2\,(x^4+3x^2+2)}+98x-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 98*x - 9*x^3 + x^5 + (x*(102 + 103*x^2))/(2*(2 + 3*x^2 + x^4)) -
(11*ArcTan[x])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0866325, size = 58, normalized size = 1.02

$$x^5-9x^3+\frac{103x^3+102x}{2\,(x^4+3x^2+2)}+98x-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 98*x - 9*x^3 + x^5 + (102*x + 103*x^3)/(2*(2 + 3*x^2 + x^4)) - (1
1*ArcTan[x])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] time = 0.016, size = 49, normalized size = 0.9

$$x^5-9\,x^3+98\,x+52\,\frac{x}{x^2+2}-118\,\arctan\left(1/2\,\sqrt{2}x\right)\sqrt{2}-\frac{x}{2\,x^2+2}-\frac{11\,\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] x^5-9*x^3+98*x+52*x/(x^2+2)-118*arctan(1/2*2^(1/2)*x)*2^(1/2)-1/2
*x/(x^2+1)-11/2*arctan(x)
```

Maxima [A] time = 0.792695, size = 69, normalized size = 1.21

$$x^5 - 9\,x^3 - 118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 98\,x + \frac{103\,x^3 + 102\,x}{2\,(x^4 + 3\,x^2 + 2)} - \frac{11}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 3*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103
*x^3 + 102*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

Fricas [A] time = 0.265662, size = 100, normalized size = 1.75

$$\frac{2\,x^9 - 12\,x^7 + 146\,x^5 + 655\,x^3 - 236\,\sqrt{2}\left(x^4 + 3\,x^2 + 2\right)\,\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 11\,\left(x^4 + 3\,x^2 + 2\right)\,\arctan(x) + 494\,x}{2\,(x^4 + 3\,x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 3*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^9 - 12*x^7 + 146*x^5 + 655*x^3 - 236*sqrt(2)*(x^4 + 3*x^
2 + 2)*arctan(1/2*sqrt(2)*x) - 11*(x^4 + 3*x^2 + 2)*arctan(x) + 4
94*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.585739, size = 54, normalized size = 0.95

$$x^5 - 9x^3 + 98x + \frac{103x^3 + 102x}{2x^4 + 6x^2 + 4} - \frac{11\,\mathrm{atan}\,(x)}{2} - 118\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] x**5 - 9*x**3 + 98*x + (103*x**3 + 102*x)/(2*x**4 + 6*x**2 + 4) -
 11*atan(x)/2 - 118*sqrt(2)*atan(sqrt(2)*x/2)
```

GIAC/XCAS [A] time = 0.275186, size = 69, normalized size = 1.21

$$x^5 - 9\,x^3 - 118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 98\,x + \frac{103\,x^3 + 102\,x}{2\,(x^4 + 3\,x^2 + 2)} - \frac{11}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 3*x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103
*x^3 + 102*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

3.84 $\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=56

$$\frac{5x^3}{3}-\frac{\left(51x^2+50\right)x}{2\left(x^4+3x^2+2\right)}-27x+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -27*x + (5*x^3)/3 - (x*(50 + 51*x^2))/(2*(2 + 3*x^2 + x^4)) + (13
*ArcTan[x])/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] time = 0.121756, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{5x^3}{3}-\frac{\left(51x^2+50\right)x}{2\left(x^4+3x^2+2\right)}-27x+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -27*x + (5*x^3)/3 - (x*(50 + 51*x^2))/(2*(2 + 3*x^2 + x^4)) + (13
*ArcTan[x])/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi in Sympy [A] time = 21.6324, size = 53, normalized size = 0.95

$$\frac{5x^3}{3}-\frac{x\left(24786x^2+24300\right)}{972\left(x^4+3x^2+2\right)}-27x+\frac{13\operatorname{atan}(x)}{2}+33\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**3/3 - x*(24786*x**2 + 24300)/(972*(x**4 + 3*x**2 + 2)) - 27*
x + 13*atan(x)/2 + 33*sqrt(2)*atan(sqrt(2)*x/2)
```

Mathematica [A] time = 0.08301, size = 57, normalized size = 1.02

$$\frac{5x^3}{3}+\frac{-51x^3-50x}{2\left(x^4+3x^2+2\right)}-27x+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -27*x + (5*x^3)/3 + (-50*x - 51*x^3)/(2*(2 + 3*x^2 + x^4)) + (13*
ArcTan[x])/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] time = 0.019, size = 46, normalized size = 0.8

$$\frac{5\,x^3}{3}-27\,x-26\,\frac{x}{x^2+2}+33\,\arctan\left(1/2\,\sqrt{2}x\right)\sqrt{2}+\frac{x}{2\,x^2+2}+\frac{13\,\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/3*x^3-27*x-26*x/(x^2+2)+33*arctan(1/2*2^(1/2)*x)*2^(1/2)+1/2*x/
(x^2+1)+13/2*arctan(x)
```

Maxima [A] time = 0.797375, size = 65, normalized size = 1.16

$$\frac{5}{3}\,x^{3} + 33\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 27\,x - \frac{51\,x^{3} + 50\,x}{2\,(x^{4} + 3\,x^{2} + 2)} + \frac{13}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 3*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 +
 50*x)/(x^4 + 3*x^2 + 2) + 13/2*arctan(x)
```

Fricas [A] time = 0.264796, size = 93, normalized size = 1.66

$$\frac{10\,x^{7} - 132\,x^{5} - 619\,x^{3} + 198\,\sqrt{2}\left(x^{4} + 3\,x^{2} + 2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 39\,\left(x^{4} + 3\,x^{2} + 2\right)\arctan\left(x\right) - 474\,x}{6\,(x^{4} + 3\,x^{2} + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 3*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(10*x^7 - 132*x^5 - 619*x^3 + 198*sqrt(2)*(x^4 + 3*x^2 + 2)*a
rctan(1/2*sqrt(2)*x) + 39*(x^4 + 3*x^2 + 2)*arctan(x) - 474*x)/(x
^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.578609, size = 53, normalized size = 0.95

$$\frac{5x^{3}}{3} - 27x - \frac{51x^{3} + 50x}{2x^{4} + 6x^{2} + 4} + \frac{13\operatorname{atan}{\left(x\right)}}{2} + 33\sqrt{2}\operatorname{atan}{\left(\frac{\sqrt{2}x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**3/3 - 27*x - (51*x**3 + 50*x)/(2*x**4 + 6*x**2 + 4) + 13*ata
n(x)/2 + 33*sqrt(2)*atan(sqrt(2)*x/2)
```

GIAC/XCAS [A] time = 0.27167, size = 65, normalized size = 1.16

$$\frac{5}{3}\,x^{3} + 33\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 27\,x - \frac{51\,x^{3} + 50\,x}{2\,(x^{4} + 3\,x^{2} + 2)} + \frac{13}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 3*x^2 + 2)^2,x, algorithm="giac")

[Out] 5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 +
 50*x)/(x^4 + 3*x^2 + 2) + 13/2*arctan(x)
```

3.85 $\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{\left(25x^2+24\right)x}{2\left(x^4+3x^2+2\right)}+5x-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 5*x + (x*(24 + 25*x^2))/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2
- (7*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rubi [A] time = 0.114363, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{\left(25x^2+24\right)x}{2\left(x^4+3x^2+2\right)}+5x-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (x*(24 + 25*x^2))/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2
- (7*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rubi in Sympy [A] time = 19.5035, size = 48, normalized size = 0.98

$$\frac{x\left(4050x^2+3888\right)}{324\left(x^4+3x^2+2\right)}+5x-\frac{15\operatorname{atan}(x)}{2}-\frac{7\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] x*(4050*x**2 + 3888)/(324*(x**4 + 3*x**2 + 2)) + 5*x - 15*atan(x)
/2 - 7*sqrt(2)*atan(sqrt(2)*x/2)/2
```

Mathematica [A] time = 0.0698881, size = 50, normalized size = 1.02

$$\frac{25x^3+24x}{2\left(x^4+3x^2+2\right)}+5x-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (24*x + 25*x^3)/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 -
(7*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Maple [A] time = 0.016, size = 41, normalized size = 0.8

$$5\,x+13\,\frac{x}{x^2+2}-\frac{7\,\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)-\frac{x}{2\,x^2+2}-\frac{15\,\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5*x+13*x/(x^2+2)-7/2*arctan(1/2*2^(1/2)*x)*2^(1/2)-1/2*x/(x^2+1)-
15/2*arctan(x)
```

Maxima [A] time = 0.793351, size = 58, normalized size = 1.18

$$-\frac{7}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5x+\frac{25x^3+24x}{2(x^4+3x^2+2)}-\frac{15}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 3*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x
^4 + 3*x^2 + 2) - 15/2*arctan(x)
```

Fricas [A] time = 0.271361, size = 99, normalized size = 2.02

$$-\frac{\sqrt{2}\left(15\sqrt{2}\left(x^4+3x^2+2\right)\arctan(x)+14\left(x^4+3x^2+2\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\sqrt{2}\left(10x^5+55x^3+44x\right)\right)}{4\left(x^4+3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 3*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(15*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(x) + 14*(x^4 +
3*x^2 + 2)*arctan(1/2*sqrt(2)*x) - sqrt(2)*(10*x^5 + 55*x^3 + 44*
x))/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.581369, size = 48, normalized size = 0.98

$$5x+\frac{25x^3+24x}{2x^4+6x^2+4}-\frac{15\operatorname{atan}(x)}{2}-\frac{7\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x + (25*x**3 + 24*x)/(2*x**4 + 6*x**2 + 4) - 15*atan(x)/2 - 7*s
qrt(2)*atan(sqrt(2)*x/2)/2
```

GIAC/XCAS [A] time = 0.276143, size = 58, normalized size = 1.18

$$-\frac{7}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5x+\frac{25x^3+24x}{2(x^4+3x^2+2)}-\frac{15}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 3*x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x
^4 + 3*x^2 + 2) - 15/2*arctan(x)
```

3.86 $\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=48

$$-\frac{x\left(12x^2+11\right)}{2\left(x^4+3x^2+2\right)}+\frac{17}{2}\tan^{-1}(x)-\frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -(x*(11 + 12*x^2))/(2*(2 + 3*x^2 + x^4)) + (17*ArcTan[x])/2 - (19
*ArcTan[x/Sqrt[2]])/(2*Sqrt[2])
```

Rubi [A] time = 0.0583412, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$-\frac{x\left(12x^2+11\right)}{2\left(x^4+3x^2+2\right)}+\frac{17}{2}\tan^{-1}(x)-\frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -(x*(11 + 12*x^2))/(2*(2 + 3*x^2 + x^4)) + (17*ArcTan[x])/2 - (19
*ArcTan[x/Sqrt[2]])/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 10.4635, size = 44, normalized size = 0.92

$$-\frac{x\left(648x^2+594\right)}{108\left(x^4+3x^2+2\right)}+\frac{17\,\mathrm{atan}\,(x)}{2}-\frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] -x*(648*x**2 + 594)/(108*(x**4 + 3*x**2 + 2)) + 17*atan(x)/2 - 19
*sqrt(2)*atan(sqrt(2)*x/2)/4
```

Mathematica [A] time = 0.0718314, size = 46, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2x\left(12x^2+11\right)}{x^4+3x^2+2}+34\tan^{-1}(x)-19\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] ((-2*x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + 34*ArcTan[x] - 19*Sqrt[
2]*ArcTan[x/Sqrt[2]])/4
```

Maple [A] time = 0.015, size = 38, normalized size = 0.8

$$-\frac{13\,x}{2\,x^2+4}-\frac{19\,\sqrt{2}}{4}\arctan\left(\frac{\sqrt{2}x}{2}\right)+\frac{x}{2\,x^2+2}+\frac{17\,\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] -13/2*x/(x^2+2)-19/4*arctan(1/2*2^(1/2)*x)*2^(1/2)+1/2*x/(x^2+1)+
17/2*arctan(x)
```

Maxima [A] time = 0.792439, size = 54, normalized size = 1.12

$$-\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{12\,x^3+11\,x}{2\,(x^4+3\,x^2+2)}+\frac{17}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 3*x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 +
3*x^2 + 2) + 17/2*arctan(x)
```

Fricas [A] time = 0.269738, size = 92, normalized size = 1.92

$$\frac{\sqrt{2}\left(17\,\sqrt{2}\left(x^4+3\,x^2+2\right)\arctan\left(x\right)-19\,\left(x^4+3\,x^2+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-\sqrt{2}\left(12\,x^3+11\,x\right)\right)}{4\,(x^4+3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 3*x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(17*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(x) - 19*(x^4 + 3
*x^2 + 2)*arctan(1/2*sqrt(2)*x) - sqrt(2)*(12*x^3 + 11*x))/(x^4 +
 3*x^2 + 2)
```

Sympy [A] time = 0.579924, size = 44, normalized size = 0.92

$$-\frac{12x^3+11x}{2x^4+6x^2+4}+\frac{17\operatorname{atan}\left(x\right)}{2}-\frac{19\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] -(12*x**3 + 11*x)/(2*x**4 + 6*x**2 + 4) + 17*atan(x)/2 - 19*sqrt(
2)*atan(sqrt(2)*x/2)/4
```

GIAC/XCAS [A] time = 0.270425, size = 54, normalized size = 1.12

$$-\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{12\,x^3+11\,x}{2\,(x^4+3\,x^2+2)}+\frac{17}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 3*x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 +
3*x^2 + 2) + 17/2*arctan(x)
```

3.87 $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2} \, dx$

Optimal. Leaf size=53

$$\frac{x(11x^2+9)}{4(x^4+3x^2+2)} - \frac{1}{x} - \frac{19}{2}\tan^{-1}(x) + \frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -x^(-1) + (x*(9 + 11*x^2))/(4*(2 + 3*x^2 + x^4)) - (19*ArcTan[x])
/2 + (45*ArcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rubi [A] time = 0.118576, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{x(11x^2+9)}{4(x^4+3x^2+2)} - \frac{1}{x} - \frac{19}{2}\tan^{-1}(x) + \frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -x^(-1) + (x*(9 + 11*x^2))/(4*(2 + 3*x^2 + x^4)) - (19*ArcTan[x])
/2 + (45*ArcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 19.0254, size = 48, normalized size = 0.91

$$\frac{x(4374x^2+7290)}{216(x^4+3x^2+2)} - \frac{47\,\mathrm{atan}(x)}{2} + \frac{199\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8} + \frac{6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**2,x)
```

```
[Out] x*(4374*x**2 + 7290)/(216*(x**4 + 3*x**2 + 2)) - 47*atan(x)/2 + 1
99*sqrt(2)*atan(sqrt(2)*x/2)/8 + 6/x
```

Mathematica [A] time = 0.0980377, size = 51, normalized size = 0.96

$$\frac{1}{8}\left(\frac{2x(11x^2+9)}{x^4+3x^2+2} - \frac{8}{x} - 76\tan^{-1}(x) + 45\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-8/x + (2*x*(9 + 11*x^2))/(2 + 3*x^2 + x^4) - 76*ArcTan[x] + 45*
Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Maple [A] time = 0.018, size = 43, normalized size = 0.8

$$-x^{-1} + \frac{13\,x}{4\,x^2+8} + \frac{45\,\sqrt{2}}{8}\arctan\left(\frac{\sqrt{2}x}{2}\right) - \frac{x}{2\,x^2+2} - \frac{19\,\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/x+13/4*x/(x^2+2)+45/8*arctan(1/2*2^(1/2)*x)*2^(1/2)-1/2*x/(x^2
+1)-19/2*arctan(x)
```

Maxima [A] time = 0.792478, size = 61, normalized size = 1.15

$$\frac{45}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{7x^4-3x^2-8}{4\left(x^5+3x^3+2x\right)}-\frac{19}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^2),x, algorithm="maxima
```

```
[Out] 45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5
 + 3*x^3 + 2*x) - 19/2*arctan(x)
```

Fricas [A] time = 0.27703, size = 104, normalized size = 1.96

$$-\frac{\sqrt{2}\left(38\sqrt{2}\left(x^5+3x^3+2x\right)\arctan\left(x\right)-45\left(x^5+3x^3+2x\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\sqrt{2}\left(7x^4-3x^2-8\right)\right)}{8\left(x^5+3x^3+2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^2),x, algorithm="fricas
```

```
[Out] -1/8*sqrt(2)*(38*sqrt(2)*(x^5 + 3*x^3 + 2*x)*arctan(x) - 45*(x^5
+ 3*x^3 + 2*x)*arctan(1/2*sqrt(2)*x) - sqrt(2)*(7*x^4 - 3*x^2 - 8
))/(x^5 + 3*x^3 + 2*x)
```

Sympy [A] time = 0.655939, size = 49, normalized size = 0.92

$$\frac{7x^4-3x^2-8}{4x^5+12x^3+8x}-\frac{19\operatorname{atan}\left(x\right)}{2}+\frac{45\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**2,x)
```

```
[Out] (7*x**4 - 3*x**2 - 8)/(4*x**5 + 12*x**3 + 8*x) - 19*atan(x)/2 + 4
5*sqrt(2)*atan(sqrt(2)*x/2)/8
```

GIAC/XCAS [A] time = 0.270304, size = 61, normalized size = 1.15

$$\frac{45}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{7x^4-3x^2-8}{4\left(x^5+3x^3+2x\right)}-\frac{19}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^2),x, algorithm="giac")

[Out] 45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5
 + 3*x^3 + 2*x) - 19/2*arctan(x)
```

3.88 $\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2} \, dx$

Optimal. Leaf size=62

$$-\frac{1}{3x^3}-\frac{x\left(9x^2+5\right)}{8\left(x^4+3x^2+2\right)}+\frac{11}{4x}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] -1/(3*x^3) + 11/(4*x) - (x*(5 + 9*x^2))/(8*(2 + 3*x^2 + x^4)) + (
21*ArcTan[x])/2 - (71*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rubi [A] time = 0.132133, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$-\frac{1}{3x^3}-\frac{x\left(9x^2+5\right)}{8\left(x^4+3x^2+2\right)}+\frac{11}{4x}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(3*x^3) + 11/(4*x) - (x*(5 + 9*x^2))/(8*(2 + 3*x^2 + x^4)) + (
21*ArcTan[x])/2 - (71*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rubi in Sympy [A] time = 18.9232, size = 32, normalized size = 0.52

$$-17\,\mathrm{atan}(x)+\frac{11\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}-\frac{23}{2x}+\frac{2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**2,x)
```

```
[Out] -17*atan(x) + 11*sqrt(2)*atan(sqrt(2)*x/2)/4 - 23/(2*x) + 2/x**3
```

Mathematica [A] time = 0.111851, size = 56, normalized size = 0.9

$$\frac{1}{48}\left(-\frac{16}{x^3}-\frac{6x\left(9x^2+5\right)}{x^4+3x^2+2}+\frac{132}{x}+504\tan^{-1}(x)-213\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-16/x^3 + 132/x - (6*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4) + 504*ArcT
an[x] - 213*Sqrt[2]*ArcTan[x/Sqrt[2]])/48
```

Maple [A] time = 0.021, size = 48, normalized size = 0.8

$$-\frac{1}{3\,x^3}+\frac{11}{4\,x}-\frac{13\,x}{8\,x^2+16}-\frac{71\,\sqrt{2}}{16}\arctan\left(\frac{\sqrt{2}x}{2}\right)+\frac{x}{2\,x^2+2}+\frac{21\,\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/3/x^3+11/4/x-13/8*x/(x^2+2)-71/16*arctan(1/2*2^(1/2)*x)*2^(1/2
)+1/2*x/(x^2+1)+21/2*arctan(x)
```

Maxima [A] time = 0.794324, size = 70, normalized size = 1.13

$$-\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{39\,x^{6}+175\,x^{4}+108\,x^{2}-16}{24\,(x^{7}+3\,x^{5}+2\,x^{3})}+\frac{21}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^4),x, algorithm="maxima
```

```
[Out] -71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/24*(39*x^6 + 175*x^4 + 1
08*x^2 - 16)/(x^7 + 3*x^5 + 2*x^3) + 21/2*arctan(x)
```

Fricas [A] time = 0.271057, size = 117, normalized size = 1.89

$$\frac{\sqrt{2}\left(252\,\sqrt{2}(x^{7}+3\,x^{5}+2\,x^{3})\arctan\left(x\right)-213\,\left(x^{7}+3\,x^{5}+2\,x^{3}\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\sqrt{2}(39\,x^{6}+175\,x^{4}+108\,x^{2}-16)\right)}{48\,(x^{7}+3\,x^{5}+2\,x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^4),x, algorithm="fricas
```

```
[Out] 1/48*sqrt(2)*(252*sqrt(2)*(x^7 + 3*x^5 + 2*x^3)*arctan(x) - 213*(
x^7 + 3*x^5 + 2*x^3)*arctan(1/2*sqrt(2)*x) + sqrt(2)*(39*x^6 + 17
5*x^4 + 108*x^2 - 16))/(x^7 + 3*x^5 + 2*x^3)
```

Sympy [A] time = 0.731493, size = 56, normalized size = 0.9

$$\frac{21\operatorname{atan}\left(x\right)}{2}-\frac{71\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}+\frac{39x^{6}+175x^{4}+108x^{2}-16}{24x^{7}+72x^{5}+48x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**2,x)
```

```
[Out] 21*atan(x)/2 - 71*sqrt(2)*atan(sqrt(2)*x/2)/16 + (39*x**6 + 175*x
**4 + 108*x**2 - 16)/(24*x**7 + 72*x**5 + 48*x**3)
```

GIAC/XCAS [A] time = 0.27081, size = 70, normalized size = 1.13

$$-\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{9\,x^{3}+5\,x}{8\,(x^{4}+3\,x^{2}+2)}+\frac{33\,x^{2}-4}{12\,x^{3}}+\frac{21}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^4),x, algorithm="giac")
```

```
[Out] -71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(9*x^3 + 5*x)/(x^4 + 3
*x^2 + 2) + 1/12*(33*x^2 - 4)/x^3 + 21/2*arctan(x)
```

3.89 $\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{x\left(3-5x^2\right)}{16\left(x^4+3x^2+2\right)}-\frac{23}{4x}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

```
[Out] -1/(5*x^5) + 11/(12*x^3) - 23/(4*x) - (x*(3 - 5*x^2))/(16*(2 + 3*
x^2 + x^4)) - (23*ArcTan[x])/2 + (97*ArcTan[x/Sqrt[2]])/(16*Sqrt[
2])
```

Rubi [A] time = 0.143821, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{x\left(3-5x^2\right)}{16\left(x^4+3x^2+2\right)}-\frac{23}{4x}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(5*x^5) + 11/(12*x^3) - 23/(4*x) - (x*(3 - 5*x^2))/(16*(2 + 3*
x^2 + x^4)) - (23*ArcTan[x])/2 + (97*ArcTan[x/Sqrt[2]])/(16*Sqrt[
2])
```

Rubi in Sympy [A] time = 22.4563, size = 41, normalized size = 0.59

$$17\operatorname{atan}(x)-\frac{11\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}+\frac{57}{4x}-\frac{23}{6x^3}+\frac{6}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**2,x)
```

```
[Out] 17*atan(x) - 11*sqrt(2)*atan(sqrt(2)*x/2)/8 + 57/(4*x) - 23/(6*x*
*3) + 6/(5*x**5)
```

Mathematica [A] time = 0.111535, size = 61, normalized size = 0.88

$$\frac{1}{480}\left(-\frac{96}{x^5}+\frac{440}{x^3}+\frac{30x\left(5x^2-3\right)}{x^4+3x^2+2}-\frac{2760}{x}-5520\tan^{-1}(x)+1455\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-96/x^5 + 440/x^3 - 2760/x + (30*x*(-3 + 5*x^2))/(2 + 3*x^2 + x^
4) - 5520*ArcTan[x] + 1455*Sqrt[2]*ArcTan[x/Sqrt[2]])/480
```

Maple [A] time = 0.023, size = 53, normalized size = 0.8

$$-\frac{1}{5\,x^{5}}+\frac{11}{12\,x^{3}}-\frac{23}{4\,x}+\frac{13\,x}{16\,x^{2}+32}+\frac{97\,\sqrt{2}}{32}\arctan\left(\frac{\sqrt{2}x}{2}\right)-\frac{x}{2\,x^{2}+2}-\frac{23\,\arctan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/5/x^5+11/12/x^3-23/4/x+13/16*x/(x^2+2)+97/32*arctan(1/2*2^(1/2
)*x)*2^(1/2)-1/2*x/(x^2+1)-23/2*arctan(x)
```

Maxima [A] time = 0.797223, size = 77, normalized size = 1.12

$$\frac{97}{32}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-\frac{1305\,x^{8}+3965\,x^{6}+2148\,x^{4}-296\,x^{2}+96}{240\,\left(x^{9}+3\,x^{7}+2\,x^{5}\right)}-\frac{23}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^6),x, algorithm="maxima
```

```
[Out] 97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/240*(1305*x^8 + 3965*x^6
+ 2148*x^4 - 296*x^2 + 96)/(x^9 + 3*x^7 + 2*x^5) - 23/2*arctan(x)
```

Fricas [A] time = 0.282272, size = 124, normalized size = 1.8

$$-\frac{\sqrt{2}\left(2760\,\sqrt{2}\left(x^{9}+3\,x^{7}+2\,x^{5}\right)\arctan\left(x\right)-1455\,\left(x^{9}+3\,x^{7}+2\,x^{5}\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+\sqrt{2}\left(1305\,x^{8}+3965\,x^{6}+2148\,x^{4}-29\right.\right.}{480\,\left(x^{9}+3\,x^{7}+2\,x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^6),x, algorithm="fricas
```

```
[Out] -1/480*sqrt(2)*(2760*sqrt(2)*(x^9 + 3*x^7 + 2*x^5)*arctan(x) - 14
55*(x^9 + 3*x^7 + 2*x^5)*arctan(1/2*sqrt(2)*x) + sqrt(2)*(1305*x^
8 + 3965*x^6 + 2148*x^4 - 296*x^2 + 96))/(x^9 + 3*x^7 + 2*x^5)
```

Sympy [A] time = 0.806293, size = 61, normalized size = 0.88

$$-\frac{23\,\operatorname{atan}\left(x\right)}{2}+\frac{97\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}-\frac{1305x^{8}+3965x^{6}+2148x^{4}-296x^{2}+96}{240x^{9}+720x^{7}+480x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**2,x)
```

```
[Out] -23*atan(x)/2 + 97*sqrt(2)*atan(sqrt(2)*x/2)/32 - (1305*x**8 + 39
65*x**6 + 2148*x**4 - 296*x**2 + 96)/(240*x**9 + 720*x**7 + 480*x
**5)
```

GIAC/XCAS [A] time = 0.270049, size = 77, normalized size = 1.12

$$\frac{97}{32}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{5\,x^{3} - 3\,x}{16\,(x^{4} + 3\,x^{2} + 2)} - \frac{345\,x^{4} - 55\,x^{2} + 12}{60\,x^{5}} - \frac{23}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^6),x, algorithm="giac")
```

```
[Out] 97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(5*x^3 - 3*x)/(x^4 + 3
*x^2 + 2) - 1/60*(345*x^4 - 55*x^2 + 12)/x^5 - 23/2*arctan(x)
```

3.90 $\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=76

$$-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}+\frac{137}{16x}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[Out] -1/(7*x^7) + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (x*(19 + 3*
x^2))/(32*(2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/S
qrt[2]])/(32*Sqrt[2])
```

Rubi [A] time = 0.154096, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}+\frac{137}{16x}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(7*x^7) + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (x*(19 + 3*
x^2))/(32*(2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/S
qrt[2]])/(32*Sqrt[2])
```

Rubi in Sympy [A] time = 25.9088, size = 48, normalized size = 0.63

$$-17\,\mathrm{atan}\,(x)+\frac{11\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}-\frac{125}{8x}+\frac{19}{4x^3}-\frac{23}{10x^5}+\frac{6}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**8/(x**4+3*x**2+2)**2,x)
```

```
[Out] -17*atan(x) + 11*sqrt(2)*atan(sqrt(2)*x/2)/16 - 125/(8*x) + 19/(4
*x**3) - 23/(10*x**5) + 6/(7*x**7)
```

Mathematica [A] time = 0.126884, size = 77, normalized size = 1.01

$$-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{3x^3+19x}{32\left(x^4+3x^2+2\right)}+\frac{137}{16x}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(7*x^7) + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (19*x + 3*x
^3)/(32*(2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqr
t[2]])/(32*Sqrt[2])
```

Maple [A] time = 0.023, size = 58, normalized size = 0.8

$$-\frac{1}{7x^{7}}+\frac{11}{20x^{5}}-\frac{23}{12x^{3}}+\frac{137}{16x}-\frac{13x}{32x^{2}+64}-\frac{123\sqrt{2}}{64}\arctan\left(\frac{\sqrt{2}x}{2}\right)+\frac{x}{2x^{2}+2}+\frac{25\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/7/x^7+11/20/x^5-23/12/x^3+137/16/x-13/32*x/(x^2+2)-123/64*arct
an(1/2*2^(1/2)*x)*2^(1/2)+1/2*x/(x^2+1)+25/2*arctan(x)
```

Maxima [A] time = 0.794469, size = 84, normalized size = 1.11

$$-\frac{123}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{29085x^{10}+81865x^{8}+40068x^{6}-7816x^{4}+2256x^{2}-960}{3360(x^{11}+3x^{9}+2x^{7})}+\frac{25}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^8),x, algorithm="maxima
```

```
[Out] -123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/3360*(29085*x^10 + 8186
5*x^8 + 40068*x^6 - 7816*x^4 + 2256*x^2 - 960)/(x^11 + 3*x^9 + 2*
x^7) + 25/2*arctan(x)
```

Fricas [A] time = 0.291305, size = 131, normalized size = 1.72

$$\frac{\sqrt{2}\left(42000\sqrt{2}(x^{11}+3x^{9}+2x^{7})\arctan(x)-12915(x^{11}+3x^{9}+2x^{7})\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\sqrt{2}(29085x^{10}+81865x^{8}+40068\ldots}{6720(x^{11}+3x^{9}+2x^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^8),x, algorithm="fricas
```

```
[Out] 1/6720*sqrt(2)*(42000*sqrt(2)*(x^11 + 3*x^9 + 2*x^7)*arctan(x) -
12915*(x^11 + 3*x^9 + 2*x^7)*arctan(1/2*sqrt(2)*x) + sqrt(2)*(290
85*x^10 + 81865*x^8 + 40068*x^6 - 7816*x^4 + 2256*x^2 - 960))/(x^
11 + 3*x^9 + 2*x^7)
```

Sympy [A] time = 0.904628, size = 66, normalized size = 0.87

$$\frac{25\operatorname{atan}(x)}{2}-\frac{123\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64}+\frac{29085x^{10}+81865x^{8}+40068x^{6}-7816x^{4}+2256x^{2}-960}{3360x^{11}+10080x^{9}+6720x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**8/(x**4+3*x**2+2)**2,x)
```

```
[Out] 25*atan(x)/2 - 123*sqrt(2)*atan(sqrt(2)*x/2)/64 + (29085*x**10 +
81865*x**8 + 40068*x**6 - 7816*x**4 + 2256*x**2 - 960)/(3360*x**1
1 + 10080*x**9 + 6720*x**7)
```

GIAC/XCAS [A] time = 0.269846, size = 84, normalized size = 1.11

$$-\frac{123}{64}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+\frac{3\,x^{3}+19\,x}{32\,(x^{4}+3\,x^{2}+2)}+\frac{14385\,x^{6}-3220\,x^{4}+924\,x^{2}-240}{1680\,x^{7}}+\frac{25}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^2*x^8),x, algorithm="giac")
```

```
[Out] -123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/32*(3*x^3 + 19*x)/(x^4
+ 3*x^2 + 2) + 1/1680*(14385*x^6 - 3220*x^4 + 924*x^2 - 240)/x^7
+ 25/2*arctan(x)
```

3.91 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=81

$$x^5-14x^3+\frac{(1669x^2+824)\,x}{8\,(x^4+3x^2+2)}+\frac{(415x^2+414)\,x}{4\,(x^4+3x^2+2)^2}+214x+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 214*x - 14*x^3 + x^5 + (x*(414 + 415*x^2))/(4*(2 + 3*x^2 + x^4)^2
) + (x*(824 + 1669*x^2))/(8*(2 + 3*x^2 + x^4)) + (477*ArcTan[x])/
8 - 351*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] time = 0.177441, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$x^5-14x^3+\frac{(1669x^2+824)\,x}{8\,(x^4+3x^2+2)}+\frac{(415x^2+414)\,x}{4\,(x^4+3x^2+2)^2}+214x+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] 214*x - 14*x^3 + x^5 + (x*(414 + 415*x^2))/(4*(2 + 3*x^2 + x^4)^2
) + (x*(824 + 1669*x^2))/(8*(2 + 3*x^2 + x^4)) + (477*ArcTan[x])/
8 - 351*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi in Sympy [A] time = 37.3971, size = 76, normalized size = 0.94

$$x^5-14x^3+\frac{x\,(5445630x^2+5432508)}{52488\,(x^4+3x^2+2)^2}+\frac{x\,(31931101044x^2+15764665824)}{153055008\,(x^4+3x^2+2)}+214x+\frac{477\,\mathrm{atan}\,(x)}{8}-351\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] x**5 - 14*x**3 + x*(5445630*x**2 + 5432508)/(52488*(x**4 + 3*x**2
 + 2)**2) + x*(31931101044*x**2 + 15764665824)/(153055008*(x**4 +
 3*x**2 + 2)) + 214*x + 477*atan(x)/8 - 351*sqrt(2)*atan(sqrt(2)*
x/2)
```

Mathematica [A] time = 0.120437, size = 71, normalized size = 0.88

$$\frac{x\,(8x^{12}-64x^{10}+1144x^8+10581x^6+26775x^4+26736x^2+9324)}{8\,(x^4+3x^2+2)^2}+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(9324 + 26736*x^2 + 26775*x^4 + 10581*x^6 + 1144*x^8 - 64*x^10
 + 8*x^12))/(8*(2 + 3*x^2 + x^4)^2) + (477*ArcTan[x])/8 - 351*Sqr
```

```
t[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] time = 0.018, size = 64, normalized size = 0.8

$$x^5 - 14\,x^3 + 214\,x - 16\,\frac{1}{\left(x^2+2\right)^2}\left(-\frac{105\,x^3}{8} - \frac{79\,x}{4}\right) - 351\,\arctan\left(1/2\,\sqrt{2}x\right)\sqrt{2} + \frac{1}{\left(x^2+1\right)^2}\left(-\frac{11\,x^3}{8} - \frac{13\,x}{8}\right) + \frac{477\,\arctan\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] x^5-14*x^3+214*x-16*(-105/8*x^3-79/4*x)/(x^2+2)^2-351*arctan(1/2*
2^(1/2)*x)*2^(1/2)+(-11/8*x^3-13/8*x)/(x^2+1)^2+477/8*arctan(x)
```

Maxima [A] time = 0.796205, size = 96, normalized size = 1.19

$$x^5 - 14\,x^3 - 351\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 214\,x + \frac{1669\,x^7 + 5831\,x^5 + 6640\,x^3 + 2476\,x}{8\left(x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4\right)} + \frac{477}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^10/(x^4 + 3*x^2 + 2)^3,x, algorithm="maxima'
```

```
[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1
669*x^7 + 5831*x^5 + 6640*x^3 + 2476*x)/(x^8 + 6*x^6 + 13*x^4 + 1
2*x^2 + 4) + 477/8*arctan(x)
```

Fricas [A] time = 0.264596, size = 154, normalized size = 1.9

$$\frac{8\,x^{13} - 64\,x^{11} + 1144\,x^9 + 10581\,x^7 + 26775\,x^5 + 26736\,x^3 - 2808\,\sqrt{2}\left(x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 477\,(\ldots}{8\left(x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^10/(x^4 + 3*x^2 + 2)^3,x, algorithm="fricas'
```

```
[Out] 1/8*(8*x^13 - 64*x^11 + 1144*x^9 + 10581*x^7 + 26775*x^5 + 26736*
x^3 - 2808*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2
*sqrt(2)*x) + 477*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) +
 9324*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] time = 0.800088, size = 75, normalized size = 0.93

$$x^5 - 14x^3 + 214x + \frac{1669x^7 + 5831x^5 + 6640x^3 + 2476x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} + \frac{477\operatorname{atan}\left(x\right)}{8} - 351\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] x**5 - 14*x**3 + 214*x + (1669*x**7 + 5831*x**5 + 6640*x**3 + 247
6*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) + 477*atan(x)/8
 - 351*sqrt(2)*atan(sqrt(2)*x/2)
```

GIAC/XCAS [A] time = 0.271586, size = 82, normalized size = 1.01

$$x^5 - 14\,x^3 - 351\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 214\,x + \frac{1669\,x^7 + 5831\,x^5 + 6640\,x^3 + 2476\,x}{8\,(x^4 + 3\,x^2 + 2)^2} + \frac{477}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^10/(x^4 + 3*x^2 + 2)^3,x, algorithm="giac")
```

```
[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1
669*x^7 + 5831*x^5 + 6640*x^3 + 2476*x)/(x^4 + 3*x^2 + 2)^2 + 477
/8*arctan(x)
```

3.92 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=80

$$\frac{5x^3}{3}+\frac{(24-409x^2)\,x}{8\,(x^4+3x^2+2)}-\frac{(207x^2+206)\,x}{4\,(x^4+3x^2+2)^2}-42x-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -42*x + (5*x^3)/3 - (x*(206 + 207*x^2))/(4*(2 + 3*x^2 + x^4)^2) +
 (x*(24 - 409*x^2))/(8*(2 + 3*x^2 + x^4)) - (449*ArcTan[x])/8 + (
219*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rubi [A] time = 0.165412, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{5x^3}{3}+\frac{(24-409x^2)\,x}{8\,(x^4+3x^2+2)}-\frac{(207x^2+206)\,x}{4\,(x^4+3x^2+2)^2}-42x-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -42*x + (5*x^3)/3 - (x*(206 + 207*x^2))/(4*(2 + 3*x^2 + x^4)^2) +
 (x*(24 - 409*x^2))/(8*(2 + 3*x^2 + x^4)) - (449*ArcTan[x])/8 + (
219*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rubi in Sympy [A] time = 32.119, size = 76, normalized size = 0.95

$$\frac{5x^3}{3}+\frac{x\,(-869437476x^2+51018336)}{17006112\,(x^4+3x^2+2)}-\frac{x\,(905418x^2+901044)}{17496\,(x^4+3x^2+2)^2}-42x-\frac{449\,\text{atan}\,(x)}{8}+\frac{219\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x**3/3 + x*(-869437476*x**2 + 51018336)/(17006112*(x**4 + 3*x**
2 + 2)) - x*(905418*x**2 + 901044)/(17496*(x**4 + 3*x**2 + 2)**2)
 - 42*x - 449*atan(x)/8 + 219*sqrt(2)*atan(sqrt(2)*x/2)/2
```

Mathematica [A] time = 0.100847, size = 66, normalized size = 0.82

$$\frac{x\,(40x^{10}-768x^8-6755x^6-16233x^4-15416x^2-5124)}{24\,(x^4+3x^2+2)^2}-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(-5124 - 15416*x^2 - 16233*x^4 - 6755*x^6 - 768*x^8 + 40*x^10)
)/(24*(2 + 3*x^2 + x^4)^2) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sq
rt[2]])/Sqrt[2]
```

Maple [A] time = 0.018, size = 62, normalized size = 0.8

$$\frac{5x^3}{3} - 42x + 16\frac{1}{(x^2+2)^2}\left(-\frac{53x^3}{16} - \frac{27x}{8}\right) + \frac{219\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}x}{2}\right) - \frac{1}{(x^2+1)^2}\left(-\frac{15x^3}{8} - \frac{17x}{8}\right) - \frac{449\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] 5/3*x^3-42*x+16*(-53/16*x^3-27/8*x)/(x^2+2)^2+219/2*arctan(1/2*2^
(1/2)*x)*2^(1/2)-(-15/8*x^3-17/8*x)/(x^2+1)^2-449/8*arctan(x)
```

Maxima [A] time = 0.796956, size = 92, normalized size = 1.15

$$\frac{5}{3}x^3 + \frac{219}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 42x - \frac{409x^7 + 1203x^5 + 1160x^3 + 364x}{8(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)} - \frac{449}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 3*x^2 + 2)^3,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x
^7 + 1203*x^5 + 1160*x^3 + 364*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2
+ 4) - 449/8*arctan(x)
```

Fricas [A] time = 0.284211, size = 159, normalized size = 1.99

$$-\frac{\sqrt{2}\left(1347\sqrt{2}(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan(x) - 5256(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \sqrt{2}(40x^{11}\right.}{48(x^8 + 6x^6 + 13x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 3*x^2 + 2)^3,x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(2)*(1347*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*a
rctan(x) - 5256*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sq
rt(2)*x) - sqrt(2)*(40*x^11 - 768*x^9 - 6755*x^7 - 16233*x^5 - 15
416*x^3 - 5124*x))/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] time = 0.774693, size = 75, normalized size = 0.94

$$\frac{5x^3}{3} - 42x - \frac{409x^7 + 1203x^5 + 1160x^3 + 364x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} - \frac{449\operatorname{atan}(x)}{8} + \frac{219\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x**3/3 - 42*x - (409*x**7 + 1203*x**5 + 1160*x**3 + 364*x)/(8*x
**8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) - 449*atan(x)/8 + 219*sq
```

```
rt(2)*atan(sqrt(2)*x/2)/2
```

GIAC/XCAS [A] time = 0.271521, size = 78, normalized size = 0.98

$$\frac{5}{3}x^3 + \frac{219}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 42x - \frac{409x^7 + 1203x^5 + 1160x^3 + 364x}{8(x^4 + 3x^2 + 2)^2} - \frac{449}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 3*x^2 + 2)^3,x, algorithm="giac")
```

```
[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x
^7 + 1203*x^5 + 1160*x^3 + 364*x)/(x^4 + 3*x^2 + 2)^2 - 449/8*arc
tan(x)
```

3.93 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$

Optimal. Leaf size=75

$$-\frac{(15x^2+244)\,x}{8\,(x^4+3x^2+2)}+\frac{(103x^2+102)\,x}{4\,(x^4+3x^2+2)^2}+5x+\frac{413}{8}\tan^{-1}(x)-\frac{191\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 5*x + (x*(102 + 103*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(244 + 15*
x^2))/(8*(2 + 3*x^2 + x^4)) + (413*ArcTan[x])/8 - (191*ArcTan[x/S
qrt[2]])/(2*Sqrt[2])
```

Rubi [A] time = 0.156005, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$-\frac{(15x^2+244)\,x}{8\,(x^4+3x^2+2)}+\frac{(103x^2+102)\,x}{4\,(x^4+3x^2+2)^2}+5x+\frac{413}{8}\tan^{-1}(x)-\frac{191\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] 5*x + (x*(102 + 103*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(244 + 15*
x^2))/(8*(2 + 3*x^2 + x^4)) + (413*ArcTan[x])/8 - (191*ArcTan[x/S
qrt[2]])/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 27.4393, size = 70, normalized size = 0.93

$$\frac{x\,(150174x^2+148716)}{5832\,(x^4+3x^2+2)^2}-\frac{x\,(3542940x^2+57631824)}{1889568\,(x^4+3x^2+2)}+5x+\frac{413\,\mathrm{atan}\,(x)}{8}-\frac{191\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] x*(150174*x**2 + 148716)/(5832*(x**4 + 3*x**2 + 2)**2) - x*(35429
40*x**2 + 57631824)/(1889568*(x**4 + 3*x**2 + 2)) + 5*x + 413*ata
n(x)/8 - 191*sqrt(2)*atan(sqrt(2)*x/2)/4
```

Mathematica [A] time = 0.112113, size = 60, normalized size = 0.8

$$\frac{1}{8}\left(\frac{x\,(40x^8+225x^6+231x^4-76x^2-124)}{(x^4+3x^2+2)^2}+413\tan^{-1}(x)-382\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((x*(-124 - 76*x^2 + 231*x^4 + 225*x^6 + 40*x^8))/(2 + 3*x^2 + x^
4)^2 + 413*ArcTan[x] - 382*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Maple [A] time = 0.018, size = 56, normalized size = 0.8

$$5\,x-16\,\frac{1}{(x^2+2)^2}\left(-1/32\,x^3+\frac{25\,x}{16}\right)-\frac{191\,\sqrt{2}}{4}\arctan\left(\frac{\sqrt{2}x}{2}\right)+\frac{1}{(x^2+1)^2}\left(-\frac{19\,x^3}{8}-\frac{21\,x}{8}\right)+\frac{413\,\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] 5*x-16*(-1/32*x^3+25/16*x)/(x^2+2)^2-191/4*arctan(1/2*2^(1/2)*x)*
2^(1/2)+(-19/8*x^3-21/8*x)/(x^2+1)^2+413/8*arctan(x)
```

Maxima [A] time = 0.785785, size = 85, normalized size = 1.13

$$-\frac{191}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+5\,x-\frac{15\,x^7+289\,x^5+556\,x^3+284\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}+\frac{413}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 3*x^2 + 2)^3,x, algorithm="maxima")
```

```
[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^
5 + 556*x^3 + 284*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 413/8*
arctan(x)
```

Fricas [A] time = 0.269459, size = 151, normalized size = 2.01

$$\frac{\sqrt{2}\left(413\,\sqrt{2}\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan(x)-764\,\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+\sqrt{2}\left(40\,x^9+225\right.\right.}{16\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 3*x^2 + 2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*(413*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arc
tan(x) - 764*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(
2)*x) + sqrt(2)*(40*x^9 + 225*x^7 + 231*x^5 - 76*x^3 - 124*x))/(x
^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] time = 0.79104, size = 68, normalized size = 0.91

$$5x-\frac{15x^7+289x^5+556x^3+284x}{8x^8+48x^6+104x^4+96x^2+32}+\frac{413\,\mathrm{atan}\,(x)}{8}-\frac{191\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x - (15*x**7 + 289*x**5 + 556*x**3 + 284*x)/(8*x**8 + 48*x**6 +
 104*x**4 + 96*x**2 + 32) + 413*atan(x)/8 - 191*sqrt(2)*atan(sqrt
(2)*x/2)/4
```

GIAC/XCAS [A] time = 0.270034, size = 72, normalized size = 0.96

$$-\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5\,x-\frac{15\,x^{7}+289\,x^{5}+556\,x^{3}+284\,x}{8\,(x^{4}+3\,x^{2}+2)^{2}}+\frac{413}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 3*x^2 + 2)^3,x, algorithm="giac")
```

```
[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^
5 + 556*x^3 + 284*x)/(x^4 + 3*x^2 + 2)^2 + 413/8*arctan(x)
```

3.94 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$

Optimal. Leaf size=72

$$-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}+\frac{x(125x^2+254)}{8(x^4+3x^2+2)}-\frac{369}{8}\tan^{-1}(x)+\frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -(x*(50 + 51*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(254 + 125*x^2))/
(8*(2 + 3*x^2 + x^4)) - (369*ArcTan[x])/8 + (267*ArcTan[x/Sqrt[2]
])/(4*Sqrt[2])
```

Rubi [A] time = 0.122325, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}+\frac{x(125x^2+254)}{8(x^4+3x^2+2)}-\frac{369}{8}\tan^{-1}(x)+\frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -(x*(50 + 51*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(254 + 125*x^2))/
(8*(2 + 3*x^2 + x^4)) - (369*ArcTan[x])/8 + (267*ArcTan[x/Sqrt[2]
])/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 20.7682, size = 66, normalized size = 0.92

$$-\frac{x(24786x^2+24300)}{1944(x^4+3x^2+2)^2}+\frac{x(3280500x^2+6665976)}{209952(x^4+3x^2+2)}-\frac{369\operatorname{atan}(x)}{8}+\frac{267\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] -x*(24786*x**2 + 24300)/(1944*(x**4 + 3*x**2 + 2)**2) + x*(328050
0*x**2 + 6665976)/(209952*(x**4 + 3*x**2 + 2)) - 369*atan(x)/8 +
267*sqrt(2)*atan(sqrt(2)*x/2)/8
```

Mathematica [A] time = 0.10914, size = 55, normalized size = 0.76

$$\frac{1}{8}\left(\frac{x(125x^6+629x^4+910x^2+408)}{(x^4+3x^2+2)^2}-369\tan^{-1}(x)+267\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((x*(408 + 910*x^2 + 629*x^4 + 125*x^6))/(2 + 3*x^2 + x^4)^2 - 36
9*ArcTan[x] + 267*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Maple [A] time = 0.016, size = 54, normalized size = 0.8

$$2\frac{1}{(x^2+2)^2}\left(\frac{51x^3}{8}+\frac{77x}{4}\right)+\frac{267\sqrt{2}}{8}\arctan\left(\frac{\sqrt{2}x}{2}\right)-\frac{1}{(x^2+1)^2}\left(-\frac{23x^3}{8}-\frac{25x}{8}\right)-\frac{369\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] 2*(51/8*x^3+77/4*x)/(x^2+2)^2+267/8*arctan(1/2*2^(1/2)*x)*2^(1/2)
-(-23/8*x^3-25/8*x)/(x^2+1)^2-369/8*arctan(x)
```

Maxima [A] time = 0.798318, size = 81, normalized size = 1.12

$$\frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{125x^7+629x^5+910x^3+408x}{8(x^8+6x^6+13x^4+12x^2+4)}-\frac{369}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 3*x^2 + 2)^3,x, algorithm="maxima")
```

```
[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 91
0*x^3 + 408*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 369/8*arctan
(x)
```

Fricas [A] time = 0.280222, size = 146, normalized size = 2.03

$$-\frac{\sqrt{2}\left(369\sqrt{2}(x^8+6x^6+13x^4+12x^2+4)\arctan(x)-534\left(x^8+6x^6+13x^4+12x^2+4\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\sqrt{2}(125x^7+6\right.}{16(x^8+6x^6+13x^4+12x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 3*x^2 + 2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*(369*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*ar
ctan(x) - 534*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt
(2)*x) - sqrt(2)*(125*x^7 + 629*x^5 + 910*x^3 + 408*x))/(x^8 + 6*
x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] time = 0.763954, size = 65, normalized size = 0.9

$$\frac{125x^7+629x^5+910x^3+408x}{8x^8+48x^6+104x^4+96x^2+32}-\frac{369\operatorname{atan}(x)}{8}+\frac{267\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (125*x**7 + 629*x**5 + 910*x**3 + 408*x)/(8*x**8 + 48*x**6 + 104*
x**4 + 96*x**2 + 32) - 369*atan(x)/8 + 267*sqrt(2)*atan(sqrt(2)*x
/2)/8
```

GIAC/XCAS [A] time = 0.273797, size = 68, normalized size = 0.94

$$\frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{125\,x^{7}+629\,x^{5}+910\,x^{3}+408\,x}{8\,(x^{4}+3\,x^{2}+2)^{2}}-\frac{369}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 3*x^2 + 2)^3,x, algorithm="giac")
```

```
[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 91
0*x^3 + 408*x)/(x^4 + 3*x^2 + 2)^2 - 369/8*arctan(x)
```

3.95 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} \, dx$

Optimal. Leaf size=72

$$\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2} - \frac{x(130x^2+211)}{8(x^4+3x^2+2)} + \frac{317}{8}\tan^{-1}(x) - \frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] (x*(24 + 25*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(211 + 130*x^2))/(
8*(2 + 3*x^2 + x^4)) + (317*ArcTan[x])/8 - (447*ArcTan[x/Sqrt[2]]
)/(8*Sqrt[2])
```

Rubi [A] time = 0.118906, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2} - \frac{x(130x^2+211)}{8(x^4+3x^2+2)} + \frac{317}{8}\tan^{-1}(x) - \frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(24 + 25*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(211 + 130*x^2))/(
8*(2 + 3*x^2 + x^4)) + (317*ArcTan[x])/8 - (447*ArcTan[x/Sqrt[2]]
)/(8*Sqrt[2])
```

Rubi in Sympy [A] time = 19.4126, size = 66, normalized size = 0.92

$$\frac{x(4050x^2+3888)}{648(x^4+3x^2+2)^2} - \frac{x(379080x^2+615276)}{23328(x^4+3x^2+2)} + \frac{317\,\mathrm{atan}(x)}{8} - \frac{447\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] x*(4050*x**2 + 3888)/(648*(x**4 + 3*x**2 + 2)**2) - x*(379080*x**
2 + 615276)/(23328*(x**4 + 3*x**2 + 2)) + 317*atan(x)/8 - 447*sqr
t(2)*atan(sqrt(2)*x/2)/16
```

Mathematica [A] time = 0.104772, size = 56, normalized size = 0.78

$$\frac{1}{16}\left(-\frac{2x(130x^6+601x^4+843x^2+374)}{(x^4+3x^2+2)^2} + 634\tan^{-1}(x) - 447\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((-2*x*(374 + 843*x^2 + 601*x^4 + 130*x^6))/(2 + 3*x^2 + x^4)^2 +
 634*ArcTan[x] - 447*Sqrt[2]*ArcTan[x/Sqrt[2]])/16
```

Maple [A] time = 0.017, size = 53, normalized size = 0.7

$$-\frac{1}{(x^2+2)^2}\left(\frac{103\,x^3}{8}+\frac{129\,x}{4}\right)-\frac{447\,\sqrt{2}}{16}\arctan\left(\frac{\sqrt{2}x}{2}\right)+\frac{1}{(x^2+1)^2}\left(-\frac{27\,x^3}{8}-\frac{29\,x}{8}\right)+\frac{317\,\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] -(103/8*x^3+129/4*x)/(x^2+2)^2-447/16*arctan(1/2*2^(1/2)*x)*2^(1/
2)+(-27/8*x^3-29/8*x)/(x^2+1)^2+317/8*arctan(x)
```

Maxima [A] time = 0.791054, size = 81, normalized size = 1.12

$$-\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}+\frac{317}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 3*x^2 + 2)^3,x, algorithm="maxima")
```

```
[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 +
843*x^3 + 374*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 317/8*arct
an(x)
```

Fricas [A] time = 0.272438, size = 146, normalized size = 2.03

$$\frac{\sqrt{2}\left(317\,\sqrt{2}\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan(x)-447\,\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-\sqrt{2}\left(130\,x^7+60\right.\right.}{16\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 3*x^2 + 2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*(317*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arc
tan(x) - 447*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(
2)*x) - sqrt(2)*(130*x^7 + 601*x^5 + 843*x^3 + 374*x))/(x^8 + 6*x
^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] time = 0.795633, size = 65, normalized size = 0.9

$$-\frac{130x^7+601x^5+843x^3+374x}{8x^8+48x^6+104x^4+96x^2+32}+\frac{317\,\mathrm{atan}(x)}{8}-\frac{447\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] -(130*x**7 + 601*x**5 + 843*x**3 + 374*x)/(8*x**8 + 48*x**6 + 104
*x**4 + 96*x**2 + 32) + 317*atan(x)/8 - 447*sqrt(2)*atan(sqrt(2)*
x/2)/16
```

GIAC/XCAS [A] time = 0.271162, size = 68, normalized size = 0.94

$$-\frac{447}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - \frac{130\,x^7 + 601\,x^5 + 843\,x^3 + 374\,x}{8\,(x^4 + 3\,x^2 + 2)^2} + \frac{317}{8}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 3*x^2 + 2)^3,x, algorithm="giac")
```

```
[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 +
843*x^3 + 374*x)/(x^4 + 3*x^2 + 2)^2 + 317/8*arctan(x)
```

3.96 $\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=72

$$-\frac{x\left(12x^2+11\right)}{4\left(x^4+3x^2+2\right)^2}+\frac{x\left(217x^2+335\right)}{16\left(x^4+3x^2+2\right)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

```
[Out] -(x*(11 + 12*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(335 + 217*x^2))/
(16*(2 + 3*x^2 + x^4)) - (257*ArcTan[x])/8 + (731*ArcTan[x/Sqrt[2
]])/(16*Sqrt[2])
```

Rubi [A] time = 0.0836836, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{x\left(12x^2+11\right)}{4\left(x^4+3x^2+2\right)^2}+\frac{x\left(217x^2+335\right)}{16\left(x^4+3x^2+2\right)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -(x*(11 + 12*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(335 + 217*x^2))/
(16*(2 + 3*x^2 + x^4)) - (257*ArcTan[x])/8 + (731*ArcTan[x/Sqrt[2
]])/(16*Sqrt[2])
```

Rubi in Sympy [A] time = 13.3934, size = 66, normalized size = 0.92

$$-\frac{x\left(648x^2+594\right)}{216\left(x^4+3x^2+2\right)^2}+\frac{x\left(11718x^2+18090\right)}{864\left(x^4+3x^2+2\right)}-\frac{257\operatorname{atan}(x)}{8}+\frac{731\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] -x*(648*x**2 + 594)/(216*(x**4 + 3*x**2 + 2)**2) + x*(11718*x**2
+ 18090)/(864*(x**4 + 3*x**2 + 2)) - 257*atan(x)/8 + 731*sqrt(2)*
atan(sqrt(2)*x/2)/32
```

Mathematica [A] time = 0.104213, size = 56, normalized size = 0.78

$$\frac{1}{32}\left(\frac{2x\left(217x^6+986x^4+1391x^2+626\right)}{\left(x^4+3x^2+2\right)^2}-1028\tan^{-1}(x)+731\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((2*x*(626 + 1391*x^2 + 986*x^4 + 217*x^6))/(2 + 3*x^2 + x^4)^2 -
 1028*ArcTan[x] + 731*Sqrt[2]*ArcTan[x/Sqrt[2]])/32
```

Maple [A] time = 0.016, size = 53, normalized size = 0.7

$$\frac{1}{(x^2+2)^2}\left(\frac{155x^3}{16}+\frac{181x}{8}\right)+\frac{731\sqrt{2}}{32}\arctan\left(\frac{\sqrt{2}x}{2}\right)-\frac{1}{(x^2+1)^2}\left(-\frac{31x^3}{8}-\frac{33x}{8}\right)-\frac{257\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] (155/16*x^3+181/8*x)/(x^2+2)^2+731/32*arctan(1/2*2^(1/2)*x)*2^(1/
2)-(-31/8*x^3-33/8*x)/(x^2+1)^2-257/8*arctan(x)
```

Maxima [A] time = 0.796606, size = 81, normalized size = 1.12

$$\frac{731}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{217x^7+986x^5+1391x^3+626x}{16(x^8+6x^6+13x^4+12x^2+4)}-\frac{257}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 3*x^2 + 2)^3,x, algorithm="maxima")
```

```
[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 +
1391*x^3 + 626*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 257/8*arc
tan(x)
```

Fricas [A] time = 0.266398, size = 146, normalized size = 2.03

$$-\frac{\sqrt{2}\left(514\sqrt{2}(x^8+6x^6+13x^4+12x^2+4)\arctan(x)-731(x^8+6x^6+13x^4+12x^2+4)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\sqrt{2}(217x^7+ \cdots\right.}{32(x^8+6x^6+13x^4+12x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 3*x^2 + 2)^3,x, algorithm="fricas")
```

```
[Out] -1/32*sqrt(2)*(514*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*ar
ctan(x) - 731*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt
(2)*x) - sqrt(2)*(217*x^7 + 986*x^5 + 1391*x^3 + 626*x))/(x^8 + 6
*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] time = 0.805529, size = 65, normalized size = 0.9

$$\frac{217x^7+986x^5+1391x^3+626x}{16x^8+96x^6+208x^4+192x^2+64}-\frac{257\operatorname{atan}(x)}{8}+\frac{731\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (217*x**7 + 986*x**5 + 1391*x**3 + 626*x)/(16*x**8 + 96*x**6 + 20
8*x**4 + 192*x**2 + 64) - 257*atan(x)/8 + 731*sqrt(2)*atan(sqrt(2
)*x/2)/32
```

GIAC/XCAS [A] time = 0.272371, size = 68, normalized size = 0.94

$$\frac{731}{32}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+\frac{217\,x^{7}+986\,x^{5}+1391\,x^{3}+626\,x}{16\,{\left(x^{4}+3\,x^{2}+2\right)}^{2}}-\frac{257}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 3*x^2 + 2)^3,x, algorithm="giac")
```

```
[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 +
1391*x^3 + 626*x)/(x^4 + 3*x^2 + 2)^2 - 257/8*arctan(x)
```

3.97 $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=79

$$\frac{x(11x^2+9)}{8(x^4+3x^2+2)^2}-\frac{x(347x^2+547)}{32(x^4+3x^2+2)}-\frac{1}{2x}+\frac{189}{8}\tan^{-1}(x)-\frac{1119\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[Out] -1/(2*x) + (x*(9 + 11*x^2))/(8*(2 + 3*x^2 + x^4)^2) - (x*(547 + 3
47*x^2))/(32*(2 + 3*x^2 + x^4)) + (189*ArcTan[x])/8 - (1119*ArcTa
n[x/Sqrt[2]])/(32*Sqrt[2])
```

Rubi [A] time = 0.170293, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{x(11x^2+9)}{8(x^4+3x^2+2)^2}-\frac{x(347x^2+547)}{32(x^4+3x^2+2)}-\frac{1}{2x}+\frac{189}{8}\tan^{-1}(x)-\frac{1119\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/(2*x) + (x*(9 + 11*x^2))/(8*(2 + 3*x^2 + x^4)^2) - (x*(547 + 3
47*x^2))/(32*(2 + 3*x^2 + x^4)) + (189*ArcTan[x])/8 - (1119*ArcTa
n[x/Sqrt[2]])/(32*Sqrt[2])
```

Rubi in Sympy [A] time = 26.9304, size = 71, normalized size = 0.9

$$\frac{x(4374x^2+7290)}{432(x^4+3x^2+2)^2}-\frac{x(1752516x^2+2920860)}{15552(x^4+3x^2+2)}+\frac{699\,\mathrm{atan}(x)}{4}-\frac{1251\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}-\frac{405}{16x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**3,x)
```

```
[Out] x*(4374*x**2 + 7290)/(432*(x**4 + 3*x**2 + 2)**2) - x*(1752516*x*
*2 + 2920860)/(15552*(x**4 + 3*x**2 + 2)) + 699*atan(x)/4 - 1251*
sqrt(2)*atan(sqrt(2)*x/2)/8 - 405/(16*x)
```

Mathematica [A] time = 0.11313, size = 63, normalized size = 0.8

$$\frac{1}{64}\left(-\frac{2(363x^8+1684x^6+2499x^4+1250x^2+64)}{x(x^4+3x^2+2)^2}+1512\tan^{-1}(x)-1119\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] ((-2*(64 + 1250*x^2 + 2499*x^4 + 1684*x^6 + 363*x^8))/(x*(2 + 3*x
^2 + x^4)^2) + 1512*ArcTan[x] - 1119*Sqrt[2]*ArcTan[x/Sqrt[2]])/6
4
```

Maple [A] time = 0.021, size = 58, normalized size = 0.7

$$-\frac{1}{2x}-\frac{1}{2\left(x^{2}+2\right)^{2}}\left(\frac{207\,x^{3}}{16}+\frac{233\,x}{8}\right)-\frac{1119\,\sqrt{2}}{64}\arctan\left(\frac{\sqrt{2}x}{2}\right)+\frac{1}{\left(x^{2}+1\right)^{2}}\left(-\frac{35\,x^{3}}{8}-\frac{37\,x}{8}\right)+\frac{189\,\arctan\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x)
```

```
[Out] -1/2/x-1/2*(207/16*x^3+233/8*x)/(x^2+2)^2-1119/64*arctan(1/2*2^(1
/2)*x)*2^(1/2)+(-35/8*x^3-37/8*x)/(x^2+1)^2+189/8*arctan(x)
```

Maxima [A] time = 0.785337, size = 88, normalized size = 1.11

$$-\frac{1119}{64}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-\frac{363\,x^{8}+1684\,x^{6}+2499\,x^{4}+1250\,x^{2}+64}{32\,\left(x^{9}+6\,x^{7}+13\,x^{5}+12\,x^{3}+4\,x\right)}+\frac{189}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^3*x^2),x, algorithm="maxima
```

```
[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(363*x^8 + 1684*x^6
 + 2499*x^4 + 1250*x^2 + 64)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x
) + 189/8*arctan(x)
```

Fricas [A] time = 0.268486, size = 158, normalized size = 2.

$$\frac{\sqrt{2}\Big(756\,\sqrt{2}\left(x^{9}+6\,x^{7}+13\,x^{5}+12\,x^{3}+4\,x\right)\arctan\left(x\right)-1119\,\left(x^{9}+6\,x^{7}+13\,x^{5}+12\,x^{3}+4\,x\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-\sqrt{2}\big(363\,x^{8}}{64\,\left(x^{9}+6\,x^{7}+13\,x^{5}+12\,x^{3}+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^3*x^2),x, algorithm="fricas
```

```
[Out] 1/64*sqrt(2)*(756*sqrt(2)*(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)*a
rctan(x) - 1119*(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)*arctan(1/2*
sqrt(2)*x) - sqrt(2)*(363*x^8 + 1684*x^6 + 2499*x^4 + 1250*x^2 +
64))/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)
```

Sympy [A] time = 0.905395, size = 70, normalized size = 0.89

$$-\frac{363x^{8}+1684x^{6}+2499x^{4}+1250x^{2}+64}{32x^{9}+192x^{7}+416x^{5}+384x^{3}+128x}+\frac{189\,\mathrm{atan}\left(x\right)}{8}-\frac{1119\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**3,x)
```

```
[Out] -(363*x**8 + 1684*x**6 + 2499*x**4 + 1250*x**2 + 64)/(32*x**9 + 1
92*x**7 + 416*x**5 + 384*x**3 + 128*x) + 189*atan(x)/8 - 1119*sqr
t(2)*atan(sqrt(2)*x/2)/64
```

GIAC/XCAS [A] time = 0.272404, size = 74, normalized size = 0.94

$$-\frac{1119}{64}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-\frac{347\,x^{7}+1588\,x^{5}+2291\,x^{3}+1058\,x}{32\,{\left(x^{4}+3\,x^{2}+2\right)}^{2}}-\frac{1}{2\,x}+\frac{189}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^3*x^2),x, algorithm="giac")
```

```
[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(347*x^7 + 1588*x^5
 + 2291*x^3 + 1058*x)/(x^4 + 3*x^2 + 2)^2 - 1/2/x + 189/8*arctan(
x)
```

3.98 $\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3} dx$

Optimal. Leaf size=86

$$-\frac{1}{6x^3}-\frac{x(9x^2+5)}{16(x^4+3x^2+2)^2}+\frac{x(571x^2+951)}{64(x^4+3x^2+2)}+\frac{17}{8x}-\frac{113}{8}\tan^{-1}(x)+\frac{1611\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

```
[Out] -1/(6*x^3) + 17/(8*x) - (x*(5 + 9*x^2))/(16*(2 + 3*x^2 + x^4)^2)
+ (x*(951 + 571*x^2))/(64*(2 + 3*x^2 + x^4)) - (113*ArcTan[x])/8
+ (1611*ArcTan[x/Sqrt[2]])/(64*Sqrt[2])
```

Rubi [A] time = 0.191496, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$-\frac{1}{6x^3}-\frac{x(9x^2+5)}{16(x^4+3x^2+2)^2}+\frac{x(571x^2+951)}{64(x^4+3x^2+2)}+\frac{17}{8x}-\frac{113}{8}\tan^{-1}(x)+\frac{1611\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/(6*x^3) + 17/(8*x) - (x*(5 + 9*x^2))/(16*(2 + 3*x^2 + x^4)^2)
+ (x*(951 + 571*x^2))/(64*(2 + 3*x^2 + x^4)) - (113*ArcTan[x])/8
+ (1611*ArcTan[x/Sqrt[2]])/(64*Sqrt[2])
```

Rubi in Sympy [A] time = 22.9406, size = 58, normalized size = 0.67

$$-\frac{7\,\mathrm{atan}(x)}{2}-\frac{133\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}-\frac{161}{8x}-\frac{12096x^2+19440}{864x^3(x^4+3x^2+2)}+\frac{49}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**3,x)
```

```
[Out] -7*atan(x)/2 - 133*sqrt(2)*atan(sqrt(2)*x/2)/16 - 161/(8*x) - (12
096*x**2 + 19440)/(864*x**3*(x**4 + 3*x**2 + 2)) + 49/(4*x**3)
```

Mathematica [A] time = 0.11641, size = 78, normalized size = 0.91

$$\frac{1}{384}\left(-\frac{64}{x^3}-\frac{24x(9x^2+5)}{(x^4+3x^2+2)^2}+\frac{6x(571x^2+951)}{x^4+3x^2+2}+\frac{816}{x}-5424\tan^{-1}(x)+4833\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] (-64/x^3 + 816/x - (24*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4)^2 + (6*x*
(951 + 571*x^2))/(2 + 3*x^2 + x^4) - 5424*ArcTan[x] + 4833*Sqrt[2
]*ArcTan[x/Sqrt[2]])/384
```

Maple [A] time = 0.024, size = 64, normalized size = 0.7

$$-\frac{1}{6x^3}+\frac{17}{8x}+\frac{1}{8(x^2+2)^2}\left(\frac{259x^3}{8}+\frac{285x}{4}\right)+\frac{1611\sqrt{2}}{128}\arctan\left(\frac{\sqrt{2}x}{2}\right)-\frac{1}{(x^2+1)^2}\left(-\frac{39x^3}{8}-\frac{41x}{8}\right)-\frac{113\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x)
```

```
[Out] -1/6/x^3+17/8/x+1/8*(259/8*x^3+285/4*x)/(x^2+2)^2+1611/128*arctan
(1/2*2^(1/2)*x)*2^(1/2)-(-39/8*x^3-41/8*x)/(x^2+1)^2-113/8*arctan
(x)
```

Maxima [A] time = 0.788658, size = 97, normalized size = 1.13

$$\frac{1611}{128}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{2121x^{10}+10408x^8+16989x^6+10126x^4+1248x^2-128}{192(x^{11}+6x^9+13x^7+12x^5+4x^3)}-\frac{113}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^3*x^4),x, algorithm="maxima
```

```
[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/192*(2121*x^10 + 10408
*x^8 + 16989*x^6 + 10126*x^4 + 1248*x^2 - 128)/(x^11 + 6*x^9 + 13
*x^7 + 12*x^5 + 4*x^3) - 113/8*arctan(x)
```

Fricas [A] time = 0.272864, size = 173, normalized size = 2.01

$$-\frac{\sqrt{2}\left(2712\sqrt{2}(x^{11}+6x^9+13x^7+12x^5+4x^3)\arctan(x)-4833(x^{11}+6x^9+13x^7+12x^5+4x^3)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\sqrt{2}(\right.}{384(x^{11}+6x^9+13x^7+12x^5+4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^3*x^4),x, algorithm="fricas
```

```
[Out] -1/384*sqrt(2)*(2712*sqrt(2)*(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*
x^3)*arctan(x) - 4833*(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*ar
ctan(1/2*sqrt(2)*x) - sqrt(2)*(2121*x^10 + 10408*x^8 + 16989*x^6
+ 10126*x^4 + 1248*x^2 - 128))/(x^11 + 6*x^9 + 13*x^7 + 12*x^5 +
4*x^3)
```

Sympy [A] time = 0.952064, size = 76, normalized size = 0.88

$$-\frac{113\,\mathrm{atan}(x)}{8}+\frac{1611\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{128}+\frac{2121x^{10}+10408x^8+16989x^6+10126x^4+1248x^2-128}{192x^{11}+1152x^9+2496x^7+2304x^5+768x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**3,x)
```

```
[Out] -113*atan(x)/8 + 1611*sqrt(2)*atan(sqrt(2)*x/2)/128 + (2121*x**10
 + 10408*x**8 + 16989*x**6 + 10126*x**4 + 1248*x**2 - 128)/(192*x
```

```
**11 + 1152*x**9 + 2496*x**7 + 2304*x**5 + 768*x**3)
```

GIAC/XCAS [A] time = 0.270553, size = 84, normalized size = 0.98

$$\frac{1611}{128}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{571\,x^{7}+2664\,x^{5}+3959\,x^{3}+1882\,x}{64\,(x^{4}+3\,x^{2}+2)^{2}}+\frac{51\,x^{2}-4}{24\,x^{3}}-\frac{113}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^3*x^4),x, algorithm="giac")
```

```
[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/64*(571*x^7 + 2664*x^5
 + 3959*x^3 + 1882*x)/(x^4 + 3*x^2 + 2)^2 + 1/24*(51*x^2 - 4)/x^3
 - 113/8*arctan(x)
```

3.99 $\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=93

$$-\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{x\left(3-5x^2\right)}{32\left(x^4+3x^2+2\right)^2}-\frac{x\left(999x^2+1771\right)}{128\left(x^4+3x^2+2\right)}-\frac{93}{16x}+\frac{29}{8}\tan^{-1}(x)-\frac{2207\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

```
[Out] -1/(10*x^5) + 17/(24*x^3) - 93/(16*x) - (x*(3 - 5*x^2))/(32*(2 +
3*x^2 + x^4)^2) - (x*(1771 + 999*x^2))/(128*(2 + 3*x^2 + x^4)) +
(29*ArcTan[x])/8 - (2207*ArcTan[x/Sqrt[2]])/(128*Sqrt[2])
```

Rubi [A] time = 0.208624, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$-\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{x\left(3-5x^2\right)}{32\left(x^4+3x^2+2\right)^2}-\frac{x\left(999x^2+1771\right)}{128\left(x^4+3x^2+2\right)}-\frac{93}{16x}+\frac{29}{8}\tan^{-1}(x)-\frac{2207\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/(10*x^5) + 17/(24*x^3) - 93/(16*x) - (x*(3 - 5*x^2))/(32*(2 +
3*x^2 + x^4)^2) - (x*(1771 + 999*x^2))/(128*(2 + 3*x^2 + x^4)) +
(29*ArcTan[x])/8 - (2207*ArcTan[x/Sqrt[2]])/(128*Sqrt[2])
```

Rubi in Sympy [A] time = 26.4956, size = 65, normalized size = 0.7

$$\frac{41\operatorname{atan}(x)}{2}+\frac{155\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}+\frac{483}{16x}-\frac{319}{24x^3}-\frac{12096x^2+19440}{864x^5\left(x^4+3x^2+2\right)}+\frac{237}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**3,x)
```

```
[Out] 41*atan(x)/2 + 155*sqrt(2)*atan(sqrt(2)*x/2)/32 + 483/(16*x) - 31
9/(24*x**3) - (12096*x**2 + 19440)/(864*x**5*(x**4 + 3*x**2 + 2))
 + 237/(20*x**5)
```

Mathematica [A] time = 0.146521, size = 73, normalized size = 0.78

$$\frac{-\frac{2\left(26145x^{12}+137120x^{10}+246477x^8+170702x^6+30816x^4-3136x^2+768\right)}{x^5\left(x^4+3x^2+2\right)^2}+13920\tan^{-1}(x)-33105\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3840}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] ((-2*(768 - 3136*x^2 + 30816*x^4 + 170702*x^6 + 246477*x^8 + 1371
20*x^10 + 26145*x^12))/(x^5*(2 + 3*x^2 + x^4)^2) + 13920*ArcTan[x
] - 33105*Sqrt[2]*ArcTan[x/Sqrt[2]])/3840
```

Maple [A] time = 0.024, size = 68, normalized size = 0.7

$$-\frac{1}{10\,x^{5}}+\frac{17}{24\,x^{3}}-\frac{93}{16\,x}-\frac{1}{16\,\left(x^{2}+2\right)^{2}}\left(\frac{311\,x^{3}}{8}+\frac{337\,x}{4}\right)-\frac{2207\,\sqrt{2}}{256}\arctan\left(\frac{\sqrt{2}x}{2}\right)+\frac{1}{\left(x^{2}+1\right)^{2}}\left(-\frac{43\,x^{3}}{8}-\frac{45\,x}{8}\right)+\frac{29\,\arctan\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x)
```

```
[Out] -1/10/x^5+17/24/x^3-93/16/x-1/16*(311/8*x^3+337/4*x)/(x^2+2)^2-22
07/256*arctan(1/2*2^(1/2)*x)*2^(1/2)+(-43/8*x^3-45/8*x)/(x^2+1)^2
+29/8*arctan(x)
```

Maxima [A] time = 0.775917, size = 104, normalized size = 1.12

$$-\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{26145\,x^{12}+137120\,x^{10}+246477\,x^{8}+170702\,x^{6}+30816\,x^{4}-3136\,x^{2}+768}{1920\,\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}\right)}+\frac{29}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^3*x^6),x, algorithm="maxima
```

```
[Out] -2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1920*(26145*x^12 + 13
7120*x^10 + 246477*x^8 + 170702*x^6 + 30816*x^4 - 3136*x^2 + 768)
/(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5) + 29/8*arctan(x)
```

Fricas [A] time = 0.2784, size = 180, normalized size = 1.94

$$\frac{\sqrt{2}\left(6960\,\sqrt{2}\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}\right)\arctan\left(x\right)-33105\,\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\sqrt{2}\right.}{3840\,\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^3*x^6),x, algorithm="fricas
```

```
[Out] 1/3840*sqrt(2)*(6960*sqrt(2)*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4
*x^5)*arctan(x) - 33105*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)
*arctan(1/2*sqrt(2)*x) - sqrt(2)*(26145*x^12 + 137120*x^10 + 2464
77*x^8 + 170702*x^6 + 30816*x^4 - 3136*x^2 + 768))/(x^13 + 6*x^11
 + 13*x^9 + 12*x^7 + 4*x^5)
```

Sympy [A] time = 1.05169, size = 82, normalized size = 0.88

$$\frac{29\operatorname{atan}\left(x\right)}{8}-\frac{2207\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{256}-\frac{26145x^{12}+137120x^{10}+246477x^{8}+170702x^{6}+30816x^{4}-3136x^{2}+768}{1920x^{13}+11520x^{11}+24960x^{9}+23040x^{7}+7680x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**3,x)
```

```
[Out] 29*atan(x)/8 - 2207*sqrt(2)*atan(sqrt(2)*x/2)/256 - (26145*x**12
+ 137120*x**10 + 246477*x**8 + 170702*x**6 + 30816*x**4 - 3136*x*
*2 + 768)/(1920*x**13 + 11520*x**11 + 24960*x**9 + 23040*x**7 + 7
680*x**5)
```

GIAC/XCAS [A] time = 0.270573, size = 90, normalized size = 0.97

$$-\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{999\,x^{7}+4768\,x^{5}+7291\,x^{3}+3554\,x}{128\,(x^{4}+3\,x^{2}+2)^{2}}-\frac{1395\,x^{4}-170\,x^{2}+24}{240\,x^{5}}+\frac{29}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 3*x^2 + 2)^3*x^6),x, algorithm="giac")
```

```
[Out] -2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/128*(999*x^7 + 4768*x
^5 + 7291*x^3 + 3554*x)/(x^4 + 3*x^2 + 2)^2 - 1/240*(1395*x^4 - 1
70*x^2 + 24)/x^5 + 29/8*arctan(x)
```

3.100 $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=86

$$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2+\frac{201\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{25\left(7x^2+15\right)}{8\left(x^4+2x^2+3\right)}-\frac{183}{4}\log\left(x^4+2x^2+3\right)$$

```
[Out] 19*x^2 + (19*x^4)/4 - (17*x^6)/6 + (5*x^8)/8 - (25*(15 + 7*x^2))/
(8*(3 + 2*x^2 + x^4)) + (201*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
]) - (183*Log[3 + 2*x^2 + x^4])/4
```

Rubi [A] time = 0.239025, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2+\frac{201\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{25\left(7x^2+15\right)}{8\left(x^4+2x^2+3\right)}-\frac{183}{4}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x^2 + (19*x^4)/4 - (17*x^6)/6 + (5*x^8)/8 - (25*(15 + 7*x^2))/
(8*(3 + 2*x^2 + x^4)) + (201*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
]) - (183*Log[3 + 2*x^2 + x^4])/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{5x^{12}}{8\left(x^4+2x^2+3\right)}-\frac{19x^6}{12}+\frac{33x^2}{2}-\frac{45\left(20x^2+48\right)}{32\left(x^4+2x^2+3\right)}-\frac{183\log\left(x^4+2x^2+3\right)}{4}+\frac{201\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)}{16}+\frac{33\int^{x^2}x\,dx}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**12/(8*(x**4 + 2*x**2 + 3)) - 19*x**6/12 + 33*x**2/2 - 45*(20
*x**2 + 48)/(32*(x**4 + 2*x**2 + 3)) - 183*log(x**4 + 2*x**2 + 3)
/4 + 201*sqrt(2)*atan(sqrt(2)*(x**2/2 + 1/2))/16 + 33*Integral(x,
 (x, x**2))/4
```

Mathematica [A] time = 0.0793939, size = 78, normalized size = 0.91

$$\frac{1}{48}\left(30x^8-136x^6+228x^4+912x^2+603\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)-\frac{150\left(7x^2+15\right)}{x^4+2x^2+3}-2196\log\left(x^4+2x^2+3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (912*x^2 + 228*x^4 - 136*x^6 + 30*x^8 - (150*(15 + 7*x^2))/(3 + 2
*x^2 + x^4) + 603*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] - 2196*Log[3
```

```
+ 2*x^2 + x^4])/48
```

Maple [A] time = 0.014, size = 74, normalized size = 0.9

$$\frac{5x^8}{8} - \frac{17x^6}{6} + \frac{19x^4}{4} + 19x^2 - \frac{1}{2x^4+4x^2+6}\left(\frac{175x^2}{4} + \frac{375}{4}\right) - \frac{183\ln\left(x^4+2x^2+3\right)}{4} + \frac{201\sqrt{2}}{16}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/8*x^8-17/6*x^6+19/4*x^4+19*x^2-1/2*(175/4*x^2+375/4)/(x^4+2*x^2
+3)-183/4*ln(x^4+2*x^2+3)+201/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(
1/2))
```

Maxima [A] time = 0.790036, size = 96, normalized size = 1.12

$$\frac{5}{8}x^8 - \frac{17}{6}x^6 + \frac{19}{4}x^4 + 19x^2 + \frac{201}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) - \frac{25\left(7x^2+15\right)}{8\left(x^4+2x^2+3\right)} - \frac{183}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^9/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/
2*sqrt(2)*(x^2 + 1)) - 25/8*(7*x^2 + 15)/(x^4 + 2*x^2 + 3) - 183/
4*log(x^4 + 2*x^2 + 3)
```

Fricas [A] time = 0.270653, size = 140, normalized size = 1.63

$$-\frac{\sqrt{2}\left(1098\sqrt{2}\left(x^4+2x^2+3\right)\log\left(x^4+2x^2+3\right) - 603\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) - \sqrt{2}\left(15x^{12} - 38x^{10} + 23x^8 \right.\right.}{48\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^9/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(2)*(1098*sqrt(2)*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3
) - 603*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - sqrt(2)
*(15*x^12 - 38*x^10 + 23*x^8 + 480*x^6 + 1254*x^4 + 843*x^2 - 112
5))/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.452715, size = 85, normalized size = 0.99

$$\frac{5x^8}{8} - \frac{17x^6}{6} + \frac{19x^4}{4} + 19x^2 - \frac{175x^2+375}{8x^4+16x^2+24} - \frac{183\log\left(x^4+2x^2+3\right)}{4} + \frac{201\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**8/8 - 17*x**6/6 + 19*x**4/4 + 19*x**2 - (175*x**2 + 375)/(8*
x**4 + 16*x**2 + 24) - 183*log(x**4 + 2*x**2 + 3)/4 + 201*sqrt(2)
*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

GIAC/XCAS [A] time = 0.271471, size = 103, normalized size = 1.2

$$\frac{5}{8}x^8 - \frac{17}{6}x^6 + \frac{19}{4}x^4 + 19x^2 + \frac{201}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{366x^4 + 557x^2 + 723}{8(x^4 + 2x^2 + 3)} - \frac{183}{4}\ln\left(x^4 + 2x^2 + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^9/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")
```

```
[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/
2*sqrt(2)*(x^2 + 1)) + 1/8*(366*x^4 + 557*x^2 + 723)/(x^4 + 2*x^2
 + 3) - 183/4*ln(x^4 + 2*x^2 + 3)
```

3.101 $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=81

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}-\frac{455\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25\left(5x^2+3\right)}{8\left(x^4+2x^2+3\right)}+\frac{19}{2}\log\left(x^4+2x^2+3\right)$$

```
[Out] (19*x^2)/2 - (17*x^4)/4 + (5*x^6)/6 + (25*(3 + 5*x^2))/(8*(3 + 2*
x^2 + x^4)) - (455*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*L
og[3 + 2*x^2 + x^4])/2
```

Rubi [A] time = 0.218502, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}-\frac{455\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25\left(5x^2+3\right)}{8\left(x^4+2x^2+3\right)}+\frac{19}{2}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (19*x^2)/2 - (17*x^4)/4 + (5*x^6)/6 + (25*(3 + 5*x^2))/(8*(3 + 2*
x^2 + x^4)) - (455*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*L
og[3 + 2*x^2 + x^4])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{5x^{10}}{6\left(x^4+2x^2+3\right)}+\frac{5\left(238x^2+186\right)}{48\left(x^4+2x^2+3\right)}+\frac{19\log\left(x^4+2x^2+3\right)}{2}-\frac{455\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)}{16}+\frac{\int^{x^2}\frac{52}{3}\,dx}{2}-\frac{31\int^{x^2}x\,dx}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**10/(6*(x**4 + 2*x**2 + 3)) + 5*(238*x**2 + 186)/(48*(x**4 +
2*x**2 + 3)) + 19*log(x**4 + 2*x**2 + 3)/2 - 455*sqrt(2)*atan(sqr
t(2)*(x**2/2 + 1/2))/16 + Integral(52/3, (x, x**2))/2 - 31*Integr
al(x, (x, x**2))/6
```

Mathematica [A] time = 0.0544429, size = 73, normalized size = 0.9

$$\frac{1}{48}\left(40x^6-204x^4+456x^2-1365\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)+\frac{150\left(5x^2+3\right)}{x^4+2x^2+3}+456\log\left(x^4+2x^2+3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (456*x^2 - 204*x^4 + 40*x^6 + (150*(3 + 5*x^2))/(3 + 2*x^2 + x^4)
 - 1365*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 456*Log[3 + 2*x^2 + x
```

```
^4])/48
```

Maple [A] time = 0.013, size = 69, normalized size = 0.9

$$\frac{5 x^{6}}{6}-\frac{17 x^{4}}{4}+\frac{19 x^{2}}{2}+\frac{1}{2 x^{4}+4 x^{2}+6}\left(\frac{125 x^{2}}{4}+\frac{75}{4}\right)+\frac{19 \ln \left(x^{4}+2 x^{2}+3\right)}{2}-\frac{455 \sqrt{2}}{16}\arctan\left(\frac{\left(2 x^{2}+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/6*x^6-17/4*x^4+19/2*x^2+1/2*(125/4*x^2+75/4)/(x^4+2*x^2+3)+19/2
*ln(x^4+2*x^2+3)-455/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 0.791742, size = 89, normalized size = 1.1

$$\frac{5}{6}x^{6}-\frac{17}{4}x^{4}+\frac{19}{2}x^{2}-\frac{455}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^{2}+1\right)\right)+\frac{25\left(5 x^{2}+3\right)}{8\left(x^{4}+2 x^{2}+3\right)}+\frac{19}{2}\log\left(x^{4}+2 x^{2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^7/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)
*(x^2 + 1)) + 25/8*(5*x^2 + 3)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 +
 2*x^2 + 3)
```

Fricas [A] time = 0.276443, size = 132, normalized size = 1.63

$$\frac{\sqrt{2}\left(228 \sqrt{2}\left(x^{4}+2 x^{2}+3\right)\log\left(x^{4}+2 x^{2}+3\right)-1365\left(x^{4}+2 x^{2}+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^{2}+1\right)\right)+\sqrt{2}\left(20 x^{10}-62 x^{8}+84 x^{6}+1\right.}{48\left(x^{4}+2 x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^7/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(2)*(228*sqrt(2)*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3)
- 1365*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + sqrt(2)*
(20*x^10 - 62*x^8 + 84*x^6 + 150*x^4 + 1059*x^2 + 225))/(x^4 + 2*
x^2 + 3)
```

Sympy [A] time = 0.454654, size = 80, normalized size = 0.99

$$\frac{5x^{6}}{6}-\frac{17x^{4}}{4}+\frac{19x^{2}}{2}+\frac{125x^{2}+75}{8x^{4}+16x^{2}+24}+\frac{19\log\left(x^{4}+2x^{2}+3\right)}{2}-\frac{455\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^{2}}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**6/6 - 17*x**4/4 + 19*x**2/2 + (125*x**2 + 75)/(8*x**4 + 16*x
**2 + 24) + 19*log(x**4 + 2*x**2 + 3)/2 - 455*sqrt(2)*atan(sqrt(2
)*x**2/2 + sqrt(2)/2)/16
```

GIAC/XCAS [A] time = 0.270333, size = 96, normalized size = 1.19

$$\frac{5}{6}\,x^{6} - \frac{17}{4}\,x^{4} + \frac{19}{2}\,x^{2} - \frac{455}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^{2} + 1\right)\right) - \frac{76\,x^{4} + 27\,x^{2} + 153}{8\,\left(x^{4} + 2\,x^{2} + 3\right)} + \frac{19}{2}\,\ln\left(x^{4} + 2\,x^{2} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^7/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)
*(x^2 + 1)) - 1/8*(76*x^4 + 27*x^2 + 153)/(x^4 + 2*x^2 + 3) + 19/
2*ln(x^4 + 2*x^2 + 3)
```

3.102 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=74

$$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{203\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25(3-x^2)}{8(x^4+2x^2+3)}+\frac{19}{4}\log\left(x^4+2x^2+3\right)$$

```
[Out] (-17*x^2)/2 + (5*x^4)/4 + (25*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) +
(203*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 +
 x^4])/4
```

Rubi [A] time = 0.214933, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{203\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25(3-x^2)}{8(x^4+2x^2+3)}+\frac{19}{4}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (-17*x^2)/2 + (5*x^4)/4 + (25*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) +
(203*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 +
 x^4])/4
```

Rubi in Sympy [A] time = 29.3047, size = 78, normalized size = 1.05

$$\frac{5x^8}{4(x^4+2x^2+3)}-6x^2+\frac{5(-26x^2+42)}{16(x^4+2x^2+3)}+\frac{19\log\left(x^4+2x^2+3\right)}{4}+\frac{203\sqrt{2}\,\text{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**8/(4*(x**4 + 2*x**2 + 3)) - 6*x**2 + 5*(-26*x**2 + 42)/(16*(
x**4 + 2*x**2 + 3)) + 19*log(x**4 + 2*x**2 + 3)/4 + 203*sqrt(2)*a
tan(sqrt(2)*(x**2/2 + 1/2))/16
```

Mathematica [A] time = 0.0513093, size = 66, normalized size = 0.89

$$\frac{1}{16}\left(20x^4-136x^2+203\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)-\frac{50(x^2-3)}{x^4+2x^2+3}+76\log\left(x^4+2x^2+3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (-136*x^2 + 20*x^4 - (50*(-3 + x^2))/(3 + 2*x^2 + x^4) + 203*Sqrt
[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 76*Log[3 + 2*x^2 + x^4])/16
```

Maple [A] time = 0.013, size = 64, normalized size = 0.9

$$\frac{5\,x^{4}}{4}-\frac{17\,x^{2}}{2}+\frac{1}{2\,x^{4}+4\,x^{2}+6}\left(-\frac{25\,x^{2}}{4}+\frac{75}{4}\right)+\frac{19\,\ln\left(x^{4}+2\,x^{2}+3\right)}{4}+\frac{203\,\sqrt{2}}{16}\arctan\left(\frac{\left(2\,x^{2}+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/4*x^4-17/2*x^2+1/2*(-25/4*x^2+75/4)/(x^4+2*x^2+3)+19/4*ln(x^4+2
*x^2+3)+203/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 0.798365, size = 80, normalized size = 1.08

$$\frac{5}{4}\,x^{4}-\frac{17}{2}\,x^{2}+\frac{203}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^{2}+1\right)\right)-\frac{25\,\left(x^{2}-3\right)}{8\,\left(x^{4}+2\,x^{2}+3\right)}+\frac{19}{4}\,\log\left(x^{4}+2\,x^{2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^5/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1))
 - 25/8*(x^2 - 3)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

Fricas [A] time = 0.261783, size = 126, normalized size = 1.7

$$\frac{\sqrt{2}\left(38\,\sqrt{2}\left(x^{4}+2\,x^{2}+3\right)\log\left(x^{4}+2\,x^{2}+3\right)+203\,\left(x^{4}+2\,x^{2}+3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^{2}+1\right)\right)+\sqrt{2}\left(10\,x^{8}-48\,x^{6}-106\,x^{4}-229\right.\right.}{16\,\left(x^{4}+2\,x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^5/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*(38*sqrt(2)*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) +
 203*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + sqrt(2)*(1
0*x^8 - 48*x^6 - 106*x^4 - 229*x^2 + 75))/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.448709, size = 73, normalized size = 0.99

$$\frac{5x^{4}}{4}-\frac{17x^{2}}{2}-\frac{25x^{2}-75}{8x^{4}+16x^{2}+24}+\frac{19\log\left(x^{4}+2x^{2}+3\right)}{4}+\frac{203\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^{2}}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**4/4 - 17*x**2/2 - (25*x**2 - 75)/(8*x**4 + 16*x**2 + 24) + 1
9*log(x**4 + 2*x**2 + 3)/4 + 203*sqrt(2)*atan(sqrt(2)*x**2/2 + sq
rt(2)/2)/16
```

GIAC/XCAS [A] time = 0.270902, size = 89, normalized size = 1.2

$$\frac{5}{4}x^4 - \frac{17}{2}x^2 + \frac{203}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{38x^4 + 101x^2 + 39}{8(x^4 + 2x^2 + 3)} + \frac{19}{4}\ln\left(x^4 + 2x^2 + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^5/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")
```

```
[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1))
 - 1/8*(38*x^4 + 101*x^2 + 39)/(x^4 + 2*x^2 + 3) + 19/4*ln(x^4 +
2*x^2 + 3)
```

3.103 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=65

$$\frac{5x^2}{2}-\frac{17\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{25\,(x^2+3)}{8\,(x^4+2x^2+3)}-\frac{17}{4}\log\left(x^4+2x^2+3\right)$$

```
[Out] (5*x^2)/2 - (25*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (17*ArcTan[(1
+ x^2)/Sqrt[2]])/(8*Sqrt[2]) - (17*Log[3 + 2*x^2 + x^4])/4
```

Rubi [A] time = 0.179814, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{5x^2}{2}-\frac{17\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{25\,(x^2+3)}{8\,(x^4+2x^2+3)}-\frac{17}{4}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 - (25*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (17*ArcTan[(1
+ x^2)/Sqrt[2]])/(8*Sqrt[2]) - (17*Log[3 + 2*x^2 + x^4])/4
```

Rubi in Sympy [A] time = 26.3388, size = 73, normalized size = 1.12

$$\frac{5x^6}{2\,(x^4+2x^2+3)}-\frac{15\,(6x^2+26)}{16\,(x^4+2x^2+3)}-\frac{17\log\left(x^4+2x^2+3\right)}{4}-\frac{17\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**6/(2*(x**4 + 2*x**2 + 3)) - 15*(6*x**2 + 26)/(16*(x**4 + 2*x
**2 + 3)) - 17*log(x**4 + 2*x**2 + 3)/4 - 17*sqrt(2)*atan(sqrt(2)
*(x**2/2 + 1/2))/16
```

Mathematica [A] time = 0.0466926, size = 61, normalized size = 0.94

$$\frac{1}{16}\left(40x^2-17\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)-\frac{50\,(x^2+3)}{x^4+2x^2+3}-68\log\left(x^4+2x^2+3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (40*x^2 - (50*(3 + x^2))/(3 + 2*x^2 + x^4) - 17*Sqrt[2]*ArcTan[(1
 + x^2)/Sqrt[2]] - 68*Log[3 + 2*x^2 + x^4])/16
```

Maple [A] time = 0.012, size = 59, normalized size = 0.9

$$\frac{5\,x^2}{2}-\frac{1}{2\,x^4+4\,x^2+6}\left(\frac{25\,x^2}{4}+\frac{75}{4}\right)-\frac{17\,\ln\left(x^4+2\,x^2+3\right)}{4}-\frac{17\,\sqrt{2}}{16}\arctan\left(\frac{\left(2\,x^2+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/2*x^2-1/2*(25/4*x^2+75/4)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17
/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 0.793378, size = 73, normalized size = 1.12

$$\frac{5}{2}\,x^2-\frac{17}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-\frac{25\,\left(x^2+3\right)}{8\,\left(x^4+2\,x^2+3\right)}-\frac{17}{4}\,\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^3/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2
 + 3)/(x^4 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

Fricas [A] time = 0.271286, size = 120, normalized size = 1.85

$$-\frac{\sqrt{2}\left(34\,\sqrt{2}\left(x^4+2\,x^2+3\right)\log\left(x^4+2\,x^2+3\right)+17\,\left(x^4+2\,x^2+3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-5\,\sqrt{2}\left(4\,x^6+8\,x^4+7\,x^2-15\right)\right)}{16\,\left(x^4+2\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^3/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*(34*sqrt(2)*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3)
+ 17*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 5*sqrt(2)*
(4*x^6 + 8*x^4 + 7*x^2 - 15))/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.45492, size = 66, normalized size = 1.02

$$\frac{5x^2}{2}-\frac{25x^2+75}{8x^4+16x^2+24}-\frac{17\log\left(x^4+2x^2+3\right)}{4}-\frac{17\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**2/2 - (25*x**2 + 75)/(8*x**4 + 16*x**2 + 24) - 17*log(x**4 +
 2*x**2 + 3)/4 - 17*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

GIAC/XCAS [A] time = 0.274086, size = 73, normalized size = 1.12

$$\frac{5}{2}\,x^2-\frac{17}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-\frac{25\,\left(x^2+3\right)}{8\,\left(x^4+2\,x^2+3\right)}-\frac{17}{4}\ln\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^3/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")
```

```
[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2
 + 3)/(x^4 + 2*x^2 + 3) - 17/4*ln(x^4 + 2*x^2 + 3)
```

3.104 $\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=58

$$-\frac{23\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)$$

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[
2]])/(8*Sqrt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Rubi [A] time = 0.118904, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{23\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[
2]])/(8*Sqrt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Rubi in Sympy [A] time = 17.5046, size = 53, normalized size = 0.91

$$\frac{0.03125\left(100x^2+100\right)}{x^4+2x^2+3}+\frac{5\log\left(x^4+2x^2+3\right)}{4}-1.4375\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 0.03125*(100*x**2 + 100)/(x**4 + 2*x**2 + 3) + 5*log(x**4 + 2*x**
2 + 3)/4 - 1.4375*sqrt(2)*atan(sqrt(2)*(x**2/2 + 1/2))
```

Mathematica [A] time = 0.0383752, size = 58, normalized size = 1.

$$-\frac{23\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[
2]])/(8*Sqrt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Maple [A] time = 0.012, size = 54, normalized size = 0.9

$$\frac{1}{2\,x^4+4\,x^2+6}\left(\frac{25\,x^2}{4}+\frac{25}{4}\right)+\frac{5\,\ln\left(x^4+2\,x^2+3\right)}{4}-\frac{23\,\sqrt{2}}{16}\arctan\left(\frac{\left(2\,x^2+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/2*(25/4*x^2+25/4)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*2^(1/
2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 0.783841, size = 66, normalized size = 1.14

$$-\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] -23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^
4 + 2*x^2 + 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

Fricas [A] time = 0.274401, size = 104, normalized size = 1.79

$$\frac{\sqrt{2}\left(10\sqrt{2}\left(x^4+2x^2+3\right)\log\left(x^4+2x^2+3\right)-23\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+25\sqrt{2}\left(x^2+1\right)\right)}{16\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*(10*sqrt(2)*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) -
 23*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25*sqrt(2)*
(x^2 + 1))/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.442492, size = 60, normalized size = 1.03

$$\frac{25x^2+25}{8x^4+16x^2+24}+\frac{5\log\left(x^4+2x^2+3\right)}{4}-\frac{23\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (25*x**2 + 25)/(8*x**4 + 16*x**2 + 24) + 5*log(x**4 + 2*x**2 + 3)
/4 - 23*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

GIAC/XCAS [A] time = 0.272747, size = 66, normalized size = 1.14

$$-\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\ln\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")
```

```
[Out] -23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^
4 + 2*x^2 + 3) + 5/4*ln(x^4 + 2*x^2 + 3)
```

3.105 $\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=66

$$\frac{89\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{25\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{4\log(x)}{9}$$

```
[Out] (25*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) + (89*ArcTan[(1 + x^2)/Sqrt
[2]])/(72*Sqrt[2]) + (4*Log[x])/9 - Log[3 + 2*x^2 + x^4]/9
```

Rubi [A] time = 0.199302, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{89\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{25\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (25*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) + (89*ArcTan[(1 + x^2)/Sqrt
[2]])/(72*Sqrt[2]) + (4*Log[x])/9 - Log[3 + 2*x^2 + x^4]/9
```

Rubi in Sympy [A] time = 25.8509, size = 80, normalized size = 1.21

$$-\frac{5x^2}{2\left(x^4+2x^2+3\right)}+\frac{5\left(14x^2+10\right)}{48\left(x^4+2x^2+3\right)}+\frac{2\log\left(x^2\right)}{9}-\frac{\log\left(x^4+2x^2+3\right)}{9}+\frac{89\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+2*x**2+3)**2,x)
```

```
[Out] -5*x**2/(2*(x**4 + 2*x**2 + 3)) + 5*(14*x**2 + 10)/(48*(x**4 + 2*
x**2 + 3)) + 2*log(x**2)/9 - log(x**4 + 2*x**2 + 3)/9 + 89*sqrt(2
)*atan(sqrt(2)*(x**2/2 + 1/2))/144
```

Mathematica [C] time = 0.102723, size = 93, normalized size = 1.41

$$\frac{1}{288}\left(-\sqrt{2}\left(16\sqrt{2}+89i\right)\log\left(x^2-i\sqrt{2}+1\right)+\sqrt{2}\left(-16\sqrt{2}+89i\right)\log\left(x^2+i\sqrt{2}+1\right)-\frac{300\left(x^2-1\right)}{x^4+2x^2+3}+128\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((-300*(-1 + x^2))/(3 + 2*x^2 + x^4) + 128*Log[x] - Sqrt[2]*(89*I
 + 16*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(89*I - 16*Sqrt
[2])*Log[1 + I*Sqrt[2] + x^2])/288
```

Maple [A] time = 0.016, size = 58, normalized size = 0.9

$$\frac{4\ln(x)}{9}-\frac{1}{18x^4+36x^2+54}\left(\frac{75x^2}{4}-\frac{75}{4}\right)-\frac{\ln(x^4+2x^2+3)}{9}+\frac{89\sqrt{2}}{144}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x)
```

```
[Out] 4/9*ln(x)-1/18*(75/4*x^2-75/4)/(x^4+2*x^2+3)-1/9*ln(x^4+2*x^2+3)+
89/144*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 0.784352, size = 74, normalized size = 1.12

$$\frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-\frac{25(x^2-1)}{24(x^4+2x^2+3)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{2}{9}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x),x, algorithm="maxima")
```

```
[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/24*(x^2 - 1)/(x
^4 + 2*x^2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

Fricas [A] time = 0.267618, size = 127, normalized size = 1.92

$$-\frac{\sqrt{2}\left(8\sqrt{2}(x^4+2x^2+3)\log\left(x^4+2x^2+3\right)-32\sqrt{2}(x^4+2x^2+3)\log(x)-89\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+75\right.}{144(x^4+2x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x),x, algorithm="fricas")
```

```
[Out] -1/144*sqrt(2)*(8*sqrt(2)*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3)
- 32*sqrt(2)*(x^4 + 2*x^2 + 3)*log(x) - 89*(x^4 + 2*x^2 + 3)*arct
an(1/2*sqrt(2)*(x^2 + 1)) + 75*sqrt(2)*(x^2 - 1))/(x^4 + 2*x^2 +
3)
```

Sympy [A] time = 0.486967, size = 65, normalized size = 0.98

$$-\frac{25x^2-25}{24x^4+48x^2+72}+\frac{4\log(x)}{9}-\frac{\log\left(x^4+2x^2+3\right)}{9}+\frac{89\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+2*x**2+3)**2,x)
```

```
[Out] -(25*x**2 - 25)/(24*x**4 + 48*x**2 + 72) + 4*log(x)/9 - log(x**4
+ 2*x**2 + 3)/9 + 89*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/144
```

GIAC/XCAS [A] time = 0.271836, size = 84, normalized size = 1.27

$$\frac{89}{144}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^{2}+1\right)\right)+\frac{8\,x^{4}-59\,x^{2}+99}{72\,\left(x^{4}+2\,x^{2}+3\right)}-\frac{1}{9}\ln\left(x^{4}+2\,x^{2}+3\right)+\frac{2}{9}\ln\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x),x, algorithm="giac")
```

```
[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(8*x^4 - 59*x
^2 + 99)/(x^4 + 2*x^2 + 3) - 1/9*ln(x^4 + 2*x^2 + 3) + 2/9*ln(x^2
)
```

3.106 $\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{2}{9x^2}-\frac{71\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{25\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}+\frac{13}{108}\log\left(x^4+2x^2+3\right)-\frac{13\log(x)}{27}$$

```
[Out] -2/(9*x^2) - (25*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - (71*ArcTan[(
1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) - (13*Log[x])/27 + (13*Log[3 + 2
*x^2 + x^4])/108
```

Rubi [A] time = 0.219128, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$

$$-\frac{2}{9x^2}-\frac{71\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{25\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}+\frac{13}{108}\log\left(x^4+2x^2+3\right)-\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -2/(9*x^2) - (25*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - (71*ArcTan[(
1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) - (13*Log[x])/27 + (13*Log[3 + 2
*x^2 + x^4])/108
```

Rubi in Sympy [A] time = 26.4935, size = 85, normalized size = 1.2

$$-\frac{5\left(10x^2+14\right)}{144\left(x^4+2x^2+3\right)}-\frac{13\log\left(x^2\right)}{54}+\frac{13\log\left(x^4+2x^2+3\right)}{108}-\frac{71\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)}{432}-\frac{5}{4\left(x^4+2x^2+3\right)}-\frac{2}{9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+2*x**2+3)**2,x)
```

```
[Out] -5*(10*x**2 + 14)/(144*(x**4 + 2*x**2 + 3)) - 13*log(x**2)/54 + 1
3*log(x**4 + 2*x**2 + 3)/108 - 71*sqrt(2)*atan(sqrt(2)*(x**2/2 +
1/2))/432 - 5/(4*(x**4 + 2*x**2 + 3)) - 2/(9*x**2)
```

Mathematica [C] time = 0.0856761, size = 97, normalized size = 1.37

$$\frac{1}{864}\left(-\frac{192}{x^2}+\sqrt{2}\left(52\sqrt{2}+71i\right)\log\left(x^2-i\sqrt{2}+1\right)+\sqrt{2}\left(52\sqrt{2}-71i\right)\log\left(x^2+i\sqrt{2}+1\right)-\frac{300\left(x^2+5\right)}{x^4+2x^2+3}-416\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-192/x^2 - (300*(5 + x^2))/(3 + 2*x^2 + x^4) - 416*Log[x] + Sqrt
[2]*(71*I + 52*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(-71*I
 + 52*Sqrt[2])*Log[1 + I*Sqrt[2] + x^2])/864
```

Maple [A] time = 0.02, size = 63, normalized size = 0.9

$$-\frac{2}{9x^2}-\frac{13\ln(x)}{27}+\frac{1}{54x^4+108x^2+162}\left(-\frac{75x^2}{4}-\frac{375}{4}\right)+\frac{13\ln(x^4+2x^2+3)}{108}-\frac{71\sqrt{2}}{432}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x)
```

```
[Out] -2/9/x^2-13/27*ln(x)+1/54*(-75/4*x^2-375/4)/(x^4+2*x^2+3)+13/108*
ln(x^4+2*x^2+3)-71/432*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 0.798573, size = 89, normalized size = 1.25

$$-\frac{71}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-\frac{41x^4+157x^2+48}{72(x^6+2x^4+3x^2)}+\frac{13}{108}\log(x^4+2x^2+3)-\frac{13}{54}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^3),x, algorithm="maxima
```

```
[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 15
7*x^2 + 48)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) -
 13/54*log(x^2)
```

Fricas [A] time = 0.271167, size = 158, normalized size = 2.23

$$\frac{\sqrt{2}\Big(26\sqrt{2}(x^6+2x^4+3x^2)\log(x^4+2x^2+3)-104\sqrt{2}(x^6+2x^4+3x^2)\log(x)-71(x^6+2x^4+3x^2)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+\right.}{432(x^6+2x^4+3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^3),x, algorithm="fricas
```

```
[Out] 1/432*sqrt(2)*(26*sqrt(2)*(x^6 + 2*x^4 + 3*x^2)*log(x^4 + 2*x^2 +
 3) - 104*sqrt(2)*(x^6 + 2*x^4 + 3*x^2)*log(x) - 71*(x^6 + 2*x^4
+ 3*x^2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 3*sqrt(2)*(41*x^4 + 157*
x^2 + 48))/(x^6 + 2*x^4 + 3*x^2)
```

Sympy [A] time = 0.583051, size = 75, normalized size = 1.06

$$-\frac{41x^4+157x^2+48}{72x^6+144x^4+216x^2}-\frac{13\log(x)}{27}+\frac{13\log(x^4+2x^2+3)}{108}-\frac{71\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+2*x**2+3)**2,x)
```

```
[Out] -(41*x**4 + 157*x**2 + 48)/(72*x**6 + 144*x**4 + 216*x**2) - 13*l
og(x)/27 + 13*log(x**4 + 2*x**2 + 3)/108 - 71*sqrt(2)*atan(sqrt(2
)*x**2/2 + sqrt(2)/2)/432
```

GIAC/XCAS [A] time = 0.271676, size = 89, normalized size = 1.25

$$-\frac{71}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{41\,x^4+157\,x^2+48}{72\,(x^6+2\,x^4+3\,x^2)}+\frac{13}{108}\ln\left(x^4+2\,x^2+3\right)-\frac{13}{54}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^3),x, algorithm="giac")
```

```
[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 15
7*x^2 + 48)/(x^6 + 2*x^4 + 3*x^2) + 13/108*ln(x^4 + 2*x^2 + 3) -
13/54*ln(x^2)
```

3.107 $\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{125\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{25\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}-\frac{13}{108}\log\left(x^4+2x^2+3\right)+\frac{13\log(x)}{27}$$

```
[Out] -1/(9*x^4) + 13/(54*x^2) + (25*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4
)) + (125*ArcTan[(1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) + (13*Log[x])/
27 - (13*Log[3 + 2*x^2 + x^4])/108
```

Rubi [A] time = 0.230945, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{125\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{25\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}-\frac{13}{108}\log\left(x^4+2x^2+3\right)+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -1/(9*x^4) + 13/(54*x^2) + (25*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4
)) + (125*ArcTan[(1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) + (13*Log[x])/
27 - (13*Log[3 + 2*x^2 + x^4])/108
```

Rubi in Sympy [A] time = 27.0219, size = 95, normalized size = 1.19

$$\frac{5\left(26x^2+22\right)}{432\left(x^4+2x^2+3\right)}+\frac{13\log\left(x^2\right)}{54}-\frac{13\log\left(x^4+2x^2+3\right)}{108}+\frac{125\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)}{432}+\frac{14}{27x^2}-\frac{5}{6x^2\left(x^4+2x^2+3\right)}-\frac{1}{9x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*(26*x**2 + 22)/(432*(x**4 + 2*x**2 + 3)) + 13*log(x**2)/54 - 13
*log(x**4 + 2*x**2 + 3)/108 + 125*sqrt(2)*atan(sqrt(2)*(x**2/2 +
1/2))/432 + 14/(27*x**2) - 5/(6*x**2*(x**4 + 2*x**2 + 3)) - 1/(9*
x**4)
```

Mathematica [C] time = 0.10529, size = 105, normalized size = 1.31

$$\frac{1}{864}\left(-\frac{96}{x^4}+\frac{208}{x^2}-\sqrt{2}\left(52\sqrt{2}+125i\right)\log\left(x^2-i\sqrt{2}+1\right)+\sqrt{2}\left(-52\sqrt{2}+125i\right)\log\left(x^2+i\sqrt{2}+1\right)+\frac{100\left(5x^2+7\right)}{x^4+2x^2+3}+416\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-96/x^4 + 208/x^2 + (100*(7 + 5*x^2))/(3 + 2*x^2 + x^4) + 416*Lo
g[x] - Sqrt[2]*(125*I + 52*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sq
rt[2]*(125*I - 52*Sqrt[2])*Log[1 + I*Sqrt[2] + x^2])/864
```

Maple [A] time = 0.02, size = 68, normalized size = 0.9

$$-\frac{1}{9\,x^4}+\frac{13}{54\,x^2}+\frac{13\,\ln(x)}{27}-\frac{1}{54\,x^4+108\,x^2+162}\left(-\frac{125\,x^2}{4}-\frac{175}{4}\right)-\frac{13\,\ln\left(x^4+2\,x^2+3\right)}{108}+\frac{125\,\sqrt{2}}{432}\arctan\left(\frac{\left(2\,x^2+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x)
```

```
[Out] -1/9/x^4+13/54/x^2+13/27*ln(x)-1/54*(-125/4*x^2-175/4)/(x^4+2*x^2
+3)-13/108*ln(x^4+2*x^2+3)+125/432*2^(1/2)*arctan(1/4*(2*x^2+2)*2
^(1/2))
```

Maxima [A] time = 0.804584, size = 96, normalized size = 1.2

$$\frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{59\,x^6+85\,x^4+36\,x^2-24}{72\left(x^8+2\,x^6+3\,x^4\right)}-\frac{13}{108}\log\left(x^4+2\,x^2+3\right)+\frac{13}{54}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^5),x, algorithm="maxima
```

```
[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(59*x^6 + 85
*x^4 + 36*x^2 - 24)/(x^8 + 2*x^6 + 3*x^4) - 13/108*log(x^4 + 2*x^
2 + 3) + 13/54*log(x^2)
```

Fricas [A] time = 0.288041, size = 165, normalized size = 2.06

$$-\frac{\sqrt{2}\left(26\,\sqrt{2}\left(x^8+2\,x^6+3\,x^4\right)\log\left(x^4+2\,x^2+3\right)-104\,\sqrt{2}\left(x^8+2\,x^6+3\,x^4\right)\log(x)-125\left(x^8+2\,x^6+3\,x^4\right)\arctan\left(\frac{1}{2}\sqrt{2}(x\right.\right.}{432\left(x^8+2\,x^6+3\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^5),x, algorithm="fricas
```

```
[Out] -1/432*sqrt(2)*(26*sqrt(2)*(x^8 + 2*x^6 + 3*x^4)*log(x^4 + 2*x^2
+ 3) - 104*sqrt(2)*(x^8 + 2*x^6 + 3*x^4)*log(x) - 125*(x^8 + 2*x^
6 + 3*x^4)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 3*sqrt(2)*(59*x^6 + 85
*x^4 + 36*x^2 - 24))/(x^8 + 2*x^6 + 3*x^4)
```

Sympy [A] time = 0.641964, size = 80, normalized size = 1.

$$\frac{13\log(x)}{27}-\frac{13\log\left(x^4+2x^2+3\right)}{108}+\frac{125\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}+\frac{59x^6+85x^4+36x^2-24}{72x^8+144x^6+216x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+2*x**2+3)**2,x)
```

```
[Out] 13*log(x)/27 - 13*log(x**4 + 2*x**2 + 3)/108 + 125*sqrt(2)*atan(s
qrt(2)*x**2/2 + sqrt(2)/2)/432 + (59*x**6 + 85*x**4 + 36*x**2 - 2
4)/(72*x**8 + 144*x**6 + 216*x**4)
```

GIAC/XCAS [A] time = 0.274018, size = 107, normalized size = 1.34

$$\frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{26\,x^4+177\,x^2+253}{216\,(x^4+2\,x^2+3)}-\frac{39\,x^4-26\,x^2+12}{108\,x^4}-\frac{13}{108}\ln\left(x^4+2\,x^2+3\right)+\frac{13}{54}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^5),x, algorithm="giac")
```

```
[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/216*(26*x^4 + 1
77*x^2 + 253)/(x^4 + 2*x^2 + 3) - 1/108*(39*x^4 - 26*x^2 + 12)/x^
4 - 13/108*ln(x^4 + 2*x^2 + 3) + 13/54*ln(x^2)
```

3.108 $\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}-\frac{1237\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{25\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}-\frac{61}{972}\log\left(x^4+2x^2+3\right)+\frac{61\log(x)}{243}$$

```
[Out] -2/(27*x^6) + 13/(108*x^4) - 13/(54*x^2) + (25*(1 - 7*x^2))/(648*
(3 + 2*x^2 + x^4)) - (1237*ArcTan[(1 + x^2)/Sqrt[2]])/(1944*Sqrt[
2]) + (61*Log[x])/243 - (61*Log[3 + 2*x^2 + x^4])/972
```

Rubi [A] time = 0.243756, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$

$$-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}-\frac{1237\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{25\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}-\frac{61}{972}\log\left(x^4+2x^2+3\right)+\frac{61\log(x)}{243}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -2/(27*x^6) + 13/(108*x^4) - 13/(54*x^2) + (25*(1 - 7*x^2))/(648*
(3 + 2*x^2 + x^4)) - (1237*ArcTan[(1 + x^2)/Sqrt[2]])/(1944*Sqrt[
2]) + (61*Log[x])/243 - (61*Log[3 + 2*x^2 + x^4])/972
```

Rubi in Sympy [A] time = 27.6736, size = 102, normalized size = 1.17

$$\frac{5\left(-68x^2+56\right)}{2592\left(x^4+2x^2+3\right)}+\frac{61\log\left(x^2\right)}{486}-\frac{61\log\left(x^4+2x^2+3\right)}{972}-\frac{1237\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)}{3888}-\frac{41}{108x^2}+\frac{71}{216x^4}-\frac{5}{8x^4\left(x^4+2x^2+3\right)}-\frac{2}{27x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((5*x**6+3*x**4+x**2+4)/x**7/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*(-68*x**2 + 56)/(2592*(x**4 + 2*x**2 + 3)) + 61*log(x**2)/486 -
 61*log(x**4 + 2*x**2 + 3)/972 - 1237*sqrt(2)*atan(sqrt(2)*(x**2/
2 + 1/2))/3888 - 41/(108*x**2) + 71/(216*x**4) - 5/(8*x**4*(x**4
+ 2*x**2 + 3)) - 2/(27*x**6)
```

Mathematica [C] time = 0.111973, size = 110, normalized size = 1.26

$$\frac{-\frac{576}{x^6}+\frac{936}{x^4}-\frac{1872}{x^2}+\sqrt{2}\left(-244\sqrt{2}+1237i\right)\log\left(x^2-i\sqrt{2}+1\right)-\sqrt{2}\left(244\sqrt{2}+1237i\right)\log\left(x^2+i\sqrt{2}+1\right)-\frac{300\left(7x^2-1\right)}{x^4+2x^2+3}+195}{7776}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-576/x^6 + 936/x^4 - 1872/x^2 - (300*(-1 + 7*x^2))/(3 + 2*x^2 +
x^4) + 1952*Log[x] + Sqrt[2]*(1237*I - 244*Sqrt[2])*Log[1 - I*Sqr
```

```
t[2] + x^2] - Sqrt[2]*(1237*I + 244*Sqrt[2])*Log[1 + I*Sqrt[2] +
x^2])/7776
```

Maple [A] time = 0.02, size = 73, normalized size = 0.8

$$-\frac{2}{27\,x^{6}}+\frac{13}{108\,x^{4}}-\frac{13}{54\,x^{2}}+\frac{61\,\ln\left(x\right)}{243}-\frac{1}{486\,x^{4}+972\,x^{2}+1458}\left(\frac{525\,x^{2}}{4}-\frac{75}{4}\right)-\frac{61\,\ln\left(x^{4}+2\,x^{2}+3\right)}{972}-\frac{1237\,\sqrt{2}}{3888}\arctan\left(\frac{\left(2\,x^{2}+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x)
```

```
[Out] -2/27/x^6+13/108/x^4-13/54/x^2+61/243*ln(x)-1/486*(525/4*x^2-75/4
)/(x^4+2*x^2+3)-61/972*ln(x^4+2*x^2+3)-1237/3888*2^(1/2)*arctan(1
/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 0.789975, size = 103, normalized size = 1.18

$$-\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^{2}+1\right)\right)-\frac{331\,x^{8}+209\,x^{6}+360\,x^{4}-138\,x^{2}+144}{648\,\left(x^{10}+2\,x^{8}+3\,x^{6}\right)}-\frac{61}{972}\log\left(x^{4}+2\,x^{2}+3\right)+\frac{61}{486}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^7),x, algorithm="maxima
```

```
[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/648*(331*x^8
 + 209*x^6 + 360*x^4 - 138*x^2 + 144)/(x^10 + 2*x^8 + 3*x^6) - 61
/972*log(x^4 + 2*x^2 + 3) + 61/486*log(x^2)
```

Fricas [A] time = 0.257951, size = 171, normalized size = 1.97

$$-\frac{\sqrt{2}\left(122\,\sqrt{2}\left(x^{10}+2\,x^{8}+3\,x^{6}\right)\log\left(x^{4}+2\,x^{2}+3\right)-488\,\sqrt{2}\left(x^{10}+2\,x^{8}+3\,x^{6}\right)\log\left(x\right)+1237\,\left(x^{10}+2\,x^{8}+3\,x^{6}\right)\arctan\left(\frac{1}{2}\right.}{3888\,\left(x^{10}+2\,x^{8}+3\,x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^7),x, algorithm="fricas
```

```
[Out] -1/3888*sqrt(2)*(122*sqrt(2)*(x^10 + 2*x^8 + 3*x^6)*log(x^4 + 2*x
^2 + 3) - 488*sqrt(2)*(x^10 + 2*x^8 + 3*x^6)*log(x) + 1237*(x^10
+ 2*x^8 + 3*x^6)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 3*sqrt(2)*(331*x
^8 + 209*x^6 + 360*x^4 - 138*x^2 + 144))/(x^10 + 2*x^8 + 3*x^6)
```

Sympy [A] time = 0.725467, size = 85, normalized size = 0.98

$$\frac{61\log\left(x\right)}{243}-\frac{61\log\left(x^{4}+2x^{2}+3\right)}{972}-\frac{1237\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^{2}}{2}+\frac{\sqrt{2}}{2}\right)}{3888}-\frac{331x^{8}+209x^{6}+360x^{4}-138x^{2}+144}{648x^{10}+1296x^{8}+1944x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**7/(x**4+2*x**2+3)**2,x)
```

```
[Out] 61*log(x)/243 - 61*log(x**4 + 2*x**2 + 3)/972 - 1237*sqrt(2)*atan
(sqrt(2)*x**2/2 + sqrt(2)/2)/3888 - (331*x**8 + 209*x**6 + 360*x*
*4 - 138*x**2 + 144)/(648*x**10 + 1296*x**8 + 1944*x**6)
```

GIAC/XCAS [A] time = 0.269813, size = 113, normalized size = 1.3

$$-\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+\frac{122x^4-281x^2+441}{1944(x^4+2x^2+3)}-\frac{671x^6+702x^4-351x^2+216}{2916x^6}-\frac{61}{972}\ln\left(x^4+2x^2+3\right)+\frac{61}{486}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^7),x, algorithm="giac")
```

```
[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/1944*(122*x^
4 - 281*x^2 + 441)/(x^4 + 2*x^2 + 3) - 1/2916*(671*x^6 + 702*x^4
- 351*x^2 + 216)/x^6 - 61/972*ln(x^4 + 2*x^2 + 3) + 61/486*ln(x^2
)
```

3.109 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=248

$$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}-\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25\left(5x^2+3\right)x}{8\left(x^4+2x^2+3\right)}+38x+\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(
8*(3 + 2*x^2 + x^4)) + (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[
(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt
[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*
x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-262771 + 618291*Sqrt[3])/
2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-26
2771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
+ x^2])/32
```

Rubi [A] time = 0.731425, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}-\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25\left(5x^2+3\right)x}{8\left(x^4+2x^2+3\right)}+38x+\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(
8*(3 + 2*x^2 + x^4)) + (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[
(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt
[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*
x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-262771 + 618291*Sqrt[3])/
2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-26
2771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
+ x^2])/32
```

Rubi in Sympy [A] time = 62.7685, size = 352, normalized size = 1.42

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + \frac{x\left(96000x^2 + 57600\right)}{6144\left(x^4 + 2x^2 + 3\right)} + 38x + \frac{\sqrt{6}\left(-514176\sqrt{3} + 379008\right)\log\left(x^2 - \sqrt{2}x\sqrt{-1 + \sqrt{3}} + \sqrt{3}\right)}{73728\sqrt{-1 + \sqrt{3}}} - \frac{\sqrt{6}\left(-514176\sqrt{3} + 379008\right)\log\left(x^2 + \sqrt{2}x\sqrt{-1 + \sqrt{3}} + \sqrt{3}\right)}{73728\sqrt{-1 + \sqrt{3}}} - \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-1028352\sqrt{3}+758016\right)}{2} + 758016\sqrt{2}\sqrt{-1 + \sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x - \frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{36864\sqrt{-1 + \sqrt{3}}\sqrt{1 + \sqrt{3}}} - \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-1028352\sqrt{3}+758016\right)}{2} + 758016\sqrt{2}\sqrt{-1 + \sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x + \frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{36864\sqrt{-1 + \sqrt{3}}\sqrt{1 + \sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**7/7 - 17*x**5/5 + 19*x**3/3 + x*(96000*x**2 + 57600)/(6144*(
x**4 + 2*x**2 + 3)) + 38*x + sqrt(6)*(-514176*sqrt(3) + 379008)*l
og(x**2 - sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(73728*sqrt(-1
+ sqrt(3))) - sqrt(6)*(-514176*sqrt(3) + 379008)*log(x**2 + sqrt(
2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(73728*sqrt(-1 + sqrt(3))) - s
qrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(-1028352*sqrt(3) + 758016)/2
 + 758016*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x - sqrt(-2 +
 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(36864*sqrt(-1 + sqrt(3))*sqrt(
1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(-1028352*sq
rt(3) + 758016)/2 + 758016*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(
2)*(x + sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(36864*sqrt(-1
 + sqrt(3))*sqrt(1 + sqrt(3)))
```

Mathematica [C] time = 0.337006, size = 145, normalized size = 0.58

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + \frac{25\left(5x^2 + 3\right)x}{8\left(x^4 + 2x^2 + 3\right)} + 38x - \frac{\left(1339\sqrt{2} + 352i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2 - 2i\sqrt{2}}} - \frac{\left(1339\sqrt{2} - 352i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2 + 2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(
8*(3 + 2*x^2 + x^4)) - ((352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 -
I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) - ((-352*I + 1339*Sqrt[
2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.106, size = 427, normalized size = 1.7

$$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x-\frac{1}{x^4+2x^2+3}\left(-\frac{125x^3}{8}-\frac{75x}{8}\right)+\frac{505\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{64}+\frac{11\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{4}-\frac{\left(-1010+1010\sqrt{3}\right)\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{-22+22\sqrt{3}}{2\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{329\sqrt{3}}{8\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{505\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{64}-\frac{11\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{4}-\frac{\left(-1010+1010\sqrt{3}\right)\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{-22+22\sqrt{3}}{2\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{329\sqrt{3}}{8\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/7*x^7-17/5*x^5+19/3*x^3+38*x-(-125/8*x^3-75/8*x)/(x^4+2*x^2+3)+
505/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2
)*3^(1/2)+11/4*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/
2))^(1/2)-505/32/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(
1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-11/2/(2+2*3^(1/
2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*
(-2+2*3^(1/2))-329/8/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2
))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-505/64*ln(x^2+3^(1/2)-x*(-
2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-11/4*ln(x^2+3^(1
/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-505/32/(2+2*3^(1
/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
*(-2+2*3^(1/2))*3^(1/2)-11/2/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+
2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-329/8/(2+2*
3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1
/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{5}{7}x^7-\frac{17}{5}x^5+\frac{19}{3}x^3+38x+\frac{25\left(5x^3+3x\right)}{8\left(x^4+2x^2+3\right)}-\frac{1}{8}\int\frac{1339x^2+987}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] 5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 38*x + 25/8*(5*x^3 + 3*x)/(x^4 +
2*x^2 + 3) - 1/8*integrate((1339*x^2 + 987)/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.312552, size = 1040, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] 1/230828640*sqrt(68699)*(616643160*14158657803^(1/4)*(x^4 + 2*x^2
 + 3)*arctan(2*14158657803^(1/4)*(505*sqrt(3) + 176)/(sqrt(68699)
*sqrt(1/68699)*(618291*sqrt(3)*sqrt(2) - 262771*sqrt(2))*sqrt((31
67785579422627314276 5*sqrt(3)*x^2 + 2*14158657803^(1/4)*sqrt(6869
9)*(35609345672131258954 1*sqrt(3)*x - 51528399685911398318 8*x)*sq
rt((262771*sqrt(3) - 1854873)/(162468944361*sqrt(3) - 60794994024
2)) - 86268515781092897641041*x^2 + 68699*sqrt(3)*(46111087198105
1735*sqrt(3) - 1255746310442552259))/(461110871981051735*sqrt(3)
- 1255746310442552259))*sqrt((262771*sqrt(3) - 1854873)/(16246894
4361*sqrt(3) - 607949940242)) + sqrt(68699)*(618291*sqrt(3)*sqrt(
2)*x - 262771*sqrt(2)*x)*sqrt((262771*sqrt(3) - 1854873)/(1624689
44361*sqrt(3) - 607949940242)) + 14158657803^(1/4)*(329*sqrt(3)*s
qrt(2) - 1339*sqrt(2)))) + 616643160*14158657803^(1/4)*(x^4 + 2*x
^2 + 3)*arctan(2*14158657803^(1/4)*(505*sqrt(3) + 176)/(sqrt(6869
9)*sqrt(1/68699)*(618291*sqrt(3)*sqrt(2) - 262771*sqrt(2))*sqrt((
316778557942262731427 65*sqrt(3)*x^2 - 2*14158657803^(1/4)*sqrt(68
699)*(356093456721312589541*sqrt(3)*x - 515283996859113983188*x)*
sqrt((262771*sqrt(3) - 1854873)/(162468944361*sqrt(3) - 607949940
242)) - 86268515781092897641041*x^2 + 68699*sqrt(3)*(461110871981
051735*sqrt(3) - 1255746310442552259))/(461110871981051735*sqrt(3
) - 1255746310442552259))*sqrt((262771*sqrt(3) - 1854873)/(162468
944361*sqrt(3) - 607949940242)) + sqrt(68699)*(618291*sqrt(3)*sqr
t(2)*x - 262771*sqrt(2)*x)*sqrt((262771*sqrt(3) - 1854873)/(16246
8944361*sqrt(3) - 607949940242)) - 14158657803^(1/4)*(329*sqrt(3)
*sqrt(2) - 1339*sqrt(2)))) - 105*14158657803^(1/4)*(618291*sqrt(3
)*sqrt(2)*(x^4 + 2*x^2 + 3) - 262771*sqrt(2)*(x^4 + 2*x^2 + 3))*l
og(950335673826788194282 95*sqrt(3)*x^2 + 6*14158657803^(1/4)*sqrt
(68699)*(356093456721312589541*sqrt(3)*x - 515283996859113983188*
x)*sqrt((262771*sqrt(3) - 1854873)/(162468944361*sqrt(3) - 607949
940242)) - 258805547343278692923123*x^2 + 206097*sqrt(3)*(4611108
71981051735*sqrt(3) - 1255746310442552259)) + 105*14158657803^(1/
4)*(618291*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3) - 262771*sqrt(2)*(x^
4 + 2*x^2 + 3))*log(95033567382678819428295*sqrt(3)*x^2 - 6*14158
657803^(1/4)*sqrt(68699)*(356093456721312589541*sqrt(3)*x - 51528
3996859113983188*x)*sqrt((262771*sqrt(3) - 1854873)/(162468944361
*sqrt(3) - 607949940242)) - 258805547343278692923123*x^2 + 206097
*sqrt(3)*(461110871981051735*sqrt(3) - 1255746310442552259)) + 4*
sqrt(68699)*(618291*sqrt(3)*sqrt(2)*(600*x^11 - 1656*x^9 + 1408*x
^7 + 33992*x^5 + 92925*x^3 + 103635*x) - 262771*sqrt(2)*(600*x^11
 - 1656*x^9 + 1408*x^7 + 33992*x^5 + 92925*x^3 + 103635*x))*sqrt(
(262771*sqrt(3) - 1854873)/(162468944361*sqrt(3) - 607949940242))
)/((618291*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3) - 262771*sqrt(2)*(x^
4 + 2*x^2 + 3))*sqrt((262771*sqrt(3) - 1854873)/(162468944361*sqr
t(3) - 607949940242)))
```

Sympy [A] time = 1.95431, size = 71, normalized size = 0.29

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + 38x + \frac{125x^3 + 75x}{8x^4 + 16x^2 + 24}$$
$$+\operatorname{RootSum}\left(1048576t^4 + 538155008t^2 + 1146851282043, \left(t \mapsto t\log\left(-\frac{16547840t^3}{453886804809} - \frac{11974973632t}{453886804809} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**7/7 - 17*x**5/5 + 19*x**3/3 + 38*x + (125*x**3 + 75*x)/(8*x*
*4 + 16*x**2 + 24) + RootSum(1048576*_t**4 + 538155008*_t**2 + 11
```

```
46851282043, Lambda(_t, _t*log(-16547840*_t**3/453886804809 - 119
74973632*_t/453886804809 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6+3\,x^4+x^2+4\right)x^8}{\left(x^4+2\,x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 2*x^2 + 3)^2, x)
```

3.110 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=237

$$x^5-\frac{17x^3}{3}+\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25\left(3-x^2\right)x}{8\left(x^4+2x^2+3\right)}+19x+\frac{3}{16}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{3}{16}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] 19*x - (17*x^3)/3 + x^5 + (25*x*(3 - x^2))/(8*(3 + 2*x^2 + x^4))
+ (3*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt
[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (3*Sqrt[(3*(-8669 + 501
1*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 +
Sqrt[3])]])/16 + (3*Sqrt[(3*(8669 + 5011*Sqrt[3]))/2]*Log[Sqrt[3]
 - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 - (3*Sqrt[(3*(8669 + 5011*
Sqrt[3]))/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rubi [A] time = 0.725377, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$x^5-\frac{17x^3}{3}+\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25\left(3-x^2\right)x}{8\left(x^4+2x^2+3\right)}+19x+\frac{3}{16}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{3}{16}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x - (17*x^3)/3 + x^5 + (25*x*(3 - x^2))/(8*(3 + 2*x^2 + x^4))
+ (3*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt
[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (3*Sqrt[(3*(-8669 + 501
1*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 +
Sqrt[3])]])/16 + (3*Sqrt[(3*(8669 + 5011*Sqrt[3]))/2]*Log[Sqrt[3]
 - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 - (3*Sqrt[(3*(8669 + 5011*
Sqrt[3]))/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rubi in Sympy [A] time = 50.0979, size = 342, normalized size = 1.44

$$x^5 - \frac{17x^3}{3} + \frac{x\left(-9600x^2+28800\right)}{3072\left(x^4+2x^2+3\right)} + 19x + \frac{\sqrt{6}\left(53568\sqrt{3}+101952\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{36864\sqrt{-1+\sqrt{3}}} - \frac{\sqrt{6}\left(53568\sqrt{3}+101952\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{36864\sqrt{-1+\sqrt{3}}} - \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(107136\sqrt{3}+203904\right)}{2}+203904\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{18432\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}} - \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(107136\sqrt{3}+203904\right)}{2}+203904\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{18432\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] x**5 - 17*x**3/3 + x*(-9600*x**2 + 28800)/(3072*(x**4 + 2*x**2 +
3)) + 19*x + sqrt(6)*(53568*sqrt(3) + 101952)*log(x**2 - sqrt(2)*
x*sqrt(-1 + sqrt(3)) + sqrt(3))/(36864*sqrt(-1 + sqrt(3))) - sqrt
(6)*(53568*sqrt(3) + 101952)*log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(
3)) + sqrt(3))/(36864*sqrt(-1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*sqr
t(-1 + sqrt(3))*(107136*sqrt(3) + 203904)/2 + 203904*sqrt(2)*sqrt
(-1 + sqrt(3)))*atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt(3))/2)/sqrt(1
+ sqrt(3)))/(18432*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) - sqrt(3
)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(107136*sqrt(3) + 203904)/2 + 2039
04*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x + sqrt(-2 + 2*sqrt
(3))/2)/sqrt(1 + sqrt(3)))/(18432*sqrt(-1 + sqrt(3))*sqrt(1 + sqr
t(3)))
```

Mathematica [C] time = 0.299269, size = 132, normalized size = 0.56

$$x^5 - \frac{17x^3}{3} - \frac{25\left(x^2-3\right)x}{8\left(x^4+2x^2+3\right)} + 19x + \frac{9\left(31\sqrt{2}+90i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} + \frac{9\left(31\sqrt{2}-90i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x - (17*x^3)/3 + x^5 - (25*x*(-3 + x^2))/(8*(3 + 2*x^2 + x^4))
 + (9*(90*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt
[2 - (2*I)*Sqrt[2]]) + (9*(-90*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 +
I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.043, size = 419, normalized size = 1.8

$$x^{5}-\frac{17 x^{3}}{3}+19 x+\frac{1}{x^{4}+2 x^{2}+3}\left(-\frac{25 x^{3}}{8}+\frac{75 x}{8}\right)-\frac{57 \ln\left(x^{2}+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{16}-\frac{405 \ln\left(x^{2}+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{64}+\frac{\left(-114+114\sqrt{3}\right)\sqrt{3}}{8\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{-810+810\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{177\sqrt{3}}{8\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{57 \ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{16}+\frac{405 \ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{64}+\frac{\left(-114+114\sqrt{3}\right)\sqrt{3}}{8\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{-810+810\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{177\sqrt{3}}{8\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] x^5-17/3*x^3+19*x+(-25/8*x^3+75/8*x)/(x^4+2*x^2+3)-57/16*ln(x^2+3
^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-405/6
4*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+57/
8/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1
/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+405/32/(2+2*3^(1/2))^(1/2)*arc
tan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2)
)-177/8/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+
2*3^(1/2))^(1/2))*3^(1/2)+57/16*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(
1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+405/64*ln(x^2+3^(1/2)-x*(-2+2*
3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+57/8/(2+2*3^(1/2))^(1/2)*arc
tan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2)
)*3^(1/2)+405/32/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(
1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-177/8/(2+2*3^(1/2))^(1/
2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x^{5}-\frac{17}{3}x^{3}+19x-\frac{25\left(x^{3}-3x\right)}{8\left(x^{4}+2x^{2}+3\right)}+\frac{9}{8}\int\frac{31x^{2}-59}{x^{4}+2x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] x^5 - 17/3*x^3 + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3) + 9/8*
integrate((31*x^2 - 59)/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.303413, size = 1031, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] 1/481056*sqrt(5011)*(21528*677973267^(1/4)*(x^4 + 2*x^2 + 3)*arct
an(2*677973267^(1/4)*(76*sqrt(3) + 135)/(3*sqrt(5011)*sqrt(1/1503
3)*(5011*sqrt(3)*sqrt(2) + 8669*sqrt(2))*sqrt((19622617239317025*
sqrt(3)*x^2 + 2*677973267^(1/4)*sqrt(5011)*(73549620457552*sqrt(3
)*x + 127391679512403*x)*sqrt((8669*sqrt(3) + 15033)/(43440359*sq
rt(3) + 75240962)) + 33987370050227547*x^2 + 15033*sqrt(3)*(13053
02816425*sqrt(3) + 2260850798259))/(1305302816425*sqrt(3) + 22608
50798259))*sqrt((8669*sqrt(3) + 15033)/(43440359*sqrt(3) + 752409
62)) + 3*sqrt(5011)*(5011*sqrt(3)*sqrt(2)*x + 8669*sqrt(2)*x)*sqr
t((8669*sqrt(3) + 15033)/(43440359*sqrt(3) + 75240962)) + 6779732
67^(1/4)*(59*sqrt(3)*sqrt(2) + 93*sqrt(2)))) + 21528*677973267^(1
/4)*(x^4 + 2*x^2 + 3)*arctan(2*677973267^(1/4)*(76*sqrt(3) + 135)
/(3*sqrt(5011)*sqrt(1/15033)*(5011*sqrt(3)*sqrt(2) + 8669*sqrt(2)
)*sqrt((19622617239317025*sqrt(3)*x^2 - 2*677973267^(1/4)*sqrt(50
11)*(73549620457552*sqrt(3)*x + 127391679512403*x)*sqrt((8669*sqr
t(3) + 15033)/(43440359*sqrt(3) + 75240962)) + 33987370050227547*
x^2 + 15033*sqrt(3)*(1305302816425*sqrt(3) + 2260850798259))/(130
5302816425*sqrt(3) + 2260850798259))*sqrt((8669*sqrt(3) + 15033)/
(43440359*sqrt(3) + 75240962)) + 3*sqrt(5011)*(5011*sqrt(3)*sqrt(
2)*x + 8669*sqrt(2)*x)*sqrt((8669*sqrt(3) + 15033)/(43440359*sqrt
(3) + 75240962)) - 677973267^(1/4)*(59*sqrt(3)*sqrt(2) + 93*sqrt(
2)))) - 9*677973267^(1/4)*(5011*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)
 + 8669*sqrt(2)*(x^4 + 2*x^2 + 3))*log(19622617239317025*sqrt(3)*
x^2 + 2*677973267^(1/4)*sqrt(5011)*(73549620457552*sqrt(3)*x + 12
7391679512403*x)*sqrt((8669*sqrt(3) + 15033)/(43440359*sqrt(3) +
75240962)) + 33987370050227547*x^2 + 15033*sqrt(3)*(1305302816425
*sqrt(3) + 2260850798259)) + 9*677973267^(1/4)*(5011*sqrt(3)*sqrt
(2)*(x^4 + 2*x^2 + 3) + 8669*sqrt(2)*(x^4 + 2*x^2 + 3))*log(19622
617239317025*sqrt(3)*x^2 - 2*677973267^(1/4)*sqrt(5011)*(73549620
457552*sqrt(3)*x + 127391679512403*x)*sqrt((8669*sqrt(3) + 15033)
/(43440359*sqrt(3) + 75240962)) + 33987370050227547*x^2 + 15033*s
qrt(3)*(1305302816425*sqrt(3) + 2260850798259)) + 4*sqrt(5011)*(5
011*sqrt(3)*sqrt(2)*(24*x^9 - 88*x^7 + 256*x^5 + 429*x^3 + 1593*x
) + 8669*sqrt(2)*(24*x^9 - 88*x^7 + 256*x^5 + 429*x^3 + 1593*x))*
sqrt((8669*sqrt(3) + 15033)/(43440359*sqrt(3) + 75240962)))/((501
1*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3) + 8669*sqrt(2)*(x^4 + 2*x^2 +
 3))*sqrt((8669*sqrt(3) + 15033)/(43440359*sqrt(3) + 75240962)))
```

Sympy [A] time = 1.96872, size = 63, normalized size = 0.27

$$x^5 - \frac{17x^3}{3} + 19x - \frac{25x^3 - 75x}{8x^4 + 16x^2 + 24} + 3\operatorname{RootSum}\left(1048576t^4 - 53262336t^2 + 677973267, \left(t \mapsto t\log\left(-\frac{2490368t^3}{13484601} + \frac{20518496t}{4494867} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] x**5 - 17*x**3/3 + 19*x - (25*x**3 - 75*x)/(8*x**4 + 16*x**2 + 24
) + 3*RootSum(1048576*_t**4 - 53262336*_t**2 + 677973267, Lambda(
_t, _t*log(-2490368*_t**3/13484601 + 20518496*_t/4494867 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^6 + 3x^4 + x^2 + 4\right)x^6}{\left(x^4 + 2x^2 + 3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")

[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 2*x^2 + 3)^2, x)
```

3.111 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=232

$$\frac{5x^3}{3}-\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{25\left(x^2+3\right)x}{8\left(x^4+2x^2+3\right)}-17x-\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (Sqr
t[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x
)/Sqrt[2*(1 + Sqrt[3])]])/16 + (Sqrt[(14395 + 26499*Sqrt[3])/2]*A
rcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 -
 (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqr
t[3])]*x + x^2])/32 + (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[Sqrt[
3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rubi [A] time = 0.701416, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{5x^3}{3}-\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{25\left(x^2+3\right)x}{8\left(x^4+2x^2+3\right)}-17x-\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```



```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (Sqr
t[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x
)/Sqrt[2*(1 + Sqrt[3])]])/16 + (Sqrt[(14395 + 26499*Sqrt[3])/2]*A
rcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 -
 (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqr
t[3])]*x + x^2])/32 + (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[Sqrt[
3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rubi in Sympy [A] time = 44.3172, size = 338, normalized size = 1.46

$$\frac{5x^3}{3} - \frac{x\left(4800x^2 + 14400\right)}{1536\left(x^4 + 2x^2 + 3\right)} - 17x - \frac{\sqrt{6}\left(-12192\sqrt{3} + 46368\right)\log\left(x^2 - \sqrt{2}x\sqrt{-1+\sqrt{3}} + \sqrt{3}\right)}{18432\sqrt{-1+\sqrt{3}}} + \frac{\sqrt{6}\left(-12192\sqrt{3} + 46368\right)\log\left(x^2 + \sqrt{2}x\sqrt{-1+\sqrt{3}} + \sqrt{3}\right)}{18432\sqrt{-1+\sqrt{3}}} + \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-24384\sqrt{3}+92736\right)}{2} + 92736\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x - \frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{9216\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}} + \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-24384\sqrt{3}+92736\right)}{2} + 92736\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x + \frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{9216\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**3/3 - x*(4800*x**2 + 14400)/(1536*(x**4 + 2*x**2 + 3)) - 17*
x - sqrt(6)*(-12192*sqrt(3) + 46368)*log(x**2 - sqrt(2)*x*sqrt(-1
 + sqrt(3)) + sqrt(3))/(18432*sqrt(-1 + sqrt(3))) + sqrt(6)*(-121
92*sqrt(3) + 46368)*log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqr
t(3))/(18432*sqrt(-1 + sqrt(3))) + sqrt(3)*(-sqrt(2)*sqrt(-1 + sq
rt(3))*(-24384*sqrt(3) + 92736)/2 + 92736*sqrt(2)*sqrt(-1 + sqrt(
3)))*atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))
/(9216*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) + sqrt(3)*(-sqrt(2)*
sqrt(-1 + sqrt(3))*(-24384*sqrt(3) + 92736)/2 + 92736*sqrt(2)*sqr
t(-1 + sqrt(3)))*atan(sqrt(2)*(x + sqrt(-2 + 2*sqrt(3))/2)/sqrt(1
 + sqrt(3)))/(9216*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3)))
```

Mathematica [C] time = 0.318717, size = 129, normalized size = 0.56

$$\frac{5x^3}{3} - \frac{25\left(x^2+3\right)x}{8\left(x^4+2x^2+3\right)} - 17x + \frac{\left(127\sqrt{2} - 356i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} + \frac{\left(127\sqrt{2} + 356i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) + ((-3
56*I + 127*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (
2*I)*Sqrt[2]]) + ((356*I + 127*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[
2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.037, size = 416, normalized size = 1.8

$$\frac{5x^3}{3}-17x+\frac{1}{x^4+2x^2+3}\left(-\frac{25x^3}{8}-\frac{75x}{8}\right)+\frac{17\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{64}$$
$$+\frac{89\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{32}$$
$$-\frac{\left(-34+34\sqrt{3}\right)\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$-\frac{-178+178\sqrt{3}}{16\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{161\sqrt{3}}{8\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$-\frac{17\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{64}$$
$$-\frac{89\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{32}$$
$$-\frac{\left(-34+34\sqrt{3}\right)\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$-\frac{-178+178\sqrt{3}}{16\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{161\sqrt{3}}{8\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/3*x^3-17*x+(-25/8*x^3-75/8*x)/(x^4+2*x^2+3)+17/64*ln(x^2+3^(1/2
)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+89/32*ln(x
^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-17/32/(2+
2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^
(1/2))*(-2+2*3^(1/2))*3^(1/2)-89/16/(2+2*3^(1/2))^(1/2)*arctan((2
*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+161/
8/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1
/2))^(1/2))*3^(1/2)-17/64*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*
(-2+2*3^(1/2))^(1/2)*3^(1/2)-89/32*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2)
)^(1/2))*(-2+2*3^(1/2))^(1/2)-17/32/(2+2*3^(1/2))^(1/2)*arctan((2
*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1
/2)-89/16/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(
2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+161/8/(2+2*3^(1/2))^(1/2)*arct
an((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{5}{3}x^3-17x-\frac{25\left(x^3+3x\right)}{8\left(x^4+2x^2+3\right)}+\frac{1}{8}\int\frac{127x^2+483}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 - 17*x - 25/8*(x^3 + 3*x)/(x^4 + 2*x^2 + 3) + 1/8*integra
te((127*x^2 + 483)/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.290408, size = 1013, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] -1/21024*sqrt(219)*(739608*143883^(1/4)*(x^4 + 2*x^2 + 3)*arctan(
2*143883^(1/4)*(17*sqrt(3) + 178)/(sqrt(219)*sqrt(1/219)*(26499*s
qrt(3)*sqrt(2) - 14395*sqrt(2))*sqrt((10288182782315085*sqrt(3)*x
^2 + 2*143883^(1/4)*sqrt(219)*(6077114820224831*sqrt(3)*x - 11712
797811682234*x)*sqrt((14395*sqrt(3) - 79497)/(381453105*sqrt(3) -
 1156903514)) - 23749107007222377*x^2 + 219*sqrt(3)*(469780035722
15*sqrt(3) - 108443410991883))/(46978003572215*sqrt(3) - 10844341
0991883))*sqrt((14395*sqrt(3) - 79497)/(381453105*sqrt(3) - 11569
03514)) + sqrt(219)*(26499*sqrt(3)*sqrt(2)*x - 14395*sqrt(2)*x)*s
qrt((14395*sqrt(3) - 79497)/(381453105*sqrt(3) - 1156903514)) + 1
43883^(1/4)*(161*sqrt(3)*sqrt(2) - 127*sqrt(2)))) + 739608*143883
^(1/4)*(x^4 + 2*x^2 + 3)*arctan(2*143883^(1/4)*(17*sqrt(3) + 178)
/(sqrt(219)*sqrt(1/219)*(26499*sqrt(3)*sqrt(2) - 14395*sqrt(2))*s
qrt((10288182782315085*sqrt(3)*x^2 - 2*143883^(1/4)*sqrt(219)*(60
77114820224831*sqrt(3)*x - 11712797811682234*x)*sqrt((14395*sqrt(
3) - 79497)/(381453105*sqrt(3) - 1156903514)) - 23749107007222377
*x^2 + 219*sqrt(3)*(46978003572215*sqrt(3) - 108443410991883))/(4
6978003572215*sqrt(3) - 108443410991883))*sqrt((14395*sqrt(3) - 7
9497)/(381453105*sqrt(3) - 1156903514)) + sqrt(219)*(26499*sqrt(3
)*sqrt(2)*x - 14395*sqrt(2)*x)*sqrt((14395*sqrt(3) - 79497)/(3814
53105*sqrt(3) - 1156903514)) - 143883^(1/4)*(161*sqrt(3)*sqrt(2)
- 127*sqrt(2)))) - 3*143883^(1/4)*(26499*sqrt(3)*sqrt(2)*(x^4 + 2
*x^2 + 3) - 14395*sqrt(2)*(x^4 + 2*x^2 + 3))*log(1131700106054659
35*sqrt(3)*x^2 + 22*143883^(1/4)*sqrt(219)*(6077114820224831*sqrt
(3)*x - 11712797811682234*x)*sqrt((14395*sqrt(3) - 79497)/(381453
105*sqrt(3) - 1156903514)) - 261240177079446147*x^2 + 2409*sqrt(3
)*(46978003572215*sqrt(3) - 108443410991883)) + 3*143883^(1/4)*(2
6499*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3) - 14395*sqrt(2)*(x^4 + 2*x
^2 + 3))*log(113170010605465935*sqrt(3)*x^2 - 22*143883^(1/4)*sqr
t(219)*(6077114820224831*sqrt(3)*x - 11712797811682234*x)*sqrt((1
4395*sqrt(3) - 79497)/(381453105*sqrt(3) - 1156903514)) - 2612401
77079446147*x^2 + 2409*sqrt(3)*(46978003572215*sqrt(3) - 10844341
0991883)) - 4*sqrt(219)*(26499*sqrt(3)*sqrt(2)*(40*x^7 - 328*x^5
- 771*x^3 - 1449*x) - 14395*sqrt(2)*(40*x^7 - 328*x^5 - 771*x^3 -
 1449*x))*sqrt((14395*sqrt(3) - 79497)/(381453105*sqrt(3) - 11569
03514)))/((26499*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3) - 14395*sqrt(2
)*(x^4 + 2*x^2 + 3))*sqrt((14395*sqrt(3) - 79497)/(381453105*sqrt
(3) - 1156903514)))
```

Sympy [A] time = 1.97303, size = 58, normalized size = 0.25

$$\frac{5x^3}{3} - 17x - \frac{25x^3 + 75x}{8x^4 + 16x^2 + 24} + \operatorname{RootSum}\left(1048576t^4 + 29480960t^2 + 2106591003, \left(t \mapsto t\log\left(\frac{557056t^3}{816619683} + \frac{166600064t}{816619683} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**3/3 - 17*x - (25*x**3 + 75*x)/(8*x**4 + 16*x**2 + 24) + Root
Sum(1048576*_t**4 + 29480960*_t**2 + 2106591003, Lambda(_t, _t*lo
g(557056*_t**3/816619683 + 166600064*_t/816619683 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6 + 3\,x^4 + x^2 + 4\right)x^4}{\left(x^4 + 2\,x^2 + 3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 2*x^2 + 3)^2, x)
```

3.112 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=225

$$-\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25\left(x^2+1\right)x}{8\left(x^4+2x^2+3\right)}+5x+\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] 5*x + (25*x*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) + (Sqrt[(19291 + 128
99*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 +
Sqrt[3])]])/16 - (Sqrt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*
(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-19291
 + 12899*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2
])/32 + (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-
1 + Sqrt[3])]*x + x^2])/32
```

Rubi [A] time = 0.741479, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$

$$-\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25\left(x^2+1\right)x}{8\left(x^4+2x^2+3\right)}+5x+\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```



```
[Out] 5*x + (25*x*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) + (Sqrt[(19291 + 128
99*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 +
Sqrt[3])]])/16 - (Sqrt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*
(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-19291
 + 12899*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2
])/32 + (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-
1 + Sqrt[3])]*x + x^2])/32
```

Rubi in Sympy [A] time = 36.7798, size = 332, normalized size = 1.48

$$\frac{x\left(2400x^2+2400\right)}{768\left(x^4+2x^2+3\right)}+5x+\frac{\sqrt{6}\left(-5328\sqrt{3}+6960\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{9216\sqrt{-1+\sqrt{3}}}-\frac{\sqrt{6}\left(-5328\sqrt{3}+6960\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{9216\sqrt{-1+\sqrt{3}}}-\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-10656\sqrt{3}+13920\right)}{2}+13920\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{4608\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}-\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-10656\sqrt{3}+13920\right)}{2}+13920\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{4608\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] x*(2400*x**2 + 2400)/(768*(x**4 + 2*x**2 + 3)) + 5*x + sqrt(6)*(-
5328*sqrt(3) + 6960)*log(x**2 - sqrt(2)*x*sqrt(-1 + sqrt(3)) + sq
rt(3))/(9216*sqrt(-1 + sqrt(3))) - sqrt(6)*(-5328*sqrt(3) + 6960)
*log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(9216*sqrt(-1
 + sqrt(3))) - sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(-10656*sqrt(
3) + 13920)/2 + 13920*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x
 - sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(4608*sqrt(-1 + sqr
t(3))*sqrt(1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(
-10656*sqrt(3) + 13920)/2 + 13920*sqrt(2)*sqrt(-1 + sqrt(3)))*ata
n(sqrt(2)*(x + sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(4608*s
qrt(-1 + sqrt(3))*sqrt(1 + sqrt(3)))
```

Mathematica [C] time = 0.315174, size = 121, normalized size = 0.54

$$\frac{25\left(x^3+x\right)}{8\left(x^4+2x^2+3\right)}+5x-\frac{\left(111\sqrt{2}-34i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}-\frac{\left(111\sqrt{2}+34i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (25*(x + x^3))/(8*(3 + 2*x^2 + x^4)) - ((-34*I + 111*Sqrt[2
])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) -
((34*I + 111*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 +
 (2*I)*Sqrt[2]])
```

Maple [B] time = 0.038, size = 412, normalized size = 1.8

$$5x - \frac{1}{x^4+2x^2+3}\left(-\frac{25x^3}{8}-\frac{25x}{8}\right) + \frac{47\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{96} - \frac{17\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{64} - \frac{\left(-94+94\sqrt{3}\right)\sqrt{3}}{48\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{-34+34\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{145\sqrt{3}}{24\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{47\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{96} + \frac{17\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{64} - \frac{\left(-94+94\sqrt{3}\right)\sqrt{3}}{48\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{-34+34\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{145\sqrt{3}}{24\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5*x-(-25/8*x^3-25/8*x)/(x^4+2*x^2+3)+47/96*ln(x^2+3^(1/2)+x*(-2+2
*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-17/64*ln(x^2+3^(1/2
)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-47/48/(2+2*3^(1/2)
)^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-
2+2*3^(1/2))*3^(1/2)+17/32/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*
3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-145/24/(2+2*3
^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/
2))*3^(1/2)-47/96*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^
(1/2))^(1/2)*3^(1/2)+17/64*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))
*(-2+2*3^(1/2))^(1/2)-47/48/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2
*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+17/3
2/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1
/2))^(1/2))*(-2+2*3^(1/2))-145/24/(2+2*3^(1/2))^(1/2)*arctan((2*x
-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$5x + \frac{25\left(x^3+x\right)}{8\left(x^4+2x^2+3\right)} - \frac{1}{8}\int \frac{111x^2+145}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3) - 1/8*integrate((111*x^2 +
 145)/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.300843, size = 1057, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] -1/1238304*sqrt(12899)*3^(3/4)*(63752*166384201^(1/4)*sqrt(3)*(x^
4 + 2*x^2 + 3)*arctan(6*166384201^(1/4)*(17*sqrt(3) - 94)/(sqrt(1
2899)*3^(1/4)*sqrt(1/38697)*(19291*sqrt(3)*sqrt(2) - 38697*sqrt(2
))*sqrt(sqrt(3)*(2*166384201^(1/4)*sqrt(12899)*3^(1/4)*(304228723
3823158*sqrt(3)*x - 5268815053490115*x)*sqrt((19291*sqrt(3) - 386
97)/(248834609*sqrt(3) - 435647642)) + 12899*sqrt(3)*(18033231526
295*sqrt(3)*x^2 - 31259062129131*x^2) + 697831960373037615*sqrt(3
) - 1209631927210982307)/(18033231526295*sqrt(3) - 31259062129131
))*sqrt((19291*sqrt(3) - 38697)/(248834609*sqrt(3) - 435647642))
+ sqrt(12899)*3^(1/4)*(19291*sqrt(3)*sqrt(2)*x - 38697*sqrt(2)*x)
*sqrt((19291*sqrt(3) - 38697)/(248834609*sqrt(3) - 435647642)) +
3*166384201^(1/4)*(111*sqrt(3)*sqrt(2) - 145*sqrt(2)))) + 63752*1
66384201^(1/4)*sqrt(3)*(x^4 + 2*x^2 + 3)*arctan(6*166384201^(1/4)
*(17*sqrt(3) - 94)/(sqrt(12899)*3^(1/4)*sqrt(1/38697)*(19291*sqrt
(3)*sqrt(2) - 38697*sqrt(2))*sqrt(-sqrt(3)*(2*166384201^(1/4)*sqr
t(12899)*3^(1/4)*(3042287233823158*sqrt(3)*x - 5268815053490115*x
)*sqrt((19291*sqrt(3) - 38697)/(248834609*sqrt(3) - 435647642)) -
 12899*sqrt(3)*(18033231526295*sqrt(3)*x^2 - 31259062129131*x^2)
- 697831960373037615*sqrt(3) + 1209631927210982307)/(180332315262
95*sqrt(3) - 31259062129131))*sqrt((19291*sqrt(3) - 38697)/(24883
4609*sqrt(3) - 435647642)) + sqrt(12899)*3^(1/4)*(19291*sqrt(3)*s
qrt(2)*x - 38697*sqrt(2)*x)*sqrt((19291*sqrt(3) - 38697)/(2488346
09*sqrt(3) - 435647642)) - 3*166384201^(1/4)*(111*sqrt(3)*sqrt(2)
 - 145*sqrt(2)))) - 20*sqrt(12899)*3^(1/4)*(19291*sqrt(3)*sqrt(2)
*(8*x^5 + 21*x^3 + 29*x) - 38697*sqrt(2)*(8*x^5 + 21*x^3 + 29*x))
*sqrt((19291*sqrt(3) - 38697)/(248834609*sqrt(3) - 435647642)) +
166384201^(1/4)*(19291*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3) - 38697*
sqrt(2)*(x^4 + 2*x^2 + 3))*log(2*166384201^(1/4)*sqrt(12899)*3^(1
/4)*(3042287233823158*sqrt(3)*x - 5268815053490115*x)*sqrt((19291
*sqrt(3) - 38697)/(248834609*sqrt(3) - 435647642)) + 12899*sqrt(3
)*(18033231526295*sqrt(3)*x^2 - 31259062129131*x^2) + 69783196037
3037615*sqrt(3) - 1209631927210982307) - 166384201^(1/4)*(19291*s
qrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3) - 38697*sqrt(2)*(x^4 + 2*x^2 + 3
))*log(-2*166384201^(1/4)*sqrt(12899)*3^(1/4)*(3042287233823158*s
qrt(3)*x - 5268815053490115*x)*sqrt((19291*sqrt(3) - 38697)/(2488
34609*sqrt(3) - 435647642)) + 12899*sqrt(3)*(18033231526295*sqrt(
3)*x^2 - 31259062129131*x^2) + 697831960373037615*sqrt(3) - 12096
31927210982307))/((19291*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3) - 3869
7*sqrt(2)*(x^4 + 2*x^2 + 3))*sqrt((19291*sqrt(3) - 38697)/(248834
609*sqrt(3) - 435647642)))
```

Sympy [A] time = 1.93164, size = 51, normalized size = 0.23

$$5x + \frac{25x^3 + 25x}{8x^4 + 16x^2 + 24} + \operatorname{RootSum}\left(3145728t^4 + 39507968t^2 + 166384201, \left(t \mapsto t\log\left(-\frac{9240576t^3}{102792131} - \frac{95003488t}{102792131} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x + (25*x**3 + 25*x)/(8*x**4 + 16*x**2 + 24) + RootSum(3145728*
_t**4 + 39507968*_t**2 + 166384201, Lambda(_t, _t*log(-9240576*_t
**3/102792131 - 95003488*_t/102792131 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^6 + 3x^4 + x^2 + 4)x^2}{(x^4 + 2x^2 + 3)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")

[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 2*x^2 + 3)^2, x)
```

3.113 $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=224

$$\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}-\frac{1}{48}\sqrt{\frac{1}{6}\left(12897\sqrt{3}-11567\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{48}\sqrt{\frac{1}{6}\left(12897\sqrt{3}-11567\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] (25*x*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) - (Sqrt[(-11567 + 12897*S
qrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt
[3])]])/48 + (Sqrt[(-11567 + 12897*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1
 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 + (Sqrt[(11567 + 1
2897*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/9
6 - (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + S
qrt[3])]*x + x^2])/96
```

Rubi [A] time = 0.617768, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}-\frac{1}{48}\sqrt{\frac{1}{6}\left(12897\sqrt{3}-11567\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{48}\sqrt{\frac{1}{6}\left(12897\sqrt{3}-11567\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```


```
[Out] (25*x*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) - (Sqrt[(-11567 + 12897*S
qrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt
[3])]])/48 + (Sqrt[(-11567 + 12897*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1
 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 + (Sqrt[(11567 + 1
2897*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/9
6 - (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + S
qrt[3])]*x + x^2])/96
```

Rubi in Sympy [A] time = 27.5498, size = 328, normalized size = 1.46

$$\frac{x\left(-400x^2+400\right)}{384\left(x^4+2x^2+3\right)}-\frac{\sqrt{6}\left(-760\sqrt{3}+56\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{4608\sqrt{-1+\sqrt{3}}}+\frac{\sqrt{6}\left(-760\sqrt{3}+56\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{4608\sqrt{-1+\sqrt{3}}}+\frac{\sqrt{3}\left(112\sqrt{2}\sqrt{-1+\sqrt{3}}-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-1520\sqrt{3}+112\right)}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{2304\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}+\frac{\sqrt{3}\left(112\sqrt{2}\sqrt{-1+\sqrt{3}}-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-1520\sqrt{3}+112\right)}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{2304\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] x*(-400*x**2 + 400)/(384*(x**4 + 2*x**2 + 3)) - sqrt(6)*(-760*sqr
t(3) + 56)*log(x**2 - sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(46
08*sqrt(-1 + sqrt(3))) + sqrt(6)*(-760*sqrt(3) + 56)*log(x**2 + s
qrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(4608*sqrt(-1 + sqrt(3)))
+ sqrt(3)*(112*sqrt(2)*sqrt(-1 + sqrt(3)) - sqrt(2)*sqrt(-1 + sqr
t(3))*(-1520*sqrt(3) + 112)/2)*atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt
(3))/2)/sqrt(1 + sqrt(3)))/(2304*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt
(3))) + sqrt(3)*(112*sqrt(2)*sqrt(-1 + sqrt(3)) - sqrt(2)*sqrt(-1
 + sqrt(3))*(-1520*sqrt(3) + 112)/2)*atan(sqrt(2)*(x + sqrt(-2 +
2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(2304*sqrt(-1 + sqrt(3))*sqrt(1
+ sqrt(3)))
```

Mathematica [C] time = 0.524882, size = 115, normalized size = 0.51

$$\frac{1}{48}\left(-\frac{50x\left(x^2-1\right)}{x^4+2x^2+3}+\frac{\left(95+44i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{\left(95-44i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] ((-50*x*(-1 + x^2))/(3 + 2*x^2 + x^4) + ((95 + (44*I)*Sqrt[2])*Ar
cTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((95 - (44*I)*
Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/48
```

Maple [B] time = 0.036, size = 408, normalized size = 1.8

$$\frac{1}{x^4+2x^2+3}\left(-\frac{25x^3}{24}+\frac{25x}{24}\right)-\frac{139\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{576}$$
$$-\frac{11\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{48}$$
$$+\frac{\left(-278+278\sqrt{3}\right)\sqrt{3}}{288\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{-22+22\sqrt{3}}{24\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$+\frac{7\sqrt{3}}{72\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{139\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{576}$$
$$+\frac{11\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{48}$$
$$+\frac{\left(-278+278\sqrt{3}\right)\sqrt{3}}{288\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$+\frac{-22+22\sqrt{3}}{24\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{7\sqrt{3}}{72\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] (-25/24*x^3+25/24*x)/(x^4+2*x^2+3)-139/576*ln(x^2+3^(1/2)+x*(-2+2
*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-11/48*ln(x^2+3^(1/2
)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+139/288/(2+2*3^(1/
2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*
(-2+2*3^(1/2))*3^(1/2)+11/24/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+
2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+7/72/(2+2*3
^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/
2))*3^(1/2)+139/576*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*
3^(1/2))^(1/2)*3^(1/2)+11/48*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2
))*(-2+2*3^(1/2))^(1/2)+139/288/(2+2*3^(1/2))^(1/2)*arctan((2*x-(
-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+
11/24/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*
3^(1/2))^(1/2))*(-2+2*3^(1/2))+7/72/(2+2*3^(1/2))^(1/2)*arctan((2
*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{25\left(x^3-x\right)}{24\left(x^4+2x^2+3\right)}+\frac{1}{24}\int\frac{95x^2+7}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] -25/24*(x^3 - x)/(x^4 + 2*x^2 + 3) + 1/24*integrate((95*x^2 + 7)/
(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.315086, size = 1038, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] -1/412704*sqrt(1433)*3^(3/4)*(108104*2053489^(1/4)*sqrt(3)*(x^4 +
 2*x^2 + 3)*arctan(6*2053489^(1/4)*(44*sqrt(3) + 139)/(sqrt(1433)
*3^(1/4)*sqrt(1/4299)*(11567*sqrt(3)*sqrt(2) + 38691*sqrt(2))*sqr
t(sqrt(3)*(2*2053489^(1/4)*sqrt(1433)*3^(1/4)*(794362419051925*sq
rt(3)*x + 1283048345880768*x)*sqrt((11567*sqrt(3) + 38691)/(14917
9599*sqrt(3) + 316396658)) + 1433*sqrt(3)*(9431668007995*sqrt(3)*
x^2 + 17418384359577*x^2) + 40546740766370505*sqrt(3) + 748816343
61821523)/(9431668007995*sqrt(3) + 17418384359577))*sqrt((11567*s
qrt(3) + 38691)/(149179599*sqrt(3) + 316396658)) + sqrt(1433)*3^(
1/4)*(11567*sqrt(3)*sqrt(2)*x + 38691*sqrt(2)*x)*sqrt((11567*sqrt
(3) + 38691)/(149179599*sqrt(3) + 316396658)) + 3*2053489^(1/4)*(
95*sqrt(3)*sqrt(2) - 7*sqrt(2)))) + 108104*2053489^(1/4)*sqrt(3)*
(x^4 + 2*x^2 + 3)*arctan(6*2053489^(1/4)*(44*sqrt(3) + 139)/(sqrt
(1433)*3^(1/4)*sqrt(1/4299)*(11567*sqrt(3)*sqrt(2) + 38691*sqrt(2
))*sqrt(-sqrt(3)*(2*2053489^(1/4)*sqrt(1433)*3^(1/4)*(79436241905
1925*sqrt(3)*x + 1283048345880768*x)*sqrt((11567*sqrt(3) + 38691)
/(149179599*sqrt(3) + 316396658)) - 1433*sqrt(3)*(9431668007995*s
qrt(3)*x^2 + 17418384359577*x^2) - 40546740766370505*sqrt(3) - 74
881634361821523)/(9431668007995*sqrt(3) + 17418384359577))*sqrt((
11567*sqrt(3) + 38691)/(149179599*sqrt(3) + 316396658)) + sqrt(14
33)*3^(1/4)*(11567*sqrt(3)*sqrt(2)*x + 38691*sqrt(2)*x)*sqrt((115
67*sqrt(3) + 38691)/(149179599*sqrt(3) + 316396658)) - 3*2053489^
(1/4)*(95*sqrt(3)*sqrt(2) - 7*sqrt(2)))) + 100*sqrt(1433)*3^(1/4)
*(11567*sqrt(3)*sqrt(2)*(x^3 - x) + 38691*sqrt(2)*(x^3 - x))*sqrt
((11567*sqrt(3) + 38691)/(149179599*sqrt(3) + 316396658)) + 20534
89^(1/4)*(11567*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3) + 38691*sqrt(2)
*(x^4 + 2*x^2 + 3))*log(6*2053489^(1/4)*sqrt(1433)*3^(1/4)*(79436
2419051925*sqrt(3)*x + 1283048345880768*x)*sqrt((11567*sqrt(3) +
38691)/(149179599*sqrt(3) + 316396658)) + 4299*sqrt(3)*(943166800
7995*sqrt(3)*x^2 + 17418384359577*x^2) + 121640222299111515*sqrt(
3) + 224644903085464569) - 2053489^(1/4)*(11567*sqrt(3)*sqrt(2)*(
x^4 + 2*x^2 + 3) + 38691*sqrt(2)*(x^4 + 2*x^2 + 3))*log(-6*205348
9^(1/4)*sqrt(1433)*3^(1/4)*(794362419051925*sqrt(3)*x + 128304834
5880768*x)*sqrt((11567*sqrt(3) + 38691)/(149179599*sqrt(3) + 3163
96658)) + 4299*sqrt(3)*(9431668007995*sqrt(3)*x^2 + 1741838435957
7*x^2) + 121640222299111515*sqrt(3) + 224644903085464569))/((1156
7*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3) + 38691*sqrt(2)*(x^4 + 2*x^2
+ 3))*sqrt((11567*sqrt(3) + 38691)/(149179599*sqrt(3) + 316396658
)))
```

Sympy [A] time = 1.92792, size = 48, normalized size = 0.21

$$-\frac{25x^3-25x}{24x^4+48x^2+72}+\operatorname{RootSum}\left(28311552t^4-23689216t^2+18481401,\left(t\mapsto t\log\left(\frac{40992768t^3}{19364129}-\frac{48423104t}{58092387}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] -(25*x**3 - 25*x)/(24*x**4 + 48*x**2 + 72) + RootSum(28311552*_t*
*4 - 23689216*_t**2 + 18481401, Lambda(_t, _t*log(40992768*_t**3/
19364129 - 48423104*_t/58092387 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^6+3x^4+x^2+4}{\left(x^4+2x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")

[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 2*x^2 + 3)^2, x)
```

3.114 $\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=229

$$-\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{25x\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}-\frac{4}{9x}+\frac{1}{48}\sqrt{\frac{1}{6}\left(699\sqrt{3}-965\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{48}\sqrt{\frac{1}{6}\left(699\sqrt{3}-965\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) + (Sqrt[(-965
+ 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1
 + Sqrt[3])]])/48 - (Sqrt[(-965 + 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*
(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 - (Sqrt[(965 +
699*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96
 + (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/96
```

Rubi [A] time = 0.727791, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$

$$-\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{25x\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}-\frac{4}{9x}+\frac{1}{48}\sqrt{\frac{1}{6}\left(699\sqrt{3}-965\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{48}\sqrt{\frac{1}{6}\left(699\sqrt{3}-965\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) + (Sqrt[(-965
+ 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1
 + Sqrt[3])]])/48 - (Sqrt[(-965 + 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*
(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 - (Sqrt[(965 +
699*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96
 + (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/96
```

Rubi in Sympy [A] time = 33.3954, size = 313, normalized size = 1.37

$$-\frac{\sqrt{6}\left(-1344\sqrt{3}+5568\right)\log\left(x^{2}-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{13824\sqrt{-1+\sqrt{3}}}+\frac{\sqrt{6}\left(-1344\sqrt{3}+5568\right)\log\left(x^{2}+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{13824\sqrt{-1+\sqrt{3}}}+\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-2688\sqrt{3}+11136\right)}{2}+11136\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{6912\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}+\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-2688\sqrt{3}+11136\right)}{2}+11136\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{6912\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}+\frac{7}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**2,x)
```

```
[Out] -sqrt(6)*(-1344*sqrt(3) + 5568)*log(x**2 - sqrt(2)*x*sqrt(-1 + sq
rt(3)) + sqrt(3))/(13824*sqrt(-1 + sqrt(3))) + sqrt(6)*(-1344*sqr
t(3) + 5568)*log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(
13824*sqrt(-1 + sqrt(3))) + sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*
(-2688*sqrt(3) + 11136)/2 + 11136*sqrt(2)*sqrt(-1 + sqrt(3)))*ata
n(sqrt(2)*(x - sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(6912*s
qrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) + sqrt(3)*(-sqrt(2)*sqrt(-1
+ sqrt(3))*(-2688*sqrt(3) + 11136)/2 + 11136*sqrt(2)*sqrt(-1 + sq
rt(3)))*atan(sqrt(2)*(x + sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3
)))/(6912*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) + 7/(3*x)
```

Mathematica [C] time = 0.350179, size = 126, normalized size = 0.55

$$-\frac{25x\left(x^{2}+5\right)}{72\left(x^{4}+2x^{2}+3\right)}-\frac{4}{9x}-\frac{\left(19\sqrt{2}+26i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{48\sqrt{2-2i\sqrt{2}}}-\frac{\left(19\sqrt{2}-26i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{48\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - ((26*I + 19*
Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(48*Sqrt[2 - (2*I)*Sqrt[2
]]) - ((-26*I + 19*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(48*Sq
rt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.035, size = 414, normalized size = 1.8

$$-\frac{4}{9x}-\frac{1}{9x^4+18x^2+27}\left(\frac{25x^3}{8}+\frac{125x}{8}\right)+\frac{\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{18}$$
$$+\frac{13\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{192}-\frac{\left(-2+2\sqrt{3}\right)\sqrt{3}}{9\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$-\frac{-26+26\sqrt{3}}{96\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{7\sqrt{3}}{72\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$-\frac{\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{18}-\frac{13\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{192}$$
$$-\frac{\left(-2+2\sqrt{3}\right)\sqrt{3}}{9\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$-\frac{-26+26\sqrt{3}}{96\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{7\sqrt{3}}{72\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x)
```

```
[Out] -4/9/x-1/9*(25/8*x^3+125/8*x)/(x^4+2*x^2+3)+1/18*ln(x^2+3^(1/2)+x
*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+13/192*ln(x^2
+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-1/9/(2+2*3^
(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2
))*(-2+2*3^(1/2))*3^(1/2)-13/96/(2+2*3^(1/2))^(1/2)*arctan((2*x+(
-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+7/72/(2+
2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^
(1/2))*3^(1/2)-1/18*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*
3^(1/2))^(1/2)*3^(1/2)-13/192*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/
2))*(-2+2*3^(1/2))^(1/2)-1/9/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+
2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-13/
96/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(
1/2))^(1/2))*(-2+2*3^(1/2))+7/72/(2+2*3^(1/2))^(1/2)*arctan((2*x-
(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{19x^4+63x^2+32}{24\left(x^5+2x^3+3x\right)}-\frac{1}{24}\int\frac{19x^2-7}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^2),x, algorithm="maxima
```

```
[Out] -1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x) - 1/24*integrate
((19*x^2 - 7)/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.33574, size = 1077, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^2),x, algorithm="fricas
```

```
[Out] 1/1811808*sqrt(699)*27^(3/4)*(12408*54289^(1/4)*sqrt(3)*(x^5 + 2*
x^3 + 3*x)*arctan(54*54289^(1/4)*(13*sqrt(3) + 32)/(sqrt(699)*27^
(1/4)*sqrt(1/233)*(965*sqrt(3)*sqrt(2) + 2097*sqrt(2))*sqrt(sqrt(
3)*(2*54289^(1/4)*sqrt(699)*27^(1/4)*(51426326156*sqrt(3)*x + 889
06597683*x)*sqrt((965*sqrt(3) + 2097)/(674535*sqrt(3) + 1198514))
 + 699*sqrt(3)*(2571065905*sqrt(3)*x^2 + 4466062683*x^2) + 539152
5202785*sqrt(3) + 9365333446251)/(2571065905*sqrt(3) + 4466062683
))*sqrt((965*sqrt(3) + 2097)/(674535*sqrt(3) + 1198514)) + 3*sqrt
(699)*27^(1/4)*(965*sqrt(3)*sqrt(2)*x + 2097*sqrt(2)*x)*sqrt((965
*sqrt(3) + 2097)/(674535*sqrt(3) + 1198514)) + 27*54289^(1/4)*(19
*sqrt(3)*sqrt(2) + 7*sqrt(2)))) + 12408*54289^(1/4)*sqrt(3)*(x^5
+ 2*x^3 + 3*x)*arctan(54*54289^(1/4)*(13*sqrt(3) + 32)/(sqrt(699)
*27^(1/4)*sqrt(1/233)*(965*sqrt(3)*sqrt(2) + 2097*sqrt(2))*sqrt(-
sqrt(3)*(2*54289^(1/4)*sqrt(699)*27^(1/4)*(51426326156*sqrt(3)*x
+ 88906597683*x)*sqrt((965*sqrt(3) + 2097)/(674535*sqrt(3) + 1198
514)) - 699*sqrt(3)*(2571065905*sqrt(3)*x^2 + 4466062683*x^2) - 5
391525202785*sqrt(3) - 9365333446251)/(2571065905*sqrt(3) + 44660
62683))*sqrt((965*sqrt(3) + 2097)/(674535*sqrt(3) + 1198514)) + 3
*sqrt(699)*27^(1/4)*(965*sqrt(3)*sqrt(2)*x + 2097*sqrt(2)*x)*sqrt
((965*sqrt(3) + 2097)/(674535*sqrt(3) + 1198514)) - 27*54289^(1/4
)*(19*sqrt(3)*sqrt(2) + 7*sqrt(2)))) - 4*sqrt(699)*27^(1/4)*(965*
sqrt(3)*sqrt(2)*(19*x^4 + 63*x^2 + 32) + 2097*sqrt(2)*(19*x^4 + 6
3*x^2 + 32))*sqrt((965*sqrt(3) + 2097)/(674535*sqrt(3) + 1198514)
) + 3*54289^(1/4)*(965*sqrt(3)*sqrt(2)*(x^5 + 2*x^3 + 3*x) + 2097
*sqrt(2)*(x^5 + 2*x^3 + 3*x))*log(2*54289^(1/4)*sqrt(699)*27^(1/4
)*(51426326156*sqrt(3)*x + 88906597683*x)*sqrt((965*sqrt(3) + 209
7)/(674535*sqrt(3) + 1198514)) + 699*sqrt(3)*(2571065905*sqrt(3)*
x^2 + 4466062683*x^2) + 5391525202785*sqrt(3) + 9365333446251) -
3*54289^(1/4)*(965*sqrt(3)*sqrt(2)*(x^5 + 2*x^3 + 3*x) + 2097*sqr
t(2)*(x^5 + 2*x^3 + 3*x))*log(-2*54289^(1/4)*sqrt(699)*27^(1/4)*(
51426326156*sqrt(3)*x + 88906597683*x)*sqrt((965*sqrt(3) + 2097)/
(674535*sqrt(3) + 1198514)) + 699*sqrt(3)*(2571065905*sqrt(3)*x^2
 + 4466062683*x^2) + 5391525202785*sqrt(3) + 9365333446251))/((96
5*sqrt(3)*sqrt(2)*(x^5 + 2*x^3 + 3*x) + 2097*sqrt(2)*(x^5 + 2*x^3
 + 3*x))*sqrt((965*sqrt(3) + 2097)/(674535*sqrt(3) + 1198514)))
```

Sympy [A] time = 2.00302, size = 53, normalized size = 0.23

$$-\frac{19x^4 + 63x^2 + 32}{24x^5 + 48x^3 + 72x} + \operatorname{RootSum}\left(28311552t^4 - 1976320t^2 + 54289, \left(t \mapsto t\log\left(-\frac{28311552t^3}{120461} + \frac{1103968t}{120461} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**2,x)
```

```
[Out] -(19*x**4 + 63*x**2 + 32)/(24*x**5 + 48*x**3 + 72*x) + RootSum(28
311552*_t**4 - 1976320*_t**2 + 54289, Lambda(_t, _t*log(-28311552
*_t**3/120461 + 1103968*_t/120461 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6 + 3\,x^4 + x^2 + 4}{\left(x^4 + 2\,x^2 + 3\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^2),x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^2), x)
```

3.115 $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=238

$$-\frac{4}{27x^3}+\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) -\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25x\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)} +\frac{13}{27x}-\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) +\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] -4/(27*x^3) + 13/(27*x) + (25*x*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^
4)) - (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/432 + (Sqrt[(6073 + 56673*Sqrt
[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3]
)]])/432 + (Sqrt[(-6073 + 56673*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*
(-1 + Sqrt[3])]*x + x^2])/864 - (Sqrt[(-6073 + 56673*Sqrt[3])/6]*
Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864
```

Rubi [A] time = 0.744989, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$-\frac{4}{27x^3}+\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) -\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25x\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)} +\frac{13}{27x}-\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) +\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(27*x^3) + 13/(27*x) + (25*x*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^
4)) - (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/432 + (Sqrt[(6073 + 56673*Sqrt
[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3]
)]])/432 + (Sqrt[(-6073 + 56673*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*
(-1 + Sqrt[3])]*x + x^2])/864 - (Sqrt[(-6073 + 56673*Sqrt[3])/6]*
Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864
```

Rubi in Sympy [A] time = 37.298, size = 320, normalized size = 1.34

$$\frac{\sqrt{6}\left(-16704\sqrt{3}+21312\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{124416\sqrt{-1+\sqrt{3}}} - \frac{\sqrt{6}\left(-16704\sqrt{3}+21312\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{124416\sqrt{-1+\sqrt{3}}} - \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-33408\sqrt{3}+42624\right)}{2}+42624\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{62208\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}} - \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-33408\sqrt{3}+42624\right)}{2}+42624\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{62208\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}} - \frac{29}{9x} + \frac{7}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**2,x)
```

```
[Out] sqrt(6)*(-16704*sqrt(3) + 21312)*log(x**2 - sqrt(2)*x*sqrt(-1 + s
qrt(3)) + sqrt(3))/(124416*sqrt(-1 + sqrt(3))) - sqrt(6)*(-16704*
sqrt(3) + 21312)*log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3
))/(124416*sqrt(-1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt
(3))*(-33408*sqrt(3) + 42624)/2 + 42624*sqrt(2)*sqrt(-1 + sqrt(3)
))*atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(
62208*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*s
qrt(-1 + sqrt(3))*(-33408*sqrt(3) + 42624)/2 + 42624*sqrt(2)*sqrt
(-1 + sqrt(3)))*atan(sqrt(2)*(x + sqrt(-2 + 2*sqrt(3))/2)/sqrt(1
+ sqrt(3)))/(62208*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) - 29/(9*
x) + 7/(9*x**3)
```

Mathematica [C] time = 0.555454, size = 131, normalized size = 0.55

$$\frac{1}{864}\left(\frac{4\left(229x^6+351x^4+248x^2-96\right)}{x^3\left(x^4+2x^2+3\right)}+\frac{2\left(229+46i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{2\left(229-46i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((4*(-96 + 248*x^2 + 351*x^4 + 229*x^6))/(x^3*(3 + 2*x^2 + x^4))
+ (2*(229 + (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1
 - I*Sqrt[2]] + (2*(229 - (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqr
t[2]]])/Sqrt[1 + I*Sqrt[2]])/864
```

Maple [B] time = 0.038, size = 419, normalized size = 1.8

$$-\frac{4}{27x^{3}}+\frac{13}{27x}+\frac{1}{27x^{4}+54x^{2}+81}\left(\frac{125x^{3}}{8}+\frac{175x}{8}\right) -\frac{275\ln\left(x^{2}+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{5184} -\frac{23\ln\left(x^{2}+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{864} +\frac{\left(-550+550\sqrt{3}\right)\sqrt{3}}{2592\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) +\frac{-46+46\sqrt{3}}{432\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{137\sqrt{3}}{648\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) +\frac{275\ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{5184} +\frac{23\ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{864} +\frac{\left(-550+550\sqrt{3}\right)\sqrt{3}}{2592\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) +\frac{-46+46\sqrt{3}}{432\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{137\sqrt{3}}{648\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x)
```

```
[Out] -4/27/x^3+13/27/x+1/27*(125/8*x^3+175/8*x)/(x^4+2*x^2+3)-275/5184
*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1
/2)-23/864*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^
(1/2)+275/2592/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/
2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+23/432/(2+2*3^(1/
2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*
(-2+2*3^(1/2))+137/648/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+275/5184*ln(x^2+3^(1/2)-
x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+23/864*ln(x^
2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+275/2592/(
2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2)
)^(1/2))*(-2+2*3^(1/2))*3^(1/2)+23/432/(2+2*3^(1/2))^(1/2)*arctan
((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+1
37/648/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{229x^{6}+351x^{4}+248x^{2}-96}{216\left(x^{7}+2x^{5}+3x^{3}\right)}+\frac{1}{216}\int\frac{229x^{2}+137}{x^{4}+2x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^4),x, algorithm="maxima
```

```
[Out] 1/216*(229*x^6 + 351*x^4 + 248*x^2 - 96)/(x^7 + 2*x^5 + 3*x^3) +
1/216*integrate((229*x^2 + 137)/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.300086, size = 1112, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^4),x, algorithm="fricas
```

```
[Out] -1/146896416*sqrt(6297)*27^(3/4)*(1662648*4405801^(1/4)*sqrt(3)*(
x^7 + 2*x^5 + 3*x^3)*arctan(54*4405801^(1/4)*(46*sqrt(3) + 275)/(
sqrt(6297)*27^(1/4)*sqrt(1/2099)*(6073*sqrt(3)*sqrt(2) - 170019*s
qrt(2))*sqrt(sqrt(3)*(2*4405801^(1/4)*sqrt(6297)*27^(1/4)*(100885
041419059841*sqrt(3)*x - 86942186720034978*x)*sqrt((6073*sqrt(3)
- 170019)/(344175129*sqrt(3) - 4836184058)) + 6297*sqrt(3)*(87886
457041685*sqrt(3)*x^2 - 828513704032353*x^2) + 166026305997447133
5*sqrt(3) - 15651452382875180523)/(87886457041685*sqrt(3) - 82851
3704032353))*sqrt((6073*sqrt(3) - 170019)/(344175129*sqrt(3) - 48
36184058)) + 3*sqrt(6297)*27^(1/4)*(6073*sqrt(3)*sqrt(2)*x - 1700
19*sqrt(2)*x)*sqrt((6073*sqrt(3) - 170019)/(344175129*sqrt(3) - 4
836184058)) + 27*4405801^(1/4)*(229*sqrt(3)*sqrt(2) - 137*sqrt(2)
))) + 1662648*4405801^(1/4)*sqrt(3)*(x^7 + 2*x^5 + 3*x^3)*arctan(
54*4405801^(1/4)*(46*sqrt(3) + 275)/(sqrt(6297)*27^(1/4)*sqrt(1/2
099)*(6073*sqrt(3)*sqrt(2) - 170019*sqrt(2))*sqrt(-sqrt(3)*(2*440
5801^(1/4)*sqrt(6297)*27^(1/4)*(100885041419059841*sqrt(3)*x - 86
942186720034978*x)*sqrt((6073*sqrt(3) - 170019)/(344175129*sqrt(3
) - 4836184058)) - 6297*sqrt(3)*(87886457041685*sqrt(3)*x^2 - 828
513704032353*x^2) - 1660263059974471335*sqrt(3) + 156514523828751
80523)/(87886457041685*sqrt(3) - 828513704032353))*sqrt((6073*sqr
t(3) - 170019)/(344175129*sqrt(3) - 4836184058)) + 3*sqrt(6297)*2
7^(1/4)*(6073*sqrt(3)*sqrt(2)*x - 170019*sqrt(2)*x)*sqrt((6073*sq
rt(3) - 170019)/(344175129*sqrt(3) - 4836184058)) - 27*4405801^(1
/4)*(229*sqrt(3)*sqrt(2) - 137*sqrt(2)))) - 4*sqrt(6297)*27^(1/4)
*(6073*sqrt(3)*sqrt(2)*(229*x^6 + 351*x^4 + 248*x^2 - 96) - 17001
9*sqrt(2)*(229*x^6 + 351*x^4 + 248*x^2 - 96))*sqrt((6073*sqrt(3)
- 170019)/(344175129*sqrt(3) - 4836184058)) - 3*4405801^(1/4)*(60
73*sqrt(3)*sqrt(2)*(x^7 + 2*x^5 + 3*x^3) - 170019*sqrt(2)*(x^7 +
2*x^5 + 3*x^3))*log(6*4405801^(1/4)*sqrt(6297)*27^(1/4)*(10088504
1419059841*sqrt(3)*x - 86942186720034978*x)*sqrt((6073*sqrt(3) -
170019)/(344175129*sqrt(3) - 4836184058)) + 18891*sqrt(3)*(878864
57041685*sqrt(3)*x^2 - 828513704032353*x^2) + 4980789179923414005
*sqrt(3) - 46954357148625541569) + 3*4405801^(1/4)*(6073*sqrt(3)*
sqrt(2)*(x^7 + 2*x^5 + 3*x^3) - 170019*sqrt(2)*(x^7 + 2*x^5 + 3*x
^3))*log(-6*4405801^(1/4)*sqrt(6297)*27^(1/4)*(100885041419059841
*sqrt(3)*x - 86942186720034978*x)*sqrt((6073*sqrt(3) - 170019)/(3
44175129*sqrt(3) - 4836184058)) + 18891*sqrt(3)*(87886457041685*s
qrt(3)*x^2 - 828513704032353*x^2) + 4980789179923414005*sqrt(3) -
 46954357148625541569))/((6073*sqrt(3)*sqrt(2)*(x^7 + 2*x^5 + 3*x
^3) - 170019*sqrt(2)*(x^7 + 2*x^5 + 3*x^3))*sqrt((6073*sqrt(3) -
170019)/(344175129*sqrt(3) - 4836184058)))
```

Sympy [A] time = 2.17939, size = 60, normalized size = 0.25

$$\operatorname{RootSum}\left(2293235712t^4 + 12437504t^2 + 4405801, \left(t \mapsto t\log\left(\frac{19707494400t^3}{145412423} + \frac{357152768t}{145412423} + x\right)\right)\right) + \frac{229x^6 + 351x^4 + 248x^2 - 96}{216x^7 + 432x^5 + 648x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**2,x)
```

```
[Out] RootSum(2293235712*_t**4 + 12437504*_t**2 + 4405801, Lambda(_t, _
t*log(19707494400*_t**3/145412423 + 357152768*_t/145412423 + x)))
 + (229*x**6 + 351*x**4 + 248*x**2 - 96)/(216*x**7 + 432*x**5 + 6
```

```
48*x**3)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6 + 3\,x^4 + x^2 + 4}{(x^4 + 2\,x^2 + 3)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^4),x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^4), x)
```

3.116 $\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592}+\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592}+\frac{25x\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}-\frac{13}{27x}+\frac{\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296}-\frac{\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296}$$

```
[Out] -4/(45*x^5) + 13/(81*x^3) - 13/(27*x) + (25*x*(1 - 7*x^2))/(648*(
3 + 2*x^2 + x^4)) + (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(
Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqr
t[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] +
 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqrt[(1139381 + 688419*Sqrt
[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2592 + (Sq
rt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[
3])]*x + x^2])/2592
```

Rubi [A] time = 0.734498, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$-\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592}+\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592}+\frac{25x\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}-\frac{13}{27x}+\frac{\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296}-\frac{\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```



```
[Out] -4/(45*x^5) + 13/(81*x^3) - 13/(27*x) + (25*x*(1 - 7*x^2))/(648*(
3 + 2*x^2 + x^4)) + (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(
Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqr
t[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] +
 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqrt[(1139381 + 688419*Sqrt
[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2592 + (Sq
rt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[
3])]*x + x^2])/2592
```

Rubi in Sympy [A] time = 41.3606, size = 326, normalized size = 1.33

$$\frac{\sqrt{6}\left(37440+106560\sqrt{3}\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{1866240\sqrt{-1+\sqrt{3}}} - \frac{\sqrt{6}\left(37440+106560\sqrt{3}\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{1866240\sqrt{-1+\sqrt{3}}} - \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(74880+213120\sqrt{3}\right)}{2}+74880\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{933120\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}} - \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(74880+213120\sqrt{3}\right)}{2}+74880\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{933120\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}} + \frac{37}{27x} - \frac{29}{27x^3} + \frac{7}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+2*x**2+3)**2,x)
```

```
[Out] sqrt(6)*(37440 + 106560*sqrt(3))*log(x**2 - sqrt(2)*x*sqrt(-1 + s
qrt(3)) + sqrt(3))/(1866240*sqrt(-1 + sqrt(3))) - sqrt(6)*(37440
+ 106560*sqrt(3))*log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(
3))/(1866240*sqrt(-1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*sqrt(-1 + sq
rt(3))*(74880 + 213120*sqrt(3))/2 + 74880*sqrt(2)*sqrt(-1 + sqrt(
3)))*atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))
/(933120*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) - sqrt(3)*(-sqrt(2
)*sqrt(-1 + sqrt(3))*(74880 + 213120*sqrt(3))/2 + 74880*sqrt(2)*s
qrt(-1 + sqrt(3)))*atan(sqrt(2)*(x + sqrt(-2 + 2*sqrt(3))/2)/sqrt
(1 + sqrt(3)))/(933120*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) + 37
/(27*x) - 29/(27*x**3) + 7/(15*x**5)
```

Mathematica [C] time = 0.608538, size = 140, normalized size = 0.57

$$\frac{-\frac{4\left(2435x^8+2475x^6+3928x^4-984x^2+864\right)}{x^5\left(x^4+2x^2+3\right)}-\frac{10i\left(475\sqrt{2}-487i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{10i\left(475\sqrt{2}+487i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{12960}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((-4*(864 - 984*x^2 + 3928*x^4 + 2475*x^6 + 2435*x^8))/(x^5*(3 +
2*x^2 + x^4)) - ((10*I)*(-487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 -
I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((10*I)*(487*I + 475*Sqrt[2])*
ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/12960
```

Maple [B] time = 0.037, size = 424, normalized size = 1.7

$$-\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}-\frac{1}{27x^4+54x^2+81}\left(\frac{175x^3}{24}-\frac{25x}{24}\right)$$
$$+\frac{481\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{7776}$$
$$+\frac{475\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{5184}$$
$$-\frac{\left(-962+962\sqrt{3}\right)\sqrt{3}}{3888\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$-\frac{-950+950\sqrt{3}}{2592\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{463\sqrt{3}}{1944\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$-\frac{481\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{7776}$$
$$-\frac{475\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{5184}$$
$$-\frac{\left(-962+962\sqrt{3}\right)\sqrt{3}}{3888\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$
$$-\frac{-950+950\sqrt{3}}{2592\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{463\sqrt{3}}{1944\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x)
```

```
[Out] -4/45/x^5+13/81/x^3-13/27/x-1/27*(175/24*x^3-25/24*x)/(x^4+2*x^2+
3)+481/7776*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))
^(1/2)*3^(1/2)+475/5184*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-
2+2*3^(1/2))^(1/2)-481/3888/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2
*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-475/
2592/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3
^(1/2))^(1/2))*(-2+2*3^(1/2))+463/1944/(2+2*3^(1/2))^(1/2)*arctan
((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-481/7776
*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1
/2)-475/5184*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2)
)^(1/2)-481/3888/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(
1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-475/2592/(2+2*3
^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/
2))*(-2+2*3^(1/2))+463/1944/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2
*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2435\,x^8+2475\,x^6+3928\,x^4-984\,x^2+864}{3240\,(x^9+2\,x^7+3\,x^5)}-\frac{1}{648}\int\frac{487\,x^2-463}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^6),x, algorithm="maxima
```

```
[Out] -1/3240*(2435*x^8 + 2475*x^6 + 3928*x^4 - 984*x^2 + 864)/(x^9 + 2
*x^7 + 3*x^5) - 1/648*integrate((487*x^2 - 463)/(x^4 + 2*x^2 + 3)
, x)
```

Fricas [A] time = 0.290717, size = 1126, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^6),x, algorithm="fricas
```

```
[Out] 1/2973970080*sqrt(8499)*27^(3/4)*(29828280*8025889^(1/4)*sqrt(3)*
(x^9 + 2*x^7 + 3*x^5)*arctan(54*8025889^(1/4)*(475*sqrt(3) + 962)
/(sqrt(8499)*27^(1/4)*sqrt(1/2833)*(1139381*sqrt(3)*sqrt(2) + 206
5257*sqrt(2))*sqrt(sqrt(3)*(2*8025889^(1/4)*sqrt(8499)*27^(1/4)*(
2070494440200369267506*sqrt(3)*x + 3586177043470224593529*x)*sqrt
((1139381*sqrt(3) + 2065257)/(784371528639*sqrt(3) + 135997561092
2)) + 8499*sqrt(3)*(3169459161670314505*sqrt(3)*x^2 + 54897932003
02634331*x^2) + 80811700245108008933985*sqrt(3) + 139973257228116
267537507)/(3169459161670314505*sqrt(3) + 5489793200302634331))*s
qrt((1139381*sqrt(3) + 2065257)/(784371528639*sqrt(3) + 135997561
0922)) + 3*sqrt(8499)*27^(1/4)*(1139381*sqrt(3)*sqrt(2)*x + 20652
57*sqrt(2)*x)*sqrt((1139381*sqrt(3) + 2065257)/(784371528639*sqrt
(3) + 1359975610922)) + 27*8025889^(1/4)*(487*sqrt(3)*sqrt(2) + 4
63*sqrt(2)))) + 29828280*8025889^(1/4)*sqrt(3)*(x^9 + 2*x^7 + 3*x
^5)*arctan(54*8025889^(1/4)*(475*sqrt(3) + 962)/(sqrt(8499)*27^(1
/4)*sqrt(1/2833)*(1139381*sqrt(3)*sqrt(2) + 2065257*sqrt(2))*sqrt
(-sqrt(3)*(2*8025889^(1/4)*sqrt(8499)*27^(1/4)*(207049444020036920
67506*sqrt(3)*x + 3586177043470224593529*x)*sqrt((1139381*sqrt(3)
 + 2065257)/(784371528639*sqrt(3) + 1359975610922)) - 8499*sqrt(3
)*(3169459161670314505*sqrt(3)*x^2 + 5489793200302634331*x^2) - 8
0811700245108008933985*sqrt(3) - 139973257228116267537507)/(31694
59161670314505*sqrt(3) + 5489793200302634331))*sqrt((1139381*sqrt
(3) + 2065257)/(784371528639*sqrt(3) + 1359975610922)) + 3*sqrt(8
499)*27^(1/4)*(1139381*sqrt(3)*sqrt(2)*x + 2065257*sqrt(2)*x)*sqr
t((1139381*sqrt(3) + 2065257)/(784371528639*sqrt(3) + 13599756109
22)) - 27*8025889^(1/4)*(487*sqrt(3)*sqrt(2) + 463*sqrt(2)))) - 4
*sqrt(8499)*27^(1/4)*(1139381*sqrt(3)*sqrt(2)*(2435*x^8 + 2475*x^
6 + 3928*x^4 - 984*x^2 + 864) + 2065257*sqrt(2)*(2435*x^8 + 2475*
x^6 + 3928*x^4 - 984*x^2 + 864))*sqrt((1139381*sqrt(3) + 2065257)
/(784371528639*sqrt(3) + 1359975610922)) + 15*8025889^(1/4)*(1139
381*sqrt(3)*sqrt(2)*(x^9 + 2*x^7 + 3*x^5) + 2065257*sqrt(2)*(x^9
+ 2*x^7 + 3*x^5))*log(18*8025889^(1/4)*sqrt(8499)*27^(1/4)*(20704
94440200369267506*sqrt(3)*x + 3586177043470224593529*x)*sqrt((113
9381*sqrt(3) + 2065257)/(784371528639*sqrt(3) + 1359975610922)) +
 76491*sqrt(3)*(3169459161670314505*sqrt(3)*x^2 + 548979320030263
4331*x^2) + 727305302205972080405865*sqrt(3) + 125975931505304640
7837563) - 15*8025889^(1/4)*(1139381*sqrt(3)*sqrt(2)*(x^9 + 2*x^7
 + 3*x^5) + 2065257*sqrt(2)*(x^9 + 2*x^7 + 3*x^5))*log(-18*802588
9^(1/4)*sqrt(8499)*27^(1/4)*(2070494440200369267506*sqrt(3)*x + 3
586177043470224593529*x)*sqrt((1139381*sqrt(3) + 2065257)/(784371
528639*sqrt(3) + 1359975610922)) + 76491*sqrt(3)*(316945916167031
4505*sqrt(3)*x^2 + 5489793200302634331*x^2) + 7273053022059720804
05865*sqrt(3) + 1259759315053046407837563))/((1139381*sqrt(3)*sqr
t(2)*(x^9 + 2*x^7 + 3*x^5) + 2065257*sqrt(2)*(x^9 + 2*x^7 + 3*x^5
))*sqrt((1139381*sqrt(3) + 2065257)/(784371528639*sqrt(3) + 13599
75610922)))
```

Sympy [A] time = 2.17976, size = 65, normalized size = 0.27

$$\operatorname{RootSum}\left(20639121408t^4 - 2333452288t^2 + 72233001, \left(t \mapsto t \log\left(-\frac{206821195776t^3}{704195977} + \frac{38757503008t}{2112587931} + x\right)\right)\right) - \frac{2435x^8 + 2475x^6 + 3928x^4 - 984x^2 + 864}{3240x^9 + 6480x^7 + 9720x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+2*x**2+3)**2,x)
```

```
[Out] RootSum(20639121408*_t**4 - 2333452288*_t**2 + 72233001, Lambda(_
t, _t*log(-206821195776*_t**3/704195977 + 38757503008*_t/21125879
31 + x))) - (2435*x**8 + 2475*x**6 + 3928*x**4 - 984*x**2 + 864)/
(3240*x**9 + 6480*x**7 + 9720*x**5)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6 + 3\,x^4 + x^2 + 4}{(x^4 + 2\,x^2 + 3)^2 x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^6),x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^6), x)
```

3.117 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=243

$$x^5-9x^3+\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{(252x^2+3305)x}{64(x^4+2x^2+3)}-\frac{25(7x^2+15)x}{16(x^4+2x^2+3)^2}+58x+\frac{3}{256}\sqrt{7678611\sqrt{3}-8595619}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{3}{256}\sqrt{7678611\sqrt{3}-8595619}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2)
 + (x*(3305 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*Sqrt[-8595619
 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/256 - (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*ArcTan
[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (3*
Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/512 - (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3
] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rubi [A] time = 0.854242, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258

$$x^5-9x^3+\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{(252x^2+3305)x}{64(x^4+2x^2+3)}-\frac{25(7x^2+15)x}{16(x^4+2x^2+3)^2}+58x+\frac{3}{256}\sqrt{7678611\sqrt{3}-8595619}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{3}{256}\sqrt{7678611\sqrt{3}-8595619}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2)
 + (x*(3305 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*Sqrt[-8595619
 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/256 - (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*ArcTan
[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (3*
Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/512 - (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3
] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rubi in Sympy [A] time = 65.0145, size = 362, normalized size = 1.49

$$x^5-9x^3-\frac{x\left(268800x^2+576000\right)}{24576\left(x^4+2x^2+3\right)^2}+\frac{x\left(297271296x^2+3898736640\right)}{75497472\left(x^4+2x^2+3\right)}+58x+\frac{\sqrt{6}\left(261881856\sqrt{3}+8222736384\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{905969664\sqrt{-1+\sqrt{3}}}-\frac{\sqrt{6}\left(261881856\sqrt{3}+8222736384\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{905969664\sqrt{-1+\sqrt{3}}}-\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(523763712\sqrt{3}+16445472768\right)}{2}+16445472768\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{452984832\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}-\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(523763712\sqrt{3}+16445472768\right)}{2}+16445472768\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{452984832\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] x**5 - 9*x**3 - x*(268800*x**2 + 576000)/(24576*(x**4 + 2*x**2 +
3)**2) + x*(297271296*x**2 + 3898736640)/(75497472*(x**4 + 2*x**2
 + 3)) + 58*x + sqrt(6)*(261881856*sqrt(3) + 8222736384)*log(x**2
 - sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(905969664*sqrt(-1 + s
qrt(3))) - sqrt(6)*(261881856*sqrt(3) + 8222736384)*log(x**2 + sq
rt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(905969664*sqrt(-1 + sqrt(3
))) - sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(523763712*sqrt(3) + 1
6445472768)/2 + 16445472768*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt
(2)*(x - sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(452984832*sq
rt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*sqrt(-1 +
 sqrt(3))*(523763712*sqrt(3) + 16445472768)/2 + 16445472768*sqrt(
2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x + sqrt(-2 + 2*sqrt(3))/2)/
sqrt(1 + sqrt(3)))/(452984832*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3)
))
```

Mathematica [C] time = 0.422754, size = 156, normalized size = 0.64

$$x^5-9x^3+\frac{\left(252x^2+3305\right)x}{64\left(x^4+2x^2+3\right)}-\frac{25\left(7x^2+15\right)x}{16\left(x^4+2x^2+3\right)^2}+58x+\frac{3\left(148\sqrt{2}+4795i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}}+\frac{3\left(148\sqrt{2}-4795i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2)
 + (x*(3305 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*(4795*I + 148
*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt
[2]]) + (3*(-4795*I + 148*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])
/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.033, size = 429, normalized size = 1.8

$$x^{5}-9x^{3}+58x+\frac{1}{\left(x^{4}+2x^{2}+3\right)^{2}}\left(\frac{63x^{7}}{16}+\frac{3809x^{5}}{64}+\frac{3333x^{3}}{32}+\frac{8415x}{64}\right)-\frac{5091\ln\left(x^{2}+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024}-\frac{14385\ln\left(x^{2}+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}+\frac{\left(-10182+10182\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{-28770+28770\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{4647\sqrt{3}}{64\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{5091\ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024}+\frac{14385\ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}+\frac{\left(-10182+10182\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{-28770+28770\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{4647\sqrt{3}}{64\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] x^5-9*x^3+58*x+(63/16*x^7+3809/64*x^5+3333/32*x^3+8415/64*x)/(x^4
+2*x^2+3)^2-5091/1024*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+
2*3^(1/2))^(1/2)*3^(1/2)-14385/1024*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2
))^(1/2))*(-2+2*3^(1/2))^(1/2)+5091/512/(2+2*3^(1/2))^(1/2)*arcta
n((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*
3^(1/2)+14385/512/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^
(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-4647/64/(2+2*3^(1/2))^
(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1
/2)+5091/1024*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2
))^(1/2)*3^(1/2)+14385/1024*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2)
)*(-2+2*3^(1/2))^(1/2)+5091/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-(
-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+
14385/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(
2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-4647/64/(2+2*3^(1/2))^(1/2)*ar
ctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x^{5}-9x^{3}+58x+\frac{252x^{7}+3809x^{5}+6666x^{3}+8415x}{64\left(x^{8}+4x^{6}+10x^{4}+12x^{2}+9\right)}+\frac{3}{64}\int\frac{148x^{2}-4647}{x^{4}+2x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^10/(x^4 + 2*x^2 + 3)^3,x, algorithm="maxima"
```

```
[Out] x^5 - 9*x^3 + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 8415*x
)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 3/64*integrate((148*x^2 -
 4647)/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.297943, size = 1202, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^10/(x^4 + 2*x^2 + 3)^3,x, algorithm="fricas"
```

```
[Out] 1/873655296*sqrt(853179)*4^(3/4)*(4*sqrt(853179)*4^(1/4)*(7678611
*sqrt(3)*sqrt(2)*(64*x^13 - 320*x^11 + 2048*x^9 + 10108*x^7 + 345
93*x^5 + 46026*x^3 + 41823*x) + 8595619*sqrt(2)*(64*x^13 - 320*x^
11 + 2048*x^9 + 10108*x^7 + 34593*x^5 + 46026*x^3 + 41823*x))*sqr
t((8595619*sqrt(3) + 23035833)/(66002414605209*sqrt(3) + 12538393
3330562)) + 172231176*2183743218123^(1/4)*(x^8 + 4*x^6 + 10*x^4 +
 12*x^2 + 9)*arctan(2*2183743218123^(1/4)*(1697*sqrt(3) + 4795)/(
sqrt(853179)*4^(1/4)*sqrt(1/853179)*(7678611*sqrt(3)*sqrt(2) + 85
95619*sqrt(2))*sqrt((22167067444109902599403685525*sqrt(3)*x^2 + 2
183743218123^(1/4)*sqrt(853179)*4^(1/4)*(425102585968430763293471
9*sqrt(3)*x + 7494932476650732136107491*x)*sqrt((8595619*sqrt(3)
+ 23035833)/(66002414605209*sqrt(3) + 125383933330562)) + 3916363
071222445729296887361*x^2 + 853179*sqrt(3)*(259817312007326746197
5*sqrt(3) + 4590318176165195966259))/(2598173120073267461975*sqrt
(3) + 4590318176165195966259))*sqrt((8595619*sqrt(3) + 23035833)/
(66002414605209*sqrt(3) + 125383933330562)) + sqrt(853179)*4^(1/4
)*(7678611*sqrt(3)*sqrt(2)*x + 8595619*sqrt(2)*x)*sqrt((8595619*s
qrt(3) + 23035833)/(66002414605209*sqrt(3) + 125383933330562)) +
2*2183743218123^(1/4)*(1549*sqrt(3)*sqrt(2) + 148*sqrt(2)))) + 17
2231176*2183743218123^(1/4)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*a
rctan(2*2183743218123^(1/4)*(1697*sqrt(3) + 4795)/(sqrt(853179)*4
^(1/4)*sqrt(1/853179)*(7678611*sqrt(3)*sqrt(2) + 8595619*sqrt(2))
*sqrt((22167067444109902599403685525*sqrt(3)*x^2 - 2183743218123^(
1/4)*sqrt(853179)*4^(1/4)*(4251025859684307632934719*sqrt(3)*x +
7494932476650732136107491*x)*sqrt((8595619*sqrt(3) + 23035833)/(6
6002414605209*sqrt(3) + 125383933330562)) + 391636307122244572929
6887361*x^2 + 853179*sqrt(3)*(2598173120073267461975*sqrt(3) + 45
90318176165195966259))/(2598173120073267461975*sqrt(3) + 45903181
76165195966259))*sqrt((8595619*sqrt(3) + 23035833)/(6600241460520
9*sqrt(3) + 125383933330562)) + sqrt(853179)*4^(1/4)*(7678611*sqr
t(3)*sqrt(2)*x + 8595619*sqrt(2)*x)*sqrt((8595619*sqrt(3) + 23035
833)/(66002414605209*sqrt(3) + 125383933330562)) - 2*218374321812
3^(1/4)*(1549*sqrt(3)*sqrt(2) + 148*sqrt(2)))) - 3*2183743218123^
(1/4)*(7678611*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9
) + 8595619*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*log(1330
0240466465941559642211150*sqrt(3)*x^2 + 6*2183743218123^(1/4)*sqr
t(853179)*4^(1/4)*(4251025859684307632934719*sqrt(3)*x + 74949324
76650732136107491*x)*sqrt((8595619*sqrt(3) + 23035833)/(660024146
05209*sqrt(3) + 125383933330562)) + 23498178427334674375781324166
*x^2 + 5119074*sqrt(3)*(2598173120073267461975*sqrt(3) + 45903181
76165195966259)) + 3*2183743218123^(1/4)*(7678611*sqrt(3)*sqrt(2)
*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 8595619*sqrt(2)*(x^8 + 4*x
^6 + 10*x^4 + 12*x^2 + 9))*log(13300240466465941559642211150*sqrt
(3)*x^2 - 6*2183743218123^(1/4)*sqrt(853179)*4^(1/4)*(42510258596
84307632934719*sqrt(3)*x + 7494932476650732136107491*x)*sqrt((859
5619*sqrt(3) + 23035833)/(66002414605209*sqrt(3) + 12538393333056
2)) + 23498178427334674375781324166*x^2 + 5119074*sqrt(3)*(259817
3120073267461975*sqrt(3) + 4590318176165195966259)))/((7678611*sq
rt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 8595619*sqrt(
2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt((8595619*sqrt(3) + 2
3035833)/(66002414605209*sqrt(3) + 125383933330562)))
```

Sympy [A] time = 2.15214, size = 82, normalized size = 0.34

$$x^5 - 9x^3 + 58x + \frac{252x^7 + 3809x^5 + 6666x^3 + 8415x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576}$$
$$+3\operatorname{RootSum}\left(17179869184t^4 - 2253289947136t^2 + 176883200667963, \left(t \mapsto t\log\left(-\frac{56941871104t^3}{55104008440689} - \frac{1957224667904t}{55104008440689} +\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] x**5 - 9*x**3 + 58*x + (252*x**7 + 3809*x**5 + 6666*x**3 + 8415*x
)/(64*x**8 + 256*x**6 + 640*x**4 + 768*x**2 + 576) + 3*RootSum(17
179869184*_t**4 - 2253289947136*_t**2 + 176883200667963, Lambda(_
t, _t*log(-56941871104*_t**3/55104008440689 - 1957224667904*_t/55
```

```
104008440689 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^6 + 3x^4 + x^2 + 4)x^{10}}{(x^4 + 2x^2 + 3)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^10/(x^4 + 2*x^2 + 3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^10/(x^4 + 2*x^2 + 3)^3, x)
```

3.118 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=242

$$\frac{5x^3}{3} - \frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) - \frac{(835x^2+1468)\,x}{64\,(x^4+2x^2+3)} + \frac{25\,(5x^2+3)\,x}{16\,(x^4+2x^2+3)^2} - 27x - \frac{21}{256}\sqrt{34271+22721\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{21}{256}\sqrt{34271+22721\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) -
 (x*(1468 + 835*x^2))/(64*(3 + 2*x^2 + x^4)) - (21*Sqrt[34271 + 2
2721*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + S
qrt[3])]])/256 + (21*Sqrt[34271 + 22721*Sqrt[3]]*ArcTan[(Sqrt[2*(
-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (21*Sqrt[-342
71 + 22721*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2]
)/512 + (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1
 + Sqrt[3])]*x + x^2])/512
```

Rubi [A] time = 0.757398, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$

$$\frac{5x^3}{3} - \frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) + \frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right) - \frac{(835x^2+1468)\,x}{64\,(x^4+2x^2+3)} + \frac{25\,(5x^2+3)\,x}{16\,(x^4+2x^2+3)^2} - 27x - \frac{21}{256}\sqrt{34271+22721\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{21}{256}\sqrt{34271+22721\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```


```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) -
 (x*(1468 + 835*x^2))/(64*(3 + 2*x^2 + x^4)) - (21*Sqrt[34271 + 2
2721*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + S
qrt[3])]])/256 + (21*Sqrt[34271 + 22721*Sqrt[3]]*ArcTan[(Sqrt[2*(
-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (21*Sqrt[-342
71 + 22721*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2]
)/512 + (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1
 + Sqrt[3])]*x + x^2])/512
```

Rubi in Sympy [A] time = 53.1182, size = 360, normalized size = 1.49

$$\frac{5x^3}{3}+\frac{x\left(96000x^2+57600\right)}{12288\left(x^4+2x^2+3\right)^2}-\frac{x\left(246251520x^2+432930816\right)}{18874368\left(x^4+2x^2+3\right)}-27x-\frac{\sqrt{6}\left(-424230912\sqrt{3}+966131712\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{226492416\sqrt{-1+\sqrt{3}}}+\frac{\sqrt{6}\left(-424230912\sqrt{3}+966131712\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{226492416\sqrt{-1+\sqrt{3}}}+\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-848461824\sqrt{3}+1932263424\right)}{2}+1932263424\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{113246208\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}+\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-848461824\sqrt{3}+1932263424\right)}{2}+1932263424\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{113246208\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x**3/3 + x*(96000*x**2 + 57600)/(12288*(x**4 + 2*x**2 + 3)**2)
- x*(246251520*x**2 + 432930816)/(18874368*(x**4 + 2*x**2 + 3)) -
 27*x - sqrt(6)*(-424230912*sqrt(3) + 966131712)*log(x**2 - sqrt(
2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(226492416*sqrt(-1 + sqrt(3)))
 + sqrt(6)*(-424230912*sqrt(3) + 966131712)*log(x**2 + sqrt(2)*x*
sqrt(-1 + sqrt(3)) + sqrt(3))/(226492416*sqrt(-1 + sqrt(3))) + sq
rt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(-848461824*sqrt(3) + 19322634
24)/2 + 1932263424*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x -
sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(113246208*sqrt(-1 + s
qrt(3))*sqrt(1 + sqrt(3))) + sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))
*(-848461824*sqrt(3) + 1932263424)/2 + 1932263424*sqrt(2)*sqrt(-1
 + sqrt(3)))*atan(sqrt(2)*(x + sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + s
qrt(3)))/(113246208*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3)))
```

Mathematica [C] time = 0.391872, size = 155, normalized size = 0.64

$$\frac{5x^3}{3}-\frac{\left(835x^2+1468\right)x}{64\left(x^4+2x^2+3\right)}+\frac{25\left(5x^2+3\right)x}{16\left(x^4+2x^2+3\right)^2}-27x+\frac{21\left(137\sqrt{2}-175i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}}+\frac{21\left(137\sqrt{2}+175i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) -
 (x*(1468 + 835*x^2))/(64*(3 + 2*x^2 + x^4)) + (21*(-175*I + 137*
Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[
2]]) + (21*(175*I + 137*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(
128*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.033, size = 426, normalized size = 1.8

$$\frac{5x^3}{3}-27x+\frac{1}{(x^4+2x^2+3)^2}\left(-\frac{835x^7}{64}-\frac{1569x^5}{32}-\frac{4941x^3}{64}-\frac{513x}{8}\right) -\frac{693\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024} +\frac{3675\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024} +\frac{\left(-1386+1386\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) -\frac{-7350+7350\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{273\sqrt{3}}{8\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) +\frac{693\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024} -\frac{3675\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024} +\frac{\left(-1386+1386\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) -\frac{-7350+7350\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{273\sqrt{3}}{8\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] 5/3*x^3-27*x+(-835/64*x^7-1569/32*x^5-4941/64*x^3-513/8*x)/(x^4+2
*x^2+3)^2-693/1024*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3
^(1/2))^(1/2)*3^(1/2)+3675/1024*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(
1/2))*(-2+2*3^(1/2))^(1/2)+693/512/(2+2*3^(1/2))^(1/2)*arctan((2*
x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/
2)-3675/512/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))
/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+273/8/(2+2*3^(1/2))^(1/2)*ar
ctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+693/
1024*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*
3^(1/2)-3675/1024*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^
(1/2))^(1/2)+693/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2
))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-3675/512/(2
+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))
^(1/2))*(-2+2*3^(1/2))+273/8/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+
2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{5}{3}x^3-27x-\frac{835x^7+3138x^5+4941x^3+4104x}{64(x^8+4x^6+10x^4+12x^2+9)}+\frac{21}{64}\int\frac{137x^2+312}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 2*x^2 + 3)^3,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 - 27*x - 1/64*(835*x^7 + 3138*x^5 + 4941*x^3 + 4104*x)/(x
^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 21/64*integrate((137*x^2 + 31
2)/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.316244, size = 1188, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 2*x^2 + 3)^3,x, algorithm="fricas")
```

```
[Out] 1/69798912*sqrt(22721)*4^(3/4)*(4*sqrt(22721)*4^(1/4)*(22721*sqrt
(3)*sqrt(2)*(320*x^11 - 3904*x^9 - 20041*x^7 - 57414*x^5 - 74151*
x^3 - 58968*x) - 34271*sqrt(2)*(320*x^11 - 3904*x^9 - 20041*x^7 -
 57414*x^5 - 74151*x^3 - 58968*x))*sqrt((34271*sqrt(3) - 68163)/(
778671391*sqrt(3) - 1361616482)) + 6894216*1548731523^(1/4)*(x^8
+ 4*x^6 + 10*x^4 + 12*x^2 + 9)*arctan(2*1548731523^(1/4)*(33*sqrt
(3) - 175)/(sqrt(22721)*4^(1/4)*sqrt(1/22721)*(22721*sqrt(3)*sqrt
(2) - 34271*sqrt(2))*sqrt((2266204729347424955*sqrt(3)*x^2 + 1548
731523^(1/4)*sqrt(22721)*4^(1/4)*(18267385333855091*sqrt(3)*x - 3
1642871720158331*x)*sqrt((34271*sqrt(3) - 68163)/(778671391*sqrt(
3) - 1361616482)) - 3927765773395386729*x^2 + 22721*sqrt(3)*(9974
0536479355*sqrt(3) - 172869405985449))/(99740536479355*sqrt(3) -
172869405985449))*sqrt((34271*sqrt(3) - 68163)/(778671391*sqrt(3)
 - 1361616482)) + sqrt(22721)*4^(1/4)*(22721*sqrt(3)*sqrt(2)*x -
34271*sqrt(2)*x)*sqrt((34271*sqrt(3) - 68163)/(778671391*sqrt(3)
- 1361616482)) + 2*1548731523^(1/4)*(104*sqrt(3)*sqrt(2) - 137*sq
rt(2)))) + 6894216*1548731523^(1/4)*(x^8 + 4*x^6 + 10*x^4 + 12*x^
2 + 9)*arctan(2*1548731523^(1/4)*(33*sqrt(3) - 175)/(sqrt(22721)*
4^(1/4)*sqrt(1/22721)*(22721*sqrt(3)*sqrt(2) - 34271*sqrt(2))*sqr
t((2266204729347424955*sqrt(3)*x^2 - 1548731523^(1/4)*sqrt(22721)
*4^(1/4)*(18267385333855091*sqrt(3)*x - 31642871720158331*x)*sqrt
((34271*sqrt(3) - 68163)/(778671391*sqrt(3) - 1361616482)) - 3927
765773395386729*x^2 + 22721*sqrt(3)*(99740536479355*sqrt(3) - 172
869405985449))/(99740536479355*sqrt(3) - 172869405985449))*sqrt((
34271*sqrt(3) - 68163)/(778671391*sqrt(3) - 1361616482)) + sqrt(2
2721)*4^(1/4)*(22721*sqrt(3)*sqrt(2)*x - 34271*sqrt(2)*x)*sqrt((3
4271*sqrt(3) - 68163)/(778671391*sqrt(3) - 1361616482)) - 2*15487
31523^(1/4)*(104*sqrt(3)*sqrt(2) - 137*sqrt(2)))) + 63*1548731523
^(1/4)*(22721*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
 - 34271*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*log(4532409
458694849910*sqrt(3)*x^2 + 2*1548731523^(1/4)*sqrt(22721)*4^(1/4)
*(18267385333855091*sqrt(3)*x - 31642871720158331*x)*sqrt((34271*
sqrt(3) - 68163)/(778671391*sqrt(3) - 1361616482)) - 785553154679
0773458*x^2 + 45442*sqrt(3)*(99740536479355*sqrt(3) - 17286940598
5449)) - 63*1548731523^(1/4)*(22721*sqrt(3)*sqrt(2)*(x^8 + 4*x^6
+ 10*x^4 + 12*x^2 + 9) - 34271*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12
*x^2 + 9))*log(4532409458694849910*sqrt(3)*x^2 - 2*1548731523^(1/
4)*sqrt(22721)*4^(1/4)*(18267385333855091*sqrt(3)*x - 31642871720
158331*x)*sqrt((34271*sqrt(3) - 68163)/(778671391*sqrt(3) - 13616
16482)) - 7855531546790773458*x^2 + 45442*sqrt(3)*(99740536479355
*sqrt(3) - 172869405985449)))/((22721*sqrt(3)*sqrt(2)*(x^8 + 4*x^
6 + 10*x^4 + 12*x^2 + 9) - 34271*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 +
12*x^2 + 9))*sqrt((34271*sqrt(3) - 68163)/(778671391*sqrt(3) - 13
61616482)))
```

Sympy [A] time = 2.19193, size = 80, normalized size = 0.33

$$\frac{5x^3}{3} - 27x - \frac{835x^7 + 3138x^5 + 4941x^3 + 4104x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576}$$
$$+21 \operatorname{RootSum}\left(17179869184t^4 + 8983937024t^2 + 1548731523, \left(t \mapsto t \log\left(-\frac{1107296256t^3}{310800559} + \frac{438857984t}{310800559} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x**3/3 - 27*x - (835*x**7 + 3138*x**5 + 4941*x**3 + 4104*x)/(64
*x**8 + 256*x**6 + 640*x**4 + 768*x**2 + 576) + 21*RootSum(171798
```

```
69184*_t**4 + 8983937024*_t**2 + 1548731523, Lambda(_t, _t*log(-1
107296256*_t**3/310800559 + 438857984*_t/310800559 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^6 + 3x^4 + x^2 + 4)x^8}{(x^4 + 2x^2 + 3)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 2*x^2 + 3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 2*x^2 + 3)^3, x)
```

3.119 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=235

$$-\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{7\left(58x^2+11\right)x}{64\left(x^4+2x^2+3\right)}+\frac{25\left(3-x^2\right)x}{16\left(x^4+2x^2+3\right)^2}+5x+\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] 5*x + (25*x*(3 - x^2))/(16*(3 + 2*x^2 + x^4)^2) + (7*x*(11 + 58*x
^2))/(64*(3 + 2*x^2 + x^4)) + (Sqrt[827621 + 1176531*Sqrt[3]]*Arc
Tan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 -
(Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] +
2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt[-827621 + 1176531*Sqrt[3
]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 + (Sqrt[-82
7621 + 1176531*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x +
x^2])/512
```

Rubi [A] time = 0.757798, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258

$$-\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{7\left(58x^2+11\right)x}{64\left(x^4+2x^2+3\right)}+\frac{25\left(3-x^2\right)x}{16\left(x^4+2x^2+3\right)^2}+5x+\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```


```
[Out] 5*x + (25*x*(3 - x^2))/(16*(3 + 2*x^2 + x^4)^2) + (7*x*(11 + 58*x
^2))/(64*(3 + 2*x^2 + x^4)) + (Sqrt[827621 + 1176531*Sqrt[3]]*Arc
Tan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 -
(Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] +
2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt[-827621 + 1176531*Sqrt[3
]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 + (Sqrt[-82
7621 + 1176531*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x +
x^2])/512
```

Rubi in Sympy [A] time = 43.1032, size = 354, normalized size = 1.51

$$\frac{x\left(-9600x^2+28800\right)}{6144\left(x^4+2x^2+3\right)^2}+\frac{x\left(29933568x^2+5677056\right)}{4718592\left(x^4+2x^2+3\right)}+5x$$
$$+\frac{\sqrt{6}\left(-48734208\sqrt{3}+41914368\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{56623104\sqrt{-1+\sqrt{3}}}$$
$$-\frac{\sqrt{6}\left(-48734208\sqrt{3}+41914368\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{56623104\sqrt{-1+\sqrt{3}}}$$
$$-\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-97468416\sqrt{3}+83828736\right)}{2}+83828736\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{28311552\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$
$$-\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-97468416\sqrt{3}+83828736\right)}{2}+83828736\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{28311552\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] x*(-9600*x**2 + 28800)/(6144*(x**4 + 2*x**2 + 3)**2) + x*(2993356
8*x**2 + 5677056)/(4718592*(x**4 + 2*x**2 + 3)) + 5*x + sqrt(6)*(
-48734208*sqrt(3) + 41914368)*log(x**2 - sqrt(2)*x*sqrt(-1 + sqrt
(3)) + sqrt(3))/(56623104*sqrt(-1 + sqrt(3))) - sqrt(6)*(-4873420
8*sqrt(3) + 41914368)*log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(3)) + s
qrt(3))/(56623104*sqrt(-1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*sqrt(-1
 + sqrt(3))*(-97468416*sqrt(3) + 83828736)/2 + 83828736*sqrt(2)*s
qrt(-1 + sqrt(3)))*atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt(3))/2)/sqrt
(1 + sqrt(3)))/(28311552*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) -
sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(-97468416*sqrt(3) + 8382873
6)/2 + 83828736*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x + sqr
t(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(28311552*sqrt(-1 + sqrt(
3))*sqrt(1 + sqrt(3)))
```

Mathematica [C] time = 0.699309, size = 138, normalized size = 0.59

$$\frac{1}{256}\left(\frac{4x\left(320x^8+1686x^6+4089x^4+5112x^2+3411\right)}{\left(x^4+2x^2+3\right)^2}-\frac{i\left(185\sqrt{2}-2644i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{i\left(185\sqrt{2}+2644i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(3411 + 5112*x^2 + 4089*x^4 + 1686*x^6 + 320*x^8))/(3 + 2*x
^2 + x^4)^2 - (I*(-2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt
[2]]])/Sqrt[1 - I*Sqrt[2]] + (I*(2644*I + 185*Sqrt[2])*ArcTan[x/S
qrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/256
```

Maple [B] time = 0.033, size = 422, normalized size = 1.8

$$5x - \frac{1}{(x^4+2x^2+3)^2}\left(-\frac{203x^7}{32} - \frac{889x^5}{64} - \frac{159x^3}{8} - \frac{531x}{64}\right) + \frac{943\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024} + \frac{185\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024} - \frac{\left(-1886+1886\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{-370+370\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{379\sqrt{3}}{64\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{943\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024} - \frac{185\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024} - \frac{\left(-1886+1886\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{-370+370\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{379\sqrt{3}}{64\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] 5*x-(-203/32*x^7-889/64*x^5-159/8*x^3-531/64*x)/(x^4+2*x^2+3)^2+9
43/1024*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/
2)*3^(1/2)+185/1024*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*
3^(1/2))^(1/2)-943/512/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-185/512/(
2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2)
)^(1/2))*(-2+2*3^(1/2))-379/64/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-
2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-943/1024*ln(x^2+
3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-185/
1024*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-
943/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+
2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-185/512/(2+2*3^(1/2))^(1
/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*
3^(1/2))-379/64/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1
/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$5x + \frac{406x^7 + 889x^5 + 1272x^3 + 531x}{64(x^8 + 4x^6 + 10x^4 + 12x^2 + 9)} - \frac{1}{64}\int \frac{1322x^2 + 1137}{x^4 + 2x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 2*x^2 + 3)^3,x, algorithm="maxima")
```

```
[Out] 5*x + 1/64*(406*x^7 + 889*x^5 + 1272*x^3 + 531*x)/(x^8 + 4*x^6 +
10*x^4 + 12*x^2 + 9) - 1/64*integrate((1322*x^2 + 1137)/(x^4 + 2*
x^2 + 3), x)
```

Fricas [A] time = 0.306692, size = 1173, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 2*x^2 + 3)^3,x, algorithm="fricas")
```

```
[Out] 1/1204767744*sqrt(1176531)*4^(3/4)*(4*sqrt(1176531)*4^(1/4)*(1176
531*sqrt(3)*sqrt(2)*(320*x^9 + 1686*x^7 + 4089*x^5 + 5112*x^3 + 3
411*x) - 827621*sqrt(2)*(320*x^9 + 1686*x^7 + 4089*x^5 + 5112*x^3
 + 3411*x))*sqrt((827621*sqrt(3) - 3529593)/(973721762751*sqrt(3)
 - 2418816050762)) + 10534088*4152675581883^(1/4)*(x^8 + 4*x^6 +
10*x^4 + 12*x^2 + 9)*arctan(2*4152675581883^(1/4)*(943*sqrt(3) +
185)/(sqrt(1176531)*4^(1/4)*sqrt(1/1176531)*(1176531*sqrt(3)*sqrt
(2) - 827621*sqrt(2))*sqrt((639880441644253633660639 5*sqrt(3)*x^2
 + 4152675581883^(1/4)*sqrt(1176531)*4^(1/4)*(6888796098644077434
301*sqrt(3)*x - 11341932760469461370531*x)*sqrt((827621*sqrt(3) -
 3529593)/(973721762751*sqrt(3) - 2418816050762)) - 1288896056483
0772608808849*x^2 + 1176531*sqrt(3)*(543870447650128754 5*sqrt(3)
- 10955053938086435979))/(5438704476501287545*sqrt(3) - 109550539
38086435979))*sqrt((827621*sqrt(3) - 3529593)/(973721762751*sqrt(
3) - 2418816050762)) + sqrt(1176531)*4^(1/4)*(1176531*sqrt(3)*sqr
t(2)*x - 827621*sqrt(2)*x)*sqrt((827621*sqrt(3) - 3529593)/(97372
1762751*sqrt(3) - 2418816050762)) + 2*4152675581883^(1/4)*(379*sq
rt(3)*sqrt(2) - 1322*sqrt(2)))) + 10534088*4152675581883^(1/4)*(x
^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*arctan(2*4152675581883^(1/4)*(9
43*sqrt(3) + 185)/(sqrt(1176531)*4^(1/4)*sqrt(1/1176531)*(1176531
*sqrt(3)*sqrt(2) - 827621*sqrt(2))*sqrt((639880441644253633660639
5*sqrt(3)*x^2 - 4152675581883^(1/4)*sqrt(1176531)*4^(1/4)*(688879
6098644077434301*sqrt(3)*x - 11341932760469461370531*x)*sqrt((827
621*sqrt(3) - 3529593)/(973721762751*sqrt(3) - 2418816050762)) -
12888960564830772608808849*x^2 + 1176531*sqrt(3)*(543870447650128
7545*sqrt(3) - 10955053938086435979))/(5438704476501287545*sqrt(3
) - 10955053938086435979))*sqrt((827621*sqrt(3) - 3529593)/(97372
1762751*sqrt(3) - 2418816050762)) + sqrt(1176531)*4^(1/4)*(117653
1*sqrt(3)*sqrt(2)*x - 827621*sqrt(2)*x)*sqrt((827621*sqrt(3) - 35
29593)/(973721762751*sqrt(3) - 2418816050762)) - 2*4152675581883^
(1/4)*(379*sqrt(3)*sqrt(2) - 1322*sqrt(2)))) - 4152675581883^(1/4
)*(1176531*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) -
827621*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*log(127976088
32885072673212790*sqrt(3)*x^2 + 2*4152675581883^(1/4)*sqrt(117653
1)*4^(1/4)*(6888796098644077434301*sqrt(3)*x - 113419327604694613
70531*x)*sqrt((827621*sqrt(3) - 3529593)/(973721762751*sqrt(3) -
2418816050762)) - 25777921129661545217617698*x^2 + 2353062*sqrt(3
)*(5438704476501287545*sqrt(3) - 10955053938086435979)) + 4152675
581883^(1/4)*(1176531*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*
x^2 + 9) - 827621*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*lo
g(12797608832885072673212790*sqrt(3)*x^2 - 2*4152675581883^(1/4)*
sqrt(1176531)*4^(1/4)*(6888796098644077434301*sqrt(3)*x - 1134193
2760469461370531*x)*sqrt((827621*sqrt(3) - 3529593)/(973721762751
*sqrt(3) - 2418816050762)) - 25777921129661545217617698*x^2 + 235
3062*sqrt(3)*(5438704476501287545*sqrt(3) - 10955053938086435979)
))/((1176531*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
- 827621*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt((82762
1*sqrt(3) - 3529593)/(973721762751*sqrt(3) - 2418816050762)))
```

Sympy [A] time = 2.21824, size = 71, normalized size = 0.3

$$5x + \frac{406x^7 + 889x^5 + 1272x^3 + 531x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + \operatorname{RootSum}\left(17179869184t^4 + 216955879424t^2 + 4152675581883, \left(t \mapsto t \log\left(-\frac{31641829376t^3}{1549210136091} - \frac{455309168896t}{1549210136091} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x + (406*x**7 + 889*x**5 + 1272*x**3 + 531*x)/(64*x**8 + 256*x*
*6 + 640*x**4 + 768*x**2 + 576) + RootSum(17179869184*_t**4 + 216
955879424*_t**2 + 4152675581883, Lambda(_t, _t*log(-31641829376*_
t**3/1549210136091 - 455309168896*_t/1549210136091 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6+3\,x^4+x^2+4\right)x^6}{\left(x^4+2\,x^2+3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 2*x^2 + 3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 2*x^2 + 3)^3, x)
```

3.120 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=238

$$\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{x\left(238-59x^2\right)}{64\left(x^4+2x^2+3\right)}-\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}-\frac{1}{256}\sqrt{3\left(32827\sqrt{3}-48835\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{256}\sqrt{3\left(32827\sqrt{3}-48835\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] (-25*x*(3 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(238 - 59*x^2))/(
64*(3 + 2*x^2 + x^4)) - (Sqrt[3*(-48835 + 32827*Sqrt[3])]*ArcTan[
(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqr
t[3*(-48835 + 32827*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*
x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3*(48835 + 32827*Sqrt[3])]
*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sqrt[3*(48
835 + 32827*Sqrt[3])]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^
2])/512
```

Rubi [A] time = 0.725481, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$

$$\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{x\left(238-59x^2\right)}{64\left(x^4+2x^2+3\right)}-\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}-\frac{1}{256}\sqrt{3\left(32827\sqrt{3}-48835\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{256}\sqrt{3\left(32827\sqrt{3}-48835\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (-25*x*(3 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(238 - 59*x^2))/(
64*(3 + 2*x^2 + x^4)) - (Sqrt[3*(-48835 + 32827*Sqrt[3])]*ArcTan[
(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqr
t[3*(-48835 + 32827*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*
x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3*(48835 + 32827*Sqrt[3])]
*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sqrt[3*(48
835 + 32827*Sqrt[3])]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^
2])/512
```

Rubi in Sympy [A] time = 38.5947, size = 350, normalized size = 1.47

$$\frac{x\left(-1087488x^2+4386816\right)}{1179648\left(x^4+2x^2+3\right)}-\frac{x\left(4800x^2+14400\right)}{3072\left(x^4+2x^2+3\right)^2}+\frac{\sqrt{6}\left(1271808+2405376\sqrt{3}\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{14155776\sqrt{-1+\sqrt{3}}}-\frac{\sqrt{6}\left(1271808+2405376\sqrt{3}\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{14155776\sqrt{-1+\sqrt{3}}}-\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(2543616+4810752\sqrt{3}\right)}{2}+2543616\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{7077888\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}-\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(2543616+4810752\sqrt{3}\right)}{2}+2543616\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{7077888\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] x*(-1087488*x**2 + 4386816)/(1179648*(x**4 + 2*x**2 + 3)) - x*(48
00*x**2 + 14400)/(3072*(x**4 + 2*x**2 + 3)**2) + sqrt(6)*(1271808
 + 2405376*sqrt(3))*log(x**2 - sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqr
t(3))/(14155776*sqrt(-1 + sqrt(3))) - sqrt(6)*(1271808 + 2405376*
sqrt(3))*log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(1415
5776*sqrt(-1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(
2543616 + 4810752*sqrt(3))/2 + 2543616*sqrt(2)*sqrt(-1 + sqrt(3))
)*atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(7
077888*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*
sqrt(-1 + sqrt(3))*(2543616 + 4810752*sqrt(3))/2 + 2543616*sqrt(2
)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x + sqrt(-2 + 2*sqrt(3))/2)/s
qrt(1 + sqrt(3)))/(7077888*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3)))
```

Mathematica [C] time = 0.664537, size = 129, normalized size = 0.54

$$\frac{1}{256}\left(\frac{4x\left(-59x^6+120x^4+199x^2+414\right)}{\left(x^4+2x^2+3\right)^2}+\frac{3\left(174+133i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{3\left(174-133i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(414 + 199*x^2 + 120*x^4 - 59*x^6))/(3 + 2*x^2 + x^4)^2 + (
3*(174 + (133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 -
 I*Sqrt[2]] + (3*(174 - (133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt
[2]]])/Sqrt[1 + I*Sqrt[2]])/256
```

Maple [B] time = 0.039, size = 418, normalized size = 1.8

$$\frac{1}{(x^4+2x^2+3)^2}\left(-\frac{59x^7}{64}+\frac{15x^5}{8}+\frac{199x^3}{64}+\frac{207x}{32}\right) - \frac{307\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024} - \frac{399\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024} + \frac{\left(-614+614\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{-798+798\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{23\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{307\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{1024} + \frac{399\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024} + \frac{\left(-614+614\sqrt{3}\right)\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{-798+798\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{23\sqrt{3}}{32\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] (-59/64*x^7+15/8*x^5+199/64*x^3+207/32*x)/(x^4+2*x^2+3)^2-307/102
4*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(
1/2)-399/1024*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2
))^(1/2)+307/512/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(
1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+399/512/(2+2*3^
(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2
))*(-2+2*3^(1/2))-23/32/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(
1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+307/1024*ln(x^2+3^(1/2)
-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+399/1024*ln
(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+307/512
/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/
2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+399/512/(2+2*3^(1/2))^(1/2)*arc
tan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2)
)-23/32/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+
2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{59x^7-120x^5-199x^3-414x}{64\left(x^8+4x^6+10x^4+12x^2+9\right)}+\frac{3}{64}\int\frac{87x^2-46}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 2*x^2 + 3)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^8 + 4*x^6 + 10*x^4
+ 12*x^2 + 9) + 3/64*integrate((87*x^2 - 46)/(x^4 + 2*x^2 + 3), x
)
```

Fricas [A] time = 0.302172, size = 1165, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 2*x^2 + 3)^3,x, algorithm="fricas")
```

```
[Out] -1/33614848*sqrt(32827)*4^(3/4)*(4*sqrt(32827)*4^(1/4)*(32827*sqr
t(3)*sqrt(2)*(59*x^7 - 120*x^5 - 199*x^3 - 414*x) + 48835*sqrt(2)
*(59*x^7 - 120*x^5 - 199*x^3 - 414*x))*sqrt((48835*sqrt(3) + 9848
1)/(1603106545*sqrt(3) + 2808846506)) + 164728*29095522083^(1/4)*
(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*arctan(2*29095522083^(1/4)*(3
07*sqrt(3) + 399)/(3*sqrt(32827)*4^(1/4)*sqrt(1/98481)*(32827*sqr
t(3)*sqrt(2) + 48835*sqrt(2))*sqrt((2905638128015672305*sqrt(3)*
x^2 + 29095522083^(1/4)*sqrt(32827)*4^(1/4)*(58070954450355287*sq
rt(3)*x + 100535022105329061*x)*sqrt((48835*sqrt(3) + 98481)/(160
3106545*sqrt(3) + 2808846506)) + 5037117386595666389*x^2 + 98481
*sqrt(3)*(295045554778655*sqrt(3) + 511481137132611))/(2950455547
78655*sqrt(3) + 511481137132611))*sqrt((48835*sqrt(3) + 98481)/(1
603106545*sqrt(3) + 2808846506)) + 3*sqrt(32827)*4^(1/4)*(32827*s
qrt(3)*sqrt(2)*x + 48835*sqrt(2)*x)*sqrt((48835*sqrt(3) + 98481)/
(1603106545*sqrt(3) + 2808846506)) + 2*29095522083^(1/4)*(46*sqrt
(3)*sqrt(2) + 261*sqrt(2)))) + 164728*29095522083^(1/4)*(x^8 + 4*
x^6 + 10*x^4 + 12*x^2 + 9)*arctan(2*29095522083^(1/4)*(307*sqrt(3
) + 399)/(3*sqrt(32827)*4^(1/4)*sqrt(1/98481)*(32827*sqrt(3)*sqrt
(2) + 48835*sqrt(2))*sqrt((2905638128015672305*sqrt(3)*x^2 - 290
95522083^(1/4)*sqrt(32827)*4^(1/4)*(58070954450355287*sqrt(3)*x +
 100535022105329061*x)*sqrt((48835*sqrt(3) + 98481)/(1603106545*s
qrt(3) + 2808846506)) + 5037117386595666389*x^2 + 98481*sqrt(3)*
(295045554778655*sqrt(3) + 511481137132611))/(295045554778655*sqr
t(3) + 511481137132611))*sqrt((48835*sqrt(3) + 98481)/(1603106545
*sqrt(3) + 2808846506)) + 3*sqrt(32827)*4^(1/4)*(32827*sqrt(3)*sq
rt(2)*x + 48835*sqrt(2)*x)*sqrt((48835*sqrt(3) + 98481)/(16031065
45*sqrt(3) + 2808846506)) - 2*29095522083^(1/4)*(46*sqrt(3)*sqrt(
2) + 261*sqrt(2)))) + 29095522083^(1/4)*(32827*sqrt(3)*sqrt(2)*(x
^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 48835*sqrt(2)*(x^8 + 4*x^6 +
10*x^4 + 12*x^2 + 9))*log(58112762560313446110*sqrt(3)*x^2 + 2*29
095522083^(1/4)*sqrt(32827)*4^(1/4)*(58070954450355287*sqrt(3)*x
+ 100535022105329061*x)*sqrt((48835*sqrt(3) + 98481)/(1603106545*
sqrt(3) + 2808846506)) + 100742347731913327782*x^2 + 196962*sqrt(
3)*(295045554778655*sqrt(3) + 511481137132611)) - 29095522083^(1/
4)*(32827*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 4
8835*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*log(58112762560
313446110*sqrt(3)*x^2 - 2*29095522083^(1/4)*sqrt(32827)*4^(1/4)*(
58070954450355287*sqrt(3)*x + 100535022105329061*x)*sqrt((48835*s
qrt(3) + 98481)/(1603106545*sqrt(3) + 2808846506)) + 100742347731
913327782*x^2 + 196962*sqrt(3)*(295045554778655*sqrt(3) + 5114811
37132611)))/((32827*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^
2 + 9) + 48835*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(
(48835*sqrt(3) + 98481)/(1603106545*sqrt(3) + 2808846506)))
```

Sympy [A] time = 2.12792, size = 68, normalized size = 0.29

$$-\frac{59x^7-120x^5-199x^3-414x}{64x^8+256x^6+640x^4+768x^2+576}$$
$$+\mathrm{RootSum}\left(17179869184t^4-38405406720t^2+29095522083,\left(t\mapsto t\log\left(\frac{10301210624t^3}{6083466813}-\frac{4322999552t}{2027822271}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] -(59*x**7 - 120*x**5 - 199*x**3 - 414*x)/(64*x**8 + 256*x**6 + 64
0*x**4 + 768*x**2 + 576) + RootSum(17179869184*_t**4 - 3840540672
```

```
0*_t**2 + 29095522083, Lambda(_t, _t*log(10301210624*_t**3/608346
6813 - 4322999552*_t/2027822271 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6+3\,x^4+x^2+4\right)x^4}{\left(x^4+2\,x^2+3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 2*x^2 + 3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 2*x^2 + 3)^3, x)
```

3.121 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536}+\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536}+\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}-\frac{x\left(88x^2+353\right)}{192\left(x^4+2x^2+3\right)}-\frac{11}{768}\sqrt{\frac{1}{3}\left(1089\sqrt{3}-1825\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{11}{768}\sqrt{\frac{1}{3}\left(1089\sqrt{3}-1825\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] (25*x*(1 + x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(353 + 88*x^2))/(1
92*(3 + 2*x^2 + x^4)) - (11*Sqrt[(-1825 + 1089*Sqrt[3])/3]*ArcTan
[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 + (11
*Sqrt[(-1825 + 1089*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] +
2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 - (11*Sqrt[(1825 + 1089*Sqrt[3])
/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/1536 + (11*Sqr
t[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
 + x^2])/1536
```

Rubi [A] time = 0.763148, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$-\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536}+\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536}+\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}-\frac{x\left(88x^2+353\right)}{192\left(x^4+2x^2+3\right)}-\frac{11}{768}\sqrt{\frac{1}{3}\left(1089\sqrt{3}-1825\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{11}{768}\sqrt{\frac{1}{3}\left(1089\sqrt{3}-1825\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```


```
[Out] (25*x*(1 + x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(353 + 88*x^2))/(1
92*(3 + 2*x^2 + x^4)) - (11*Sqrt[(-1825 + 1089*Sqrt[3])/3]*ArcTan
[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 + (11
*Sqrt[(-1825 + 1089*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] +
2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 - (11*Sqrt[(1825 + 1089*Sqrt[3])
/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/1536 + (11*Sqr
t[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
```

```
+ x^2])/1536
```

Rubi in Sympy [A] time = 37.2087, size = 350, normalized size = 1.42

$$\frac{x\left(2400x^2+2400\right)}{1536\left(x^4+2x^2+3\right)^2}-\frac{x\left(135168x^2+542208\right)}{294912\left(x^4+2x^2+3\right)} - \frac{\sqrt{6}\left(67584\sqrt{3}+194304\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{3538944\sqrt{-1+\sqrt{3}}} + \frac{\sqrt{6}\left(67584\sqrt{3}+194304\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{3538944\sqrt{-1+\sqrt{3}}} + \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(135168\sqrt{3}+388608\right)}{2}+388608\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{1769472\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}} + \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(135168\sqrt{3}+388608\right)}{2}+388608\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{1769472\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] x*(2400*x**2 + 2400)/(1536*(x**4 + 2*x**2 + 3)**2) - x*(135168*x*
*2 + 542208)/(294912*(x**4 + 2*x**2 + 3)) - sqrt(6)*(67584*sqrt(3
) + 194304)*log(x**2 - sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(3
538944*sqrt(-1 + sqrt(3))) + sqrt(6)*(67584*sqrt(3) + 194304)*log
(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(3538944*sqrt(-1
+ sqrt(3))) + sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(135168*sqrt(3
) + 388608)/2 + 388608*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(
x - sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(1769472*sqrt(-1 +
 sqrt(3))*sqrt(1 + sqrt(3))) + sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3
))*(135168*sqrt(3) + 388608)/2 + 388608*sqrt(2)*sqrt(-1 + sqrt(3)
))*atan(sqrt(2)*(x + sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(
1769472*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3)))
```

Mathematica [C] time = 0.606633, size = 133, normalized size = 0.54

$$\frac{1}{768}\left(-\frac{4x\left(88x^6+529x^4+670x^2+759\right)}{\left(x^4+2x^2+3\right)^2} - \frac{11i\left(31\sqrt{2}-16i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{11i\left(31\sqrt{2}+16i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((-4*x*(759 + 670*x^2 + 529*x^4 + 88*x^6))/(3 + 2*x^2 + x^4)^2 -
((11*I)*(-16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[
1 - I*Sqrt[2]] + ((11*I)*(16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*
Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/768
```

Maple [B] time = 0.038, size = 418, normalized size = 1.7

$$\frac{1}{(x^4+2x^2+3)^2}\left(-\frac{11x^7}{24}-\frac{529x^5}{192}-\frac{335x^3}{96}-\frac{253x}{64}\right)+\frac{517\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{9216}+\frac{341\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{3072}-\frac{\left(-1034+1034\sqrt{3}\right)\sqrt{3}}{4608\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{-682+682\sqrt{3}}{1536\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{253\sqrt{3}}{576\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{517\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{9216}-\frac{341\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{3072}-\frac{\left(-1034+1034\sqrt{3}\right)\sqrt{3}}{4608\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{-682+682\sqrt{3}}{1536\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{253\sqrt{3}}{576\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] (-11/24*x^7-529/192*x^5-335/96*x^3-253/64*x)/(x^4+2*x^2+3)^2+517/
9216*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*
3^(1/2)+341/3072*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(
1/2))^(1/2)-517/4608/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2
))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-341/1536/(2
+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))
^(1/2))*(-2+2*3^(1/2))+253/576/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-
2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-517/9216*ln(x^2+
3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-341/
3072*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-
517/4608/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-341/1536/(2+2*3^(1/2))^
(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+
2*3^(1/2))+253/576/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))
^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{88x^7+529x^5+670x^3+759x}{192\left(x^8+4x^6+10x^4+12x^2+9\right)}-\frac{11}{192}\int\frac{8x^2-23}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 2*x^2 + 3)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(88*x^7 + 529*x^5 + 670*x^3 + 759*x)/(x^8 + 4*x^6 + 10*x^4
 + 12*x^2 + 9) - 11/192*integrate((8*x^2 - 23)/(x^4 + 2*x^2 + 3),
 x)
```

Fricas [A] time = 0.317195, size = 1079, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 2*x^2 + 3)^3,x, algorithm="fricas")
```

```
[Out] -1/9216*12^(3/4)*(29656*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
9)*arctan(6*(31*sqrt(3) + 47)/(12^(1/4)*sqrt(1/3)*(1825*sqrt(3)*s
qrt(2) + 3267*sqrt(2))*sqrt(sqrt(3)*(12^(1/4)*(470973814459*sqrt(
3)*x + 815752391079*x)*sqrt((1825*sqrt(3) + 3267)/(1987425*sqrt(3
) + 3444194)) + sqrt(3)*(12778571525*sqrt(3)*x^2 + 22133333673*x^
2) + 38335714575*sqrt(3) + 66400001019)/(12778571525*sqrt(3) + 22
133333673))*sqrt((1825*sqrt(3) + 3267)/(1987425*sqrt(3) + 3444194
)) + 12^(1/4)*(1825*sqrt(3)*sqrt(2)*x + 3267*sqrt(2)*x)*sqrt((182
5*sqrt(3) + 3267)/(1987425*sqrt(3) + 3444194)) + 48*sqrt(3)*sqrt(
2) + 138*sqrt(2))) + 29656*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2
 + 9)*arctan(6*(31*sqrt(3) + 47)/(12^(1/4)*sqrt(1/3)*(1825*sqrt(3
)*sqrt(2) + 3267*sqrt(2))*sqrt(-sqrt(3)*(12^(1/4)*(470973814459*s
qrt(3)*x + 815752391079*x)*sqrt((1825*sqrt(3) + 3267)/(1987425*sq
rt(3) + 3444194)) - sqrt(3)*(12778571525*sqrt(3)*x^2 + 2213333367
3*x^2) - 38335714575*sqrt(3) - 66400001019)/(12778571525*sqrt(3)
+ 22133333673))*sqrt((1825*sqrt(3) + 3267)/(1987425*sqrt(3) + 344
4194)) + 12^(1/4)*(1825*sqrt(3)*sqrt(2)*x + 3267*sqrt(2)*x)*sqrt(
(1825*sqrt(3) + 3267)/(1987425*sqrt(3) + 3444194)) - 48*sqrt(3)*s
qrt(2) - 138*sqrt(2))) - 11*(1825*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 +
10*x^4 + 12*x^2 + 9) + 3267*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^
2 + 9))*log(66*12^(1/4)*(470973814459*sqrt(3)*x + 815752391079*x)
*sqrt((1825*sqrt(3) + 3267)/(1987425*sqrt(3) + 3444194)) + 66*sqr
t(3)*(12778571525*sqrt(3)*x^2 + 22133333673*x^2) + 2530157161950*
sqrt(3) + 4382400067254) + 11*(1825*sqrt(3)*sqrt(2)*(x^8 + 4*x^6
+ 10*x^4 + 12*x^2 + 9) + 3267*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*
x^2 + 9))*log(-66*12^(1/4)*(470973814459*sqrt(3)*x + 815752391079
*x)*sqrt((1825*sqrt(3) + 3267)/(1987425*sqrt(3) + 3444194)) + 66*
sqrt(3)*(12778571525*sqrt(3)*x^2 + 22133333673*x^2) + 25301571619
50*sqrt(3) + 4382400067254) + 4*12^(1/4)*(1825*sqrt(3)*sqrt(2)*(8
8*x^7 + 529*x^5 + 670*x^3 + 759*x) + 3267*sqrt(2)*(88*x^7 + 529*x
^5 + 670*x^3 + 759*x))*sqrt((1825*sqrt(3) + 3267)/(1987425*sqrt(3
) + 3444194)))/((1825*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*
x^2 + 9) + 3267*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt
((1825*sqrt(3) + 3267)/(1987425*sqrt(3) + 3444194)))
```

Sympy [A] time = 2.16307, size = 68, normalized size = 0.28

$$-\frac{88x^7 + 529x^5 + 670x^3 + 759x}{192x^8 + 768x^6 + 1920x^4 + 2304x^2 + 1728} + \operatorname{RootSum}\left(463856467968t^4 - 57887948800t^2 + 1929229929, \left(t \mapsto t\log\left(\frac{14193524736t^3}{54274187} - \frac{17989888t}{1345641} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] -(88*x**7 + 529*x**5 + 670*x**3 + 759*x)/(192*x**8 + 768*x**6 + 1
920*x**4 + 2304*x**2 + 1728) + RootSum(463856467968*_t**4 - 57887
948800*_t**2 + 1929229929, Lambda(_t, _t*log(14193524736*_t**3/54
274187 - 17989888*_t/1345641 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6+3\,x^4+x^2+4\right)x^2}{\left(x^4+2\,x^2+3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 2*x^2 + 3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 2*x^2 + 3)^3, x)
```

3.122 $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3} dx$

Optimal. Leaf size=248

$$\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25x\left(1-x^2\right)}{48\left(x^4+2x^2+3\right)^2}+\frac{x\left(51x^2+64\right)}{192\left(x^4+2x^2+3\right)}-\frac{1}{256}\sqrt{\frac{1}{3}\left(1019\sqrt{3}-1291\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{256}\sqrt{\frac{1}{3}\left(1019\sqrt{3}-1291\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] (25*x*(1 - x^2))/(48*(3 + 2*x^2 + x^4)^2) + (x*(64 + 51*x^2))/(19
2*(3 + 2*x^2 + x^4)) - (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sq
rt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(
-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sq
rt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(1291 + 1019*Sqrt[3])/3]*Log[Sq
rt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sqrt[(1291 + 1019
*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rubi [A] time = 0.704989, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25x\left(1-x^2\right)}{48\left(x^4+2x^2+3\right)^2}+\frac{x\left(51x^2+64\right)}{192\left(x^4+2x^2+3\right)}-\frac{1}{256}\sqrt{\frac{1}{3}\left(1019\sqrt{3}-1291\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{256}\sqrt{\frac{1}{3}\left(1019\sqrt{3}-1291\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (25*x*(1 - x^2))/(48*(3 + 2*x^2 + x^4)^2) + (x*(64 + 51*x^2))/(19
2*(3 + 2*x^2 + x^4)) - (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sq
rt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(
-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sq
rt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(1291 + 1019*Sqrt[3])/3]*Log[Sq
rt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sqrt[(1291 + 1019
*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rubi in Sympy [A] time = 31.1256, size = 350, normalized size = 1.41

$$\frac{x(-400x^2+400)}{768(x^4+2x^2+3)^2}+\frac{x(9792x^2+12288)}{36864(x^4+2x^2+3)}+\frac{\sqrt{6}\left(1152+4896\sqrt{3}\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{442368\sqrt{-1+\sqrt{3}}}-\frac{\sqrt{6}\left(1152+4896\sqrt{3}\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{442368\sqrt{-1+\sqrt{3}}}-\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(2304+9792\sqrt{3}\right)}{2}+2304\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{221184\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}-\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(2304+9792\sqrt{3}\right)}{2}+2304\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{221184\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] x*(-400*x**2 + 400)/(768*(x**4 + 2*x**2 + 3)**2) + x*(9792*x**2 +
 12288)/(36864*(x**4 + 2*x**2 + 3)) + sqrt(6)*(1152 + 4896*sqrt(3
))*log(x**2 - sqrt(2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(442368*sqr
t(-1 + sqrt(3))) - sqrt(6)*(1152 + 4896*sqrt(3))*log(x**2 + sqrt(
2)*x*sqrt(-1 + sqrt(3)) + sqrt(3))/(442368*sqrt(-1 + sqrt(3))) -
sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(2304 + 9792*sqrt(3))/2 + 23
04*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt
(3))/2)/sqrt(1 + sqrt(3)))/(221184*sqrt(-1 + sqrt(3))*sqrt(1 + sq
rt(3))) - sqrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(2304 + 9792*sqrt(
3))/2 + 2304*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x + sqrt(-
2 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(221184*sqrt(-1 + sqrt(3))*s
qrt(1 + sqrt(3)))
```

Mathematica [C] time = 0.615858, size = 129, normalized size = 0.52

$$\frac{1}{768}\left(\frac{4x\left(51x^6+166x^4+181x^2+292\right)}{(x^4+2x^2+3)^2}+\frac{3\left(34+21i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{3\left(34-21i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(292 + 181*x^2 + 166*x^4 + 51*x^6))/(3 + 2*x^2 + x^4)^2 + (
3*(34 + (21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I
*Sqrt[2]] + (3*(34 - (21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]
])/Sqrt[1 + I*Sqrt[2]])/768
```

Maple [B] time = 0.037, size = 418, normalized size = 1.7

$$\frac{1}{(x^4+2x^2+3)^2}\left(\frac{17x^7}{64}+\frac{83x^5}{96}+\frac{181x^3}{192}+\frac{73x}{48}\right) - \frac{55\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{3072} - \frac{21\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024} + \frac{\left(-110+110\sqrt{3}\right)\sqrt{3}}{1536\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{-42+42\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{\sqrt{3}}{48\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{55\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{3072} + \frac{21\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024} + \frac{\left(-110+110\sqrt{3}\right)\sqrt{3}}{1536\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) + \frac{-42+42\sqrt{3}}{512\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right) - \frac{\sqrt{3}}{48\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] (17/64*x^7+83/96*x^5+181/192*x^3+73/48*x)/(x^4+2*x^2+3)^2-55/3072
*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1
/2)-21/1024*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))
^(1/2)+55/1536/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/
2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+21/512/(2+2*3^(1/
2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*
(-2+2*3^(1/2))-1/48/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2)
)^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+55/3072*ln(x^2+3^(1/2)-x*(-
2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+21/1024*ln(x^2+3
^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+55/1536/(2+2*
3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1
/2))*(-2+2*3^(1/2))*3^(1/2)+21/512/(2+2*3^(1/2))^(1/2)*arctan((2*
x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-1/48/
(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2
))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{51x^7+166x^5+181x^3+292x}{192\,(x^8+4x^6+10x^4+12x^2+9)}+\frac{1}{64}\int\frac{17x^2-4}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 2*x^2 + 3)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^8 + 4*x^6 + 10*x^4
+ 12*x^2 + 9) + 1/64*integrate((17*x^2 - 4)/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.315774, size = 1177, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 2*x^2 + 3)^3,x, algorithm="fricas")
```

```
[Out] -1/9391104*sqrt(1019)*12^(3/4)*(20424*1038361^(1/4)*sqrt(3)*(x^8
+ 4*x^6 + 10*x^4 + 12*x^2 + 9)*arctan(6*1038361^(1/4)*(21*sqrt(3)
 + 55)/(sqrt(1019)*12^(1/4)*sqrt(1/3057)*(1291*sqrt(3)*sqrt(2) +
3057*sqrt(2))*sqrt(sqrt(3)*(1038361^(1/4)*sqrt(1019)*12^(1/4)*(23
9300294807*sqrt(3)*x + 412134121929*x)*sqrt((1291*sqrt(3) + 3057)
/(1315529*sqrt(3) + 2390882)) + 1019*sqrt(3)*(7108200815*sqrt(3)*
x^2 + 12403970091*x^2) + 21729769891455*sqrt(3) + 37918936568187)
/(7108200815*sqrt(3) + 12403970091))*sqrt((1291*sqrt(3) + 3057)/(
1315529*sqrt(3) + 2390882)) + sqrt(1019)*12^(1/4)*(1291*sqrt(3)*s
qrt(2)*x + 3057*sqrt(2)*x)*sqrt((1291*sqrt(3) + 3057)/(1315529*sq
rt(3) + 2390882)) + 6*1038361^(1/4)*(17*sqrt(3)*sqrt(2) + 4*sqrt(
2)))) + 20424*1038361^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^
2 + 9)*arctan(6*1038361^(1/4)*(21*sqrt(3) + 55)/(sqrt(1019)*12^(1
/4)*sqrt(1/3057)*(1291*sqrt(3)*sqrt(2) + 3057*sqrt(2))*sqrt(-sqrt
(3)*(1038361^(1/4)*sqrt(1019)*12^(1/4)*(239300294807*sqrt(3)*x +
412134121929*x)*sqrt((1291*sqrt(3) + 3057)/(1315529*sqrt(3) + 239
0882)) - 1019*sqrt(3)*(7108200815*sqrt(3)*x^2 + 12403970091*x^2)
- 21729769891455*sqrt(3) - 37918936568187)/(7108200815*sqrt(3) +
12403970091))*sqrt((1291*sqrt(3) + 3057)/(1315529*sqrt(3) + 23908
82)) + sqrt(1019)*12^(1/4)*(1291*sqrt(3)*sqrt(2)*x + 3057*sqrt(2)
*x)*sqrt((1291*sqrt(3) + 3057)/(1315529*sqrt(3) + 2390882)) - 6*1
038361^(1/4)*(17*sqrt(3)*sqrt(2) + 4*sqrt(2)))) - 4*sqrt(1019)*12
^(1/4)*(1291*sqrt(3)*sqrt(2)*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)
 + 3057*sqrt(2)*(51*x^7 + 166*x^5 + 181*x^3 + 292*x))*sqrt((1291*
sqrt(3) + 3057)/(1315529*sqrt(3) + 2390882)) + 3*1038361^(1/4)*(1
291*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 3057*sq
rt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*log(2*1038361^(1/4)*sq
rt(1019)*12^(1/4)*(239300294807*sqrt(3)*x + 412134121929*x)*sqrt(
(1291*sqrt(3) + 3057)/(1315529*sqrt(3) + 2390882)) + 2038*sqrt(3)
*(7108200815*sqrt(3)*x^2 + 12403970091*x^2) + 43459539782910*sqrt
(3) + 75837873136374) - 3*1038361^(1/4)*(1291*sqrt(3)*sqrt(2)*(x^
8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 3057*sqrt(2)*(x^8 + 4*x^6 + 10
*x^4 + 12*x^2 + 9))*log(-2*1038361^(1/4)*sqrt(1019)*12^(1/4)*(239
300294807*sqrt(3)*x + 412134121929*x)*sqrt((1291*sqrt(3) + 3057)/
(1315529*sqrt(3) + 2390882)) + 2038*sqrt(3)*(7108200815*sqrt(3)*x
^2 + 12403970091*x^2) + 43459539782910*sqrt(3) + 75837873136374))
/((1291*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 305
7*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt((1291*sqrt(3)
 + 3057)/(1315529*sqrt(3) + 2390882)))
```

Sympy [A] time = 2.09213, size = 68, normalized size = 0.27

$$\frac{51x^7 + 166x^5 + 181x^3 + 292x}{192x^8 + 768x^6 + 1920x^4 + 2304x^2 + 1728} + \operatorname{RootSum}\left(51539607552t^4 - 338427904t^2 + 1038361, \left(t \mapsto t \log\left(\frac{5536481280t^3}{867169} - \frac{19920128t}{867169} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (51*x**7 + 166*x**5 + 181*x**3 + 292*x)/(192*x**8 + 768*x**6 + 19
20*x**4 + 2304*x**2 + 1728) + RootSum(51539607552*_t**4 - 3384279
04*_t**2 + 1038361, Lambda(_t, _t*log(5536481280*_t**3/867169 - 1
9920128*_t/867169 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 2*x^2 + 3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 2*x^2 + 3)^3, x)
```

3.123 $\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=253

$$-\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}+\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}-\frac{25x\left(x^2+5\right)}{144\left(x^4+2x^2+3\right)^2}-\frac{x\left(242x^2+325\right)}{1728\left(x^4+2x^2+3\right)}-\frac{4}{27x}+\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304}-\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304}$$

```
[Out] -4/(27*x) - (25*x*(5 + x^2))/(144*(3 + 2*x^2 + x^4)^2) - (x*(325
+ 242*x^2))/(1728*(3 + 2*x^2 + x^4)) + (Sqrt[(59711 + 55161*Sqrt[
3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])
]])/2304 - (Sqrt[(59711 + 55161*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 +
Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2304 - (Sqrt[(-59711 + 5
5161*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4
608 + (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1
+ Sqrt[3])]*x + x^2])/4608
```

Rubi [A] time = 0.832625, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$

$$-\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}+\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}-\frac{25x\left(x^2+5\right)}{144\left(x^4+2x^2+3\right)^2}-\frac{x\left(242x^2+325\right)}{1728\left(x^4+2x^2+3\right)}-\frac{4}{27x}+\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304}-\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```



```
[Out] -4/(27*x) - (25*x*(5 + x^2))/(144*(3 + 2*x^2 + x^4)^2) - (x*(325
+ 242*x^2))/(1728*(3 + 2*x^2 + x^4)) + (Sqrt[(59711 + 55161*Sqrt[
3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])
]])/2304 - (Sqrt[(59711 + 55161*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 +
Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2304 - (Sqrt[(-59711 + 5
5161*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4
608 + (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1
+ Sqrt[3])]*x + x^2])/4608
```

Rubi in Sympy [A] time = 38.1595, size = 333, normalized size = 1.32

$$-\frac{\sqrt{6}\left(-36864\sqrt{3}+225792\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{1327104\sqrt{-1+\sqrt{3}}}+\frac{\sqrt{6}\left(-36864\sqrt{3}+225792\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{1327104\sqrt{-1+\sqrt{3}}}+\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-73728\sqrt{3}+451584\right)}{2}+451584\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{663552\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}+\frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-73728\sqrt{3}+451584\right)}{2}+451584\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{663552\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}}+\frac{33792x^2+12288}{36864x\left(x^4+2x^2+3\right)}+\frac{2}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**3,x)
```

```
[Out] -sqrt(6)*(-36864*sqrt(3) + 225792)*log(x**2 - sqrt(2)*x*sqrt(-1 +
 sqrt(3)) + sqrt(3))/(1327104*sqrt(-1 + sqrt(3))) + sqrt(6)*(-368
64*sqrt(3) + 225792)*log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(3)) + sq
rt(3))/(1327104*sqrt(-1 + sqrt(3))) + sqrt(3)*(-sqrt(2)*sqrt(-1 +
 sqrt(3))*(-73728*sqrt(3) + 451584)/2 + 451584*sqrt(2)*sqrt(-1 +
sqrt(3)))*atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt(3))/2)/sqrt(1 + sqrt
(3)))/(663552*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) + sqrt(3)*(-s
qrt(2)*sqrt(-1 + sqrt(3))*(-73728*sqrt(3) + 451584)/2 + 451584*sq
rt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x + sqrt(-2 + 2*sqrt(3))/
2)/sqrt(1 + sqrt(3)))/(663552*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3)
)) + (33792*x**2 + 12288)/(36864*x*(x**4 + 2*x**2 + 3)) + 2/(3*x)
```

Mathematica [C] time = 0.785148, size = 140, normalized size = 0.55

$$\frac{-\frac{12\left(166x^8+611x^6+1412x^4+1849x^2+768\right)}{x\left(x^4+2x^2+3\right)^2}+\frac{3i\left(7\sqrt{2}+332i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}-\frac{3i\left(7\sqrt{2}-332i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{6912}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] ((-12*(768 + 1849*x^2 + 1412*x^4 + 611*x^6 + 166*x^8))/(x*(3 + 2*
x^2 + x^4)^2) + ((3*I)*(332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sq
rt[2]]])/Sqrt[1 - I*Sqrt[2]] - ((3*I)*(-332*I + 7*Sqrt[2])*ArcTan
[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/6912
```

Maple [B] time = 0.037, size = 424, normalized size = 1.7

$$
\begin{aligned}
&-\frac{4}{27x}-\frac{1}{27\left(x^{4}+2x^{2}+3\right)^{2}}\left(\frac{121x^{7}}{32}+\frac{809x^{5}}{64}+\frac{419x^{3}}{16}+\frac{2475x}{64}\right)\\
&+\frac{325\ln\left(x^{2}+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\,\sqrt{3}}{27648}\\
&-\frac{7\ln\left(x^{2}+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{9216}\\
&-\frac{\left(-650+650\sqrt{3}\right)\sqrt{3}}{13824\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\\
&+\frac{-14+14\sqrt{3}}{4608\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{173\sqrt{3}}{1728\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\\
&-\frac{325\ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\,\sqrt{3}}{27648}\\
&+\frac{7\ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{9216}\\
&-\frac{\left(-650+650\sqrt{3}\right)\sqrt{3}}{13824\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\\
&+\frac{-14+14\sqrt{3}}{4608\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)-\frac{173\sqrt{3}}{1728\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x)
```

```
[Out] -4/27/x-1/27*(121/32*x^7+809/64*x^5+419/16*x^3+2475/64*x)/(x^4+2*
x^2+3)^2+325/27648*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3
^(1/2))^(1/2)*3^(1/2)-7/9216*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2
))*(-2+2*3^(1/2))^(1/2)-325/13824/(2+2*3^(1/2))^(1/2)*arctan((2*x
+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2
)+7/4608/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-173/1728/(2+2*3^(1/2))^(1/2)*ar
ctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-325/
27648*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)
*3^(1/2)+7/9216*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1
/2))^(1/2)-325/13824/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2
))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+7/4608/(2+2
*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(
1/2))*(-2+2*3^(1/2))-173/1728/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2
+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{166x^{8}+611x^{6}+1412x^{4}+1849x^{2}+768}{576\left(x^{9}+4x^{7}+10x^{5}+12x^{3}+9x\right)}-\frac{1}{576}\int\frac{166x^{2}+173}{x^{4}+2x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^3*x^2),x, algorithm="maxima
```

```
[Out] -1/576*(166*x^8 + 611*x^6 + 1412*x^4 + 1849*x^2 + 768)/(x^9 + 4*x
^7 + 10*x^5 + 12*x^3 + 9*x) - 1/576*integrate((166*x^2 + 173)/(x^
4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.296996, size = 1189, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^3*x^2),x, algorithm="fricas
```

```
[Out] -1/6276096*sqrt(681)*4^(3/4)*(4*sqrt(681)*4^(1/4)*(55161*sqrt(3)*
sqrt(2)*(166*x^8 + 611*x^6 + 1412*x^4 + 1849*x^2 + 768) - 59711*s
qrt(2)*(166*x^8 + 611*x^6 + 1412*x^4 + 1849*x^2 + 768))*sqrt((597
11*sqrt(3) - 165483)/(3293718471*sqrt(3) - 6346805642)) - 421912*
154587^(1/4)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)*arctan(2*15458
7^(1/4)*(325*sqrt(3) - 21)/(sqrt(681)*4^(1/4)*sqrt(1/681)*(55161*
sqrt(3)*sqrt(2) - 59711*sqrt(2))*sqrt((629263425815075355*sqrt(3)
*x^2 + 154587^(1/4)*sqrt(681)*4^(1/4)*(432147084979531229*sqrt(3)
*x - 743938470505411707*x)*sqrt((59711*sqrt(3) - 165483)/(3293718
471*sqrt(3) - 6346805642)) - 1117045736175016449*x^2 + 681*sqrt(3
)*(924028525425955*sqrt(3) - 1640302108920729))/(924028525425955*
sqrt(3) - 1640302108920729))*sqrt((59711*sqrt(3) - 165483)/(32937
18471*sqrt(3) - 6346805642)) + sqrt(681)*4^(1/4)*(55161*sqrt(3)*s
qrt(2)*x - 59711*sqrt(2)*x)*sqrt((59711*sqrt(3) - 165483)/(329371
8471*sqrt(3) - 6346805642)) + 2*154587^(1/4)*(173*sqrt(3)*sqrt(2)
 - 498*sqrt(2)))) - 421912*154587^(1/4)*(x^9 + 4*x^7 + 10*x^5 + 1
2*x^3 + 9*x)*arctan(2*154587^(1/4)*(325*sqrt(3) - 21)/(sqrt(681)*
4^(1/4)*sqrt(1/681)*(55161*sqrt(3)*sqrt(2) - 59711*sqrt(2))*sqrt(
(629263425815075355*sqrt(3)*x^2 - 154587^(1/4)*sqrt(681)*4^(1/4)*
(432147084979531229*sqrt(3)*x - 743938470505411707*x)*sqrt((59711
*sqrt(3) - 165483)/(3293718471*sqrt(3) - 6346805642)) - 111704573
6175016449*x^2 + 681*sqrt(3)*(924028525425955*sqrt(3) - 164030210
8920729))/(924028525425955*sqrt(3) - 1640302108920729))*sqrt((597
11*sqrt(3) - 165483)/(3293718471*sqrt(3) - 6346805642)) + sqrt(68
1)*4^(1/4)*(55161*sqrt(3)*sqrt(2)*x - 59711*sqrt(2)*x)*sqrt((5971
1*sqrt(3) - 165483)/(3293718471*sqrt(3) - 6346805642)) - 2*154587
^(1/4)*(173*sqrt(3)*sqrt(2) - 498*sqrt(2)))) + 154587^(1/4)*(5516
1*sqrt(3)*sqrt(2)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x) - 59711*s
qrt(2)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x))*log(113267416646713
56390*sqrt(3)*x^2 + 18*154587^(1/4)*sqrt(681)*4^(1/4)*(4321470849
79531229*sqrt(3)*x - 743938470505411707*x)*sqrt((59711*sqrt(3) -
165483)/(3293718471*sqrt(3) - 6346805642)) - 20106823251150296082
*x^2 + 12258*sqrt(3)*(924028525425955*sqrt(3) - 1640302108920729)
) - 154587^(1/4)*(55161*sqrt(3)*sqrt(2)*(x^9 + 4*x^7 + 10*x^5 + 1
2*x^3 + 9*x) - 59711*sqrt(2)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x
))*log(11326741664671356390*sqrt(3)*x^2 - 18*154587^(1/4)*sqrt(68
1)*4^(1/4)*(432147084979531229*sqrt(3)*x - 743938470505411707*x)*
sqrt((59711*sqrt(3) - 165483)/(3293718471*sqrt(3) - 6346805642))
- 20106823251150296082*x^2 + 12258*sqrt(3)*(924028525425955*sqrt(
3) - 1640302108920729)))/((55161*sqrt(3)*sqrt(2)*(x^9 + 4*x^7 + 1
0*x^5 + 12*x^3 + 9*x) - 59711*sqrt(2)*(x^9 + 4*x^7 + 10*x^5 + 12*
x^3 + 9*x))*sqrt((59711*sqrt(3) - 165483)/(3293718471*sqrt(3) - 6
346805642)))
```

Sympy [A] time = 2.26729, size = 73, normalized size = 0.29

$$-\frac{166x^8+611x^6+1412x^4+1849x^2+768}{576x^9+2304x^7+5760x^5+6912x^3+5184x} + \text{RootSum}\left(4174708211712t^4+15652880384t^2+37564641, \left(t\mapsto t\log\left(-\frac{98146713600t^3}{11971753}-\frac{9639364864t}{323237331}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**3,x)
```

```
[Out] -(166*x**8 + 611*x**6 + 1412*x**4 + 1849*x**2 + 768)/(576*x**9 +
2304*x**7 + 5760*x**5 + 6912*x**3 + 5184*x) + RootSum(41747082117
```

```
12*_t**4 + 15652880384*_t**2 + 37564641, Lambda(_t, _t*log(-98146
713600*_t**3/11971753 - 9639364864*_t/323237331 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6 + 3\,x^4 + x^2 + 4}{(x^4 + 2\,x^2 + 3)^3 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^3*x^2),x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^3*x^2), x)
```

3.124 $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=262

$$-\frac{4}{81x^3}+\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{41472}-\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{41472}+\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}+\frac{x\left(1025x^2+1474\right)}{5184\left(x^4+2x^2+3\right)}+\frac{7}{27x}-\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736}+\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736}$$

```
[Out] -4/(81*x^3) + 7/(27*x) + (25*x*(7 + 5*x^2))/(432*(3 + 2*x^2 + x^4
)^2) + (x*(1474 + 1025*x^2))/(5184*(3 + 2*x^2 + x^4)) - (Sqrt[(10
004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*
x)/Sqrt[2*(1 + Sqrt[3])]])/20736 + (Sqrt[(10004741 + 11240451*Sqr
t[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/20736 + (Sqrt[(-10004741 + 11240451*Sqrt[3])/3]*Log[Sqrt[3]
 - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/41472 - (Sqrt[(-10004741 + 11
240451*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])
/41472
```

Rubi [A] time = 0.871598, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$-\frac{4}{81x^3}+\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{41472}-\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{41472}+\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}+\frac{x\left(1025x^2+1474\right)}{5184\left(x^4+2x^2+3\right)}+\frac{7}{27x}-\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736}+\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] -4/(81*x^3) + 7/(27*x) + (25*x*(7 + 5*x^2))/(432*(3 + 2*x^2 + x^4
)^2) + (x*(1474 + 1025*x^2))/(5184*(3 + 2*x^2 + x^4)) - (Sqrt[(10
004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*
x)/Sqrt[2*(1 + Sqrt[3])]])/20736 + (Sqrt[(10004741 + 11240451*Sqr
t[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/20736 + (Sqrt[(-10004741 + 11240451*Sqrt[3])/3]*Log[Sqrt[3]
 - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/41472 - (Sqrt[(-10004741 + 11
240451*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])
```

```
/41472
```

Rubi in Sympy [A] time = 42.0006, size = 342, normalized size = 1.31

$$\frac{\sqrt{6}\left(-907776\sqrt{3}+1594368\right)\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{11943936\sqrt{-1+\sqrt{3}}} - \frac{\sqrt{6}\left(-907776\sqrt{3}+1594368\right)\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{3}}+\sqrt{3}\right)}{11943936\sqrt{-1+\sqrt{3}}} - \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-1815552\sqrt{3}+3188736\right)}{2}+3188736\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{5971968\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}} - \frac{\sqrt{3}\left(-\frac{\sqrt{2}\sqrt{-1+\sqrt{3}}\left(-1815552\sqrt{3}+3188736\right)}{2}+3188736\sqrt{2}\sqrt{-1+\sqrt{3}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{3}}}{2}\right)}{\sqrt{1+\sqrt{3}}}\right)}{5971968\sqrt{-1+\sqrt{3}}\sqrt{1+\sqrt{3}}} - \frac{197}{108x} + \frac{33792x^2+12288}{36864x^3\left(x^4+2x^2+3\right)} + \frac{4}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**3,x)
```

```
[Out] sqrt(6)*(-907776*sqrt(3) + 1594368)*log(x**2 - sqrt(2)*x*sqrt(-1
+ sqrt(3)) + sqrt(3))/(11943936*sqrt(-1 + sqrt(3))) - sqrt(6)*(-9
07776*sqrt(3) + 1594368)*log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(3))
+ sqrt(3))/(11943936*sqrt(-1 + sqrt(3))) - sqrt(3)*(-sqrt(2)*sqrt
(-1 + sqrt(3))*(-1815552*sqrt(3) + 3188736)/2 + 3188736*sqrt(2)*s
qrt(-1 + sqrt(3)))*atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt(3))/2)/sqrt
(1 + sqrt(3)))/(5971968*sqrt(-1 + sqrt(3))*sqrt(1 + sqrt(3))) - s
qrt(3)*(-sqrt(2)*sqrt(-1 + sqrt(3))*(-1815552*sqrt(3) + 3188736)/
2 + 3188736*sqrt(2)*sqrt(-1 + sqrt(3)))*atan(sqrt(2)*(x + sqrt(-2
 + 2*sqrt(3))/2)/sqrt(1 + sqrt(3)))/(5971968*sqrt(-1 + sqrt(3))*s
qrt(1 + sqrt(3))) - 197/(108*x) + (33792*x**2 + 12288)/(36864*x**
3*(x**4 + 2*x**2 + 3)) + 4/(27*x**3)
```

Mathematica [C] time = 0.67611, size = 139, normalized size = 0.53

$$\frac{\frac{4\left(2369x^{10}+8644x^8+19939x^6+20090x^4+9024x^2-2304\right)}{x^3\left(x^4+2x^2+3\right)^2}+\frac{\left(4738+127i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{\left(4738-127i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{20736}$$

Antiderivative was successfully verified.

```
[In]   Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] ((4*(-2304 + 9024*x^2 + 20090*x^4 + 19939*x^6 + 8644*x^8 + 2369*x
^10))/(x^3*(3 + 2*x^2 + x^4)^2) + ((4738 + (127*I)*Sqrt[2])*ArcTa
n[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((4738 - (127*I)*
Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/2073
6
```

Maple [B] time = 0.04, size = 429, normalized size = 1.6

$$
\begin{aligned}
&-\frac{4}{81x^3}+\frac{7}{27x}+\frac{1}{27\left(x^4+2x^2+3\right)^2}\left(\frac{1025x^7}{192}+\frac{881x^5}{48}+\frac{7523x^3}{192}+\frac{1087x}{32}\right)\\
&-\frac{4865\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{248832}\\
&-\frac{127\ln\left(x^2+\sqrt{3}+x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{82944}\\
&+\frac{\left(-9730+9730\sqrt{3}\right)\sqrt{3}}{124416\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\\
&+\frac{-254+254\sqrt{3}}{41472\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{1121\sqrt{3}}{7776\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\\
&+\frac{4865\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{248832}\\
&+\frac{127\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{82944}\\
&+\frac{\left(-9730+9730\sqrt{3}\right)\sqrt{3}}{124416\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)\\
&+\frac{-254+254\sqrt{3}}{41472\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)+\frac{1121\sqrt{3}}{7776\sqrt{2+2\sqrt{3}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x)
```

```
[Out] -4/81/x^3+7/27/x+1/27*(1025/192*x^7+881/48*x^5+7523/192*x^3+1087/
32*x)/(x^4+2*x^2+3)^2-4865/248832*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))
^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-127/82944*ln(x^2+3^(1/2)+x*(
-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+4865/124416/(2+2*3^(1/2
))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(
-2+2*3^(1/2))*3^(1/2)+127/41472/(2+2*3^(1/2))^(1/2)*arctan((2*x+(
-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+1121/777
6/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1
/2))^(1/2))*3^(1/2)+4865/248832*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(
1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+127/82944*ln(x^2+3^(1/2)-x*(-2
+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+4865/124416/(2+2*3^(1/2))
^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2
+2*3^(1/2))*3^(1/2)+127/41472/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2
+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+1121/7776/
(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2
))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2369x^{10}+8644x^8+19939x^6+20090x^4+9024x^2-2304}{5184\left(x^{11}+4x^9+10x^7+12x^5+9x^3\right)}+\frac{1}{5184}\int\frac{2369x^2+2242}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^3*x^4),x, algorithm="maxima
```

```
[Out] 1/5184*(2369*x^10 + 8644*x^8 + 19939*x^6 + 20090*x^4 + 9024*x^2 -
 2304)/(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3) + 1/5184*integrat
e((2369*x^2 + 2242)/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 0.292129, size = 1241, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^3*x^4),x, algorithm="fricas
```

```
[Out] -1/3836740608*sqrt(15419)*12^(3/4)*(94479352*237745561^(1/4)*sqrt
(3)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*arctan(6*237745561^(
1/4)*(127*sqrt(3) + 4865)/(sqrt(15419)*12^(1/4)*sqrt(1/46257)*(10
004741*sqrt(3)*sqrt(2) - 33721353*sqrt(2))*sqrt(sqrt(3)*(23774556
1^(1/4)*sqrt(15419)*12^(1/4)*(4101021105251275118178390l*sqrt(3)*
x - 6966531654113795879643051 3*x)*sqrt((10004741*sqrt(3) - 337213
53)/(112457800978191*sqrt(3) - 239569029263642)) + 15419*sqrt(3)*
(6189055293793482939145*sqrt(3)*x^2 - 11453925320315644758219*x^2
) + 28628713072500514031603026 5*sqrt(3) - 529824223541840779580 93
6283)/(6189055293793482939145*sqrt(3) - 11453925320315644758219))
*sqrt((10004741*sqrt(3) - 33721353)/(112457800978191*sqrt(3) - 23
9569029263642)) + sqrt(15419)*12^(1/4)*(10004741*sqrt(3)*sqrt(2)*
x - 33721353*sqrt(2)*x)*sqrt((10004741*sqrt(3) - 33721353)/(11245
7800978191*sqrt(3) - 239569029263642)) + 6*237745561^(1/4)*(2369*
sqrt(3)*sqrt(2) - 2242*sqrt(2)))) + 94479352*237745561^(1/4)*sqrt
(3)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*arctan(6*237745561^(
1/4)*(127*sqrt(3) + 4865)/(sqrt(15419)*12^(1/4)*sqrt(1/46257)*(10
004741*sqrt(3)*sqrt(2) - 33721353*sqrt(2))*sqrt(-sqrt(3)*(2377455
61^(1/4)*sqrt(15419)*12^(1/4)*(4101021105251275118178390l*sqrt(3)
*x - 6966531654113795879643051 3*x)*sqrt((10004741*sqrt(3) - 33721
353)/(112457800978191*sqrt(3) - 239569029263642)) - 15419*sqrt(3)
*(6189055293793482939145*sqrt(3)*x^2 - 11453925320315644758219*x^
2) - 28628713072500514031603026 5*sqrt(3) + 529824223541840779580 9
36283)/(6189055293793482939145*sqrt(3) - 11453925320315644758219)
)*sqrt((10004741*sqrt(3) - 33721353)/(112457800978191*sqrt(3) - 2
39569029263642)) + sqrt(15419)*12^(1/4)*(10004741*sqrt(3)*sqrt(2)
*x - 33721353*sqrt(2)*x)*sqrt((10004741*sqrt(3) - 33721353)/(1124
57800978191*sqrt(3) - 239569029263642)) - 6*237745561^(1/4)*(2369
*sqrt(3)*sqrt(2) - 2242*sqrt(2)))) - 4*sqrt(15419)*12^(1/4)*(1000
4741*sqrt(3)*sqrt(2)*(2369*x^10 + 8644*x^8 + 19939*x^6 + 20090*x^
4 + 9024*x^2 - 2304) - 33721353*sqrt(2)*(2369*x^10 + 8644*x^8 + 1
9939*x^6 + 20090*x^4 + 9024*x^2 - 2304))*sqrt((10004741*sqrt(3) -
 33721353)/(112457800978191*sqrt(3) - 239569029263642)) - 2377455
61^(1/4)*(10004741*sqrt(3)*sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^
5 + 9*x^3) - 33721353*sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9
*x^3))*log(54*237745561^(1/4)*sqrt(15419)*12^(1/4)*(4101021105251
2751181783901*sqrt(3)*x - 69665316541137958796430513*x)*sqrt((100
04741*sqrt(3) - 33721353)/(112457800978191*sqrt(3) - 239569029263
642)) + 832626*sqrt(3)*(6189055293793482939145*sqrt(3)*x^2 - 1145
3925320315644758219*x^2) + 1545950505915027757706563431 0*sqrt(3)
- 2861050807125940209737055928 2) + 237745561^(1/4)*(10004741*sqrt
(3)*sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3) - 33721353*s
qrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3))*log(-54*23774556
1^(1/4)*sqrt(15419)*12^(1/4)*(4101021105251275118178390l*sqrt(3)*
x - 6966531654113795879643051 3*x)*sqrt((10004741*sqrt(3) - 337213
53)/(112457800978191*sqrt(3) - 239569029263642)) + 832626*sqrt(3)
*(6189055293793482939145*sqrt(3)*x^2 - 11453925320315644758219*x^
2) + 1545950505915027757706563431 0*sqrt(3) - 2861050807125940209 7
370559282))/((10004741*sqrt(3)*sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 1
2*x^5 + 9*x^3) - 33721353*sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5
 + 9*x^3))*sqrt((10004741*sqrt(3) - 33721353)/(112457800978191*sq
rt(3) - 239569029263642)))
```

Sympy [A] time = 2.33789, size = 80, normalized size = 0.31

$$\operatorname{RootSum}\left(338151365148672t^4 + 2622682824704t^2 + 19257390441, \left(t \mapsto t\log\left(\frac{357010935644160t^3}{182097141061} + \frac{26016957890816t}{1638874269549} + \right.\right.\right.$$

$$+ \frac{2369x^{10} + 8644x^8 + 19939x^6 + 20090x^4 + 9024x^2 - 2304}{5184x^{11} + 20736x^9 + 51840x^7 + 62208x^5 + 46656x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**3,x)
```

```
[Out] RootSum(338151365148672*_t**4 + 2622682824704*_t**2 + 19257390441
, Lambda(_t, _t*log(357010935644160*_t**3/182097141061 + 26016957
890816*_t/1638874269549 + x))) + (2369*x**10 + 8644*x**8 + 19939*
x**6 + 20090*x**4 + 9024*x**2 - 2304)/(5184*x**11 + 20736*x**9 +
51840*x**7 + 62208*x**5 + 46656*x**3)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6 + 3\,x^4 + x^2 + 4}{(x^4 + 2\,x^2 + 3)^3 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^3*x^4),x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^3*x^4), x)
```

3.125 $\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=149

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+b^2g+c^2e\right)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(cf-bg)}{2c^2}+\frac{gx^4}{4c}$$

```
[Out] ((c*f - b*g)*x^2)/(2*c^2) + (g*x^4)/(4*c) - ((2*c^3*d - c^2*(b*e
+ 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*e + b^2*g - c*(b
*f + a*g))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 0.594175, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+b^2g+c^2e\right)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(cf-bg)}{2c^2}+\frac{gx^4}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*f - b*g)*x^2)/(2*c^2) + (g*x^4)/(4*c) - ((2*c^3*d - c^2*(b*e
+ 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*e + b^2*g - c*(b
*f + a*g))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\left(\frac{bg}{2}-\frac{cf}{2}\right)\int^{x^2}\frac{1}{c^2}\,dx+\frac{gx^4}{4c}+\frac{\left(-acg+b^2g-bcf+c^2e\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{\left(-3abcg+2ac^2f+b^3g-b^2cf+bc^2e-2c^3d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^3\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] -(b*g/2 - c*f/2)*Integral(c**(-2), (x, x**2)) + g*x**4/(4*c) + (-
a*c*g + b**2*g - b*c*f + c**2*e)*log(a + b*x**2 + c*x**4)/(4*c**3
) + (-3*a*b*c*g + 2*a*c**2*f + b**3*g - b**2*c*f + b*c**2*e - 2*c
**3*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**3*sqrt(-4*
a*c + b**2))
```

Mathematica [A] time = 0.230919, size = 142, normalized size = 0.95

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+b^2g+c^2e\right)+\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{\sqrt{4ac-b^2}}+2cx^2(cf-bg)+c^2gx^4}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(c*f - b*g)*x^2 + c^2*g*x^4 + (2*(2*c^3*d - c^2*(b*e + 2*a*f
) - b^3*g + b*c*(b*f + 3*a*g))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4
*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*e + b^2*g - c*(b*f + a*g))*Log[
a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [B] time = 0.006, size = 357, normalized size = 2.4

$$\frac{gx^4}{4c} - \frac{bx^2g}{2c^2} + \frac{fx^2}{2c} - \frac{\ln\left(cx^4+bx^2+a\right)ag}{4c^2} + \frac{\ln\left(cx^4+bx^2+a\right)b^2g}{4c^3} - \frac{\ln\left(cx^4+bx^2+a\right)bf}{4c^2} + \frac{\ln\left(cx^4+bx^2+a\right)e}{4c} + \frac{3abg}{2c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} - \frac{fa}{c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} + d\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} - \frac{b^3g}{2c^3}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} + \frac{b^2f}{2c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} - \frac{be}{2c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*g*x^4/c-1/2/c^2*x^2*b*g+1/2*f*x^2/c-1/4/c^2*ln(c*x^4+b*x^2+a)
*a*g+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2*g-1/4/c^2*ln(c*x^4+b*x^2+a)*b*
f+1/4/c*ln(c*x^4+b*x^2+a)*e+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*g-1/c/(4*a*c-b^2)^(1/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))*f*a+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*d-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2))*b^3*g+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*f-1/2/c/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^6 + f*x^4 + e*x^2 + d)*x/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.356744, size = 1, normalized size = 0.01

$$\left[\frac{\left(2c^3d - bc^2e + \left(b^2c - 2ac^2\right)f - \left(b^3 - 3abc\right)g\right)\log\left(-\frac{b^3-4abc+2\left(b^2c-4ac^2\right)x^2-\left(2c^2x^4+2bcx^2+b^2-2ac\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right) + \left(c^2gx^4 + 2\left(\right.\right.}{4\sqrt{b^2-4ac}c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)*x/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*
g)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2
*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) +
 (c^2*g*x^4 + 2*(c^2*f - b*c*g)*x^2 + (c^2*e - b*c*f + (b^2 - a*c
)*g)*log(c*x^4 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c
)*c^3), 1/4*(2*(2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 -
3*a*b*c)*g)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) + (c^2*g*x^4 + 2*(c^2*f - b*c*g)*x^2 + (c^2*e - b*c*f + (b^2 -
 a*c)*g)*log(c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
 4*a*c)*c^3)]
```

Sympy [A] time = 164.926, size = 789, normalized size = 5.3

$$\left(-\frac{\sqrt{-4ac+b^2}\left(3abcg-2ac^2f-b^3g+b^2cf-bc^2e+2c^3d\right)}{4c^3\left(4ac-b^2\right)} - \frac{acg-b^2g+bcf-c^2e}{4c^3}\right)\log\left(x^2+\frac{2a^2cg-ab^2g+abcf+8ac^3\left(-\frac{\sqrt{-4ac+b^2}(3abcg-2ac^2f-b^3g+b^2cf-bc^2e+2c^3d)}{4c^3(4ac-b^2)}-\frac{acg-b^2g+bcf-\cdots}{4c^3}\cdots\right.}{3abcg-2ac^2f-b\cdots}\right.$$

$$+\left(\frac{\sqrt{-4ac+b^2}\left(3abcg-2ac^2f-b^3g+b^2cf-bc^2e+2c^3d\right)}{4c^3\left(4ac-b^2\right)} - \frac{acg-b^2g+bcf-c^2e}{4c^3}\right)\log\left(x^2+\frac{2a^2cg-ab^2g+abcf+8ac^3\left(\frac{\sqrt{-4ac+b^2}(3abcg-2ac^2f-b^3g+b^2cf-bc^2e+2c^3d)}{4c^3(4ac-b^2)}-\frac{acg-b^2g+bcf-c^2\cdots}{4c^3}\cdots\right.}{3abcg-2ac^2f-b^3g\cdots}\right.$$

$$+\frac{gx^4}{4c}-\frac{x^2\left(bg-cf\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(3*a*b*c*g - 2*a*c**2*f - b**3*g + b**2*c*f
 - b*c**2*e + 2*c**3*d)/(4*c**3*(4*a*c - b**2)) - (a*c*g - b**2*g
 + b*c*f - c**2*e)/(4*c**3))*log(x**2 + (2*a**2*c*g - a*b**2*g +
a*b*c*f + 8*a*c**3*(-sqrt(-4*a*c + b**2)*(3*a*b*c*g - 2*a*c**2*f
- b**3*g + b**2*c*f - b*c**2*e + 2*c**3*d)/(4*c**3*(4*a*c - b**2)
) - (a*c*g - b**2*g + b*c*f - c**2*e)/(4*c**3)) - 2*a*c**2*e - 2*
b**2*c**2*(-sqrt(-4*a*c + b**2)*(3*a*b*c*g - 2*a*c**2*f - b**3*g
+ b**2*c*f - b*c**2*e + 2*c**3*d)/(4*c**3*(4*a*c - b**2)) - (a*c*
g - b**2*g + b*c*f - c**2*e)/(4*c**3)) + b*c**2*d)/(3*a*b*c*g - 2
*a*c**2*f - b**3*g + b**2*c*f - b*c**2*e + 2*c**3*d)) + (sqrt(-4*
a*c + b**2)*(3*a*b*c*g - 2*a*c**2*f - b**3*g + b**2*c*f - b*c**2*
e + 2*c**3*d)/(4*c**3*(4*a*c - b**2)) - (a*c*g - b**2*g + b*c*f -
 c**2*e)/(4*c**3))*log(x**2 + (2*a**2*c*g - a*b**2*g + a*b*c*f +
8*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*g - 2*a*c**2*f - b**3*g +
b**2*c*f - b*c**2*e + 2*c**3*d)/(4*c**3*(4*a*c - b**2)) - (a*c*g
- b**2*g + b*c*f - c**2*e)/(4*c**3)) - 2*a*c**2*e - 2*b**2*c**2*(
sqrt(-4*a*c + b**2)*(3*a*b*c*g - 2*a*c**2*f - b**3*g + b**2*c*f -
 b*c**2*e + 2*c**3*d)/(4*c**3*(4*a*c - b**2)) - (a*c*g - b**2*g +
 b*c*f - c**2*e)/(4*c**3)) + b*c**2*d)/(3*a*b*c*g - 2*a*c**2*f -
b**3*g + b**2*c*f - b*c**2*e + 2*c**3*d)) + g*x**4/(4*c) - x**2*(
b*g - c*f)/(2*c**2)
```

GIAC/XCAS [A] time = 0.298245, size = 197, normalized size = 1.32

$$\frac{cgx^4 + 2\,cfx^2 - 2\,bgx^2}{4\,c^2} - \frac{\left(bcf - b^2g + acg - c^2e\right)\ln\left(cx^4 + bx^2 + a\right)}{4\,c^3} + \frac{\left(2\,c^3d + b^2cf - 2\,ac^2f - b^3g + 3\,abcg - bc^2e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)*x/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4*(c*g*x^4 + 2*c*f*x^2 - 2*b*g*x^2)/c^2 - 1/4*(b*c*f - b^2*g +
a*c*g - c^2*e)*ln(c*x^4 + b*x^2 + a)/c^3 + 1/2*(2*c^3*d + b^2*c*f
 - 2*a*c^2*f - b^3*g + 3*a*b*c*g - b*c^2*e)*arctan((2*c*x^2 + b)/
sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.126 $\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=594

$$\frac{x\,(a\,(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d)+x^2\,(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b^4(-g)+b^3cf}{2c^3\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-b^2c(ce-24ag)-\frac{-b^3c(ce-34ag)-b^2c^2(19af+cd)+4abc^2(2ce-13ag)-4ac^3(cd-5af)-5b^5g+3b^4cf}{\sqrt{b^2-4ac}}-bc^2(13af+cd\right.}{2\sqrt{2}c^{7/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}$$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-b^2c(ce-24ag)+\frac{-b^3c(ce-34ag)-b^2c^2(19af+cd)+4abc^2(2ce-13ag)-4ac^3(cd-5af)-5b^5g+3b^4cf}{\sqrt{b^2-4ac}}-bc^2(13af+cd\right.}{2\sqrt{2}c^{7/2}\,(b^2-4ac)\,\sqrt{\sqrt{b^2-4ac}+b}}$$

$$+\frac{x(cf-2bg)}{c^3}+\frac{gx^3}{3c^2}$$

```
[Out] ((c*f - 2*b*g)*x)/c^3 + (g*x^3)/(3*c^2) + (x*(a*(2*c^3*d - c^2*(b
*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g)) + (b^3*c*f + b*c^2*(c*d
- 3*a*f) - b^4*g - b^2*c*(c*e - 4*a*g) + 2*a*c^2*(c*e - a*g))*x^2
))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*c*f - b*c^
2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*c^2*(3*c*
e - 7*a*g) - (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d +
19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*
a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^
2*c*(c*e - 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) + (3*b^4*c*f - 4*a*c
^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e
- 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(
7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 28.5386, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$

$$\frac{x\,(a\,(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d)+x^2\,(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b^4(-g)+b^3cf}{2c^3\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-b^2c(ce-24ag)-\frac{-b^3c(ce-34ag)-b^2c^2(19af+cd)+4abc^2(2ce-13ag)-4ac^3(cd-5af)-5b^5g+3b^4cf}{\sqrt{b^2-4ac}}-bc^2(13af+cd\right.}{2\sqrt{2}c^{7/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}$$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-b^2c(ce-24ag)+\frac{-b^3c(ce-34ag)-b^2c^2(19af+cd)+4abc^2(2ce-13ag)-4ac^3(cd-5af)-5b^5g+3b^4cf}{\sqrt{b^2-4ac}}-bc^2(13af+cd\right.}{2\sqrt{2}c^{7/2}\,(b^2-4ac)\,\sqrt{\sqrt{b^2-4ac}+b}}$$

$$+\frac{x(cf-2bg)}{c^3}+\frac{gx^3}{3c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*f - 2*b*g)*x)/c^3 + (g*x^3)/(3*c^2) + (x*(a*(2*c^3*d - c^2*(b
*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g)) + (b^3*c*f + b*c^2*(c*d
- 3*a*f) - b^4*g - b^2*c*(c*e - 4*a*g) + 2*a*c^2*(c*e - a*g))*x^2
))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*c*f - b*c^
2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*c^2*(3*c*
e - 7*a*g) - (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d +
19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*
a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^
```

```
2*c*(c*e - 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) + (3*b^4*c*f - 4*a*c
^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e
- 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(
7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**4*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 6.64107, size = 721, normalized size = 1.21

$$\frac{\frac{6\sqrt{c}x\left(a^2c\left(3bg-2c\left(f+gx^2\right)\right)+a\left(b^3(-g)+b^2c\left(f+4gx^2\right)-bc^2\left(e+3fx^2\right)+2c^3\left(d+ex^2\right)\right)+bx^2\left(b^3(-g)+b^2cf-bc^2e+c^3d\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-b^2c\left(-ce\ldots\right.\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (12*Sqrt[c]*(c*f - 2*b*g)*x + 4*c^(3/2)*g*x^3 + (6*Sqrt[c]*x*(b*(
c^3*d - b*c^2*e + b^2*c*f - b^3*g)*x^2 + a^2*c*(3*b*g - 2*c*(f +
g*x^2)) + a*(-(b^3*g) + 2*c^3*(d + e*x^2) - b*c^2*(e + 3*f*x^2) +
 b^2*c*(f + 4*g*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*
Sqrt[2]*(-5*b^5*g - b^3*c*(c*e + 3*Sqrt[b^2 - 4*a*c]*f - 34*a*g)
+ b^4*(3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + 2*a*c^2*(-2*c^2*d - 3*c*S
qrt[b^2 - 4*a*c]*e + 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*c]*g) - b^2*c*
(c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f + 24*a*Sqrt[b^2 - 4*a*c
]*g) + b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d + 8*a*e) + 13*a*(Sqrt[b^2 -
4*a*c]*f - 4*a*g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
3*Sqrt[2]*(5*b^5*g + b^3*c*(c*e - 3*Sqrt[b^2 - 4*a*c]*f - 34*a*g)
 + b^4*(-3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + b^2*c*(c^2*d + c*Sqrt[b
^2 - 4*a*c]*e + 19*a*c*f - 24*a*Sqrt[b^2 - 4*a*c]*g) + 2*a*c^2*(2
*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e - 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*
c]*g) + b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d - 8*a*e) + 13*a*(Sqrt[b^2 -
 4*a*c]*f + 4*a*g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
12*c^(7/2))
```

Maple [B] time = 0.115, size = 8533, normalized size = 14.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bc^3d-\left(b^2c^2-2\,ac^3\right)e+\left(b^3c-3\,abc^2\right)f-\left(b^4-4\,ab^2c+2\,a^2c^2\right)g\right)x^3+\left(2\,ac^3d-abc^2e+\left(ab^2c-2\,a^2c^2\right)f-\left(ab^3-3\,a^2\right.\right.}{2\left(ab^2c^3-4\,a^2c^4+\left(b^2c^4-4\,ac^5\right)x^4+\left(b^3c^3-4\,abc^4\right)x^2\right)}$$
$$+\frac{-\int\frac{2\,ac^3d-abc^2e-\left(bc^3d+\left(b^2c^2-6\,ac^3\right)e-\left(3\,b^3c-13\,abc^2\right)f+\left(5\,b^4-24\,ab^2c+14\,a^2c^2\right)g\right)x^2+\left(3\,ab^2c-10\,a^2c^2\right)f-\left(5\,ab^3-19\,a^2bc\right)g}{cx^4+bx^2+a}\,dx}{2\left(b^2c^3-4\,ac^4\right)}$$
$$+\frac{cgx^3+3\left(cf-2\,bg\right)x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxi
```

```
[Out] 1/2*((b*c^3*d - (b^2*c^2 - 2*a*c^3)*e + (b^3*c - 3*a*b*c^2)*f - (
b^4 - 4*a*b^2*c + 2*a^2*c^2)*g)*x^3 + (2*a*c^3*d - a*b*c^2*e + (a
*b^2*c - 2*a^2*c^2)*f - (a*b^3 - 3*a^2*b*c)*g)*x)/(a*b^2*c^3 - 4*
a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) +
1/2*integrate(-(2*a*c^3*d - a*b*c^2*e - (b*c^3*d + (b^2*c^2 - 6*a
*c^3)*e - (3*b^3*c - 13*a*b*c^2)*f + (5*b^4 - 24*a*b^2*c + 14*a^2
*c^2)*g)*x^2 + (3*a*b^2*c - 10*a^2*c^2)*f - (5*a*b^3 - 19*a^2*b*c
)*g)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 - 4*a*c^4) + 1/3*(c*g*x^3 +
 3*(c*f - 2*b*g)*x)/c^3
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="fric
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac
```

```
[Out] Exception raised: TypeError
```

3.127 $\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=471

$$-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(19ag+ce)-4bc^2(2af+cd)+4ac^2(ce-5ag)-3b^4g+b^3cf}{\sqrt{b^2-4ac}}-c^2(be-6af)-bc(13ag+bf)+3b^3g+2c^3d\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2c(19ag+ce)-4bc^2(2af+cd)+4ac^2(ce-5ag)-3b^4g+b^3cf}{\sqrt{b^2-4ac}}-c^2(be-6af)-bc(13ag+bf)+3b^3g+2c^3d\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{gx}{c^2}$$

```
[Out] (g*x)/c^2 - (x*(b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g) + (
2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*x^2))/(2
*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c^3*d - c^2*(b*e -
6*a*f) + 3*b^3*g - b*c*(b*f + 13*a*g) + (b^3*c*f - 4*b*c^2*(c*d +
 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^2*c*(c*e + 19*a*g))
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 -
4*a*c]]) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f + 1
3*a*g) - (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*
e - 5*a*g) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*
(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 15.8233, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114

$$-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(19ag+ce)-4bc^2(2af+cd)+4ac^2(ce-5ag)-3b^4g+b^3cf}{\sqrt{b^2-4ac}}-c^2(be-6af)-bc(13ag+bf)+3b^3g+2c^3d\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2c(19ag+ce)-4bc^2(2af+cd)+4ac^2(ce-5ag)-3b^4g+b^3cf}{\sqrt{b^2-4ac}}-c^2(be-6af)-bc(13ag+bf)+3b^3g+2c^3d\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{gx}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] (g*x)/c^2 - (x*(b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g) + (
2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*x^2))/(2
*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c^3*d - c^2*(b*e -
6*a*f) + 3*b^3*g - b*c*(b*f + 13*a*g) + (b^3*c*f - 4*b*c^2*(c*d +
 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^2*c*(c*e + 19*a*g))
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 -
4*a*c]]) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f + 1
3*a*g) - (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*
e - 5*a*g) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*
(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 168.249, size = 547, normalized size = 1.16

$$\frac{gx}{c^2} + \frac{x\left(-2a^2cg + ab^2g - abcf + 2ac^2e - bc^2d + x^2\left(-3abcg + 2ac^2f + b^3g - b^2cf + bc^2e - 2c^3d\right)\right)}{2c^2\left(-4ac + b^2\right)\left(a + bx^2 + cx^4\right)}$$
$$- \frac{\sqrt{2}\left(b\left(-13abcg + 6ac^2f + 3b^3g - b^2cf - bc^2e + 2c^3d\right) - 2c\left(-10a^2cg + 3ab^2g - abcf + 2ac^2e - bc^2d\right) + \sqrt{-4ac + b^2}\,(-1 \cdots}{4c^{\frac{5}{2}}\sqrt{b + \sqrt{-4ac + b^2}}\left(-4ac + b^2\right)^{\frac{3}{2}}}$$
$$+ \frac{\sqrt{2}\left(b\left(-13abcg + 6ac^2f + 3b^3g - b^2cf - bc^2e + 2c^3d\right) - 2c\left(-10a^2cg + 3ab^2g - abcf + 2ac^2e - bc^2d\right) - \sqrt{-4ac + b^2}\,(-1 \cdots}{4c^{\frac{5}{2}}\sqrt{b - \sqrt{-4ac + b^2}}\left(-4ac + b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] g*x/c**2 + x*(-2*a**2*c*g + a*b**2*g - a*b*c*f + 2*a*c**2*e - b*c
**2*d + x**2*(-3*a*b*c*g + 2*a*c**2*f + b**3*g - b**2*c*f + b*c**
2*e - 2*c**3*d))/(2*c**2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)) -
 sqrt(2)*(b*(-13*a*b*c*g + 6*a*c**2*f + 3*b**3*g - b**2*c*f - b*c
**2*e + 2*c**3*d) - 2*c*(-10*a**2*c*g + 3*a*b**2*g - a*b*c*f + 2*
a*c**2*e - b*c**2*d) + sqrt(-4*a*c + b**2)*(-13*a*b*c*g + 6*a*c**
2*f + 3*b**3*g - b**2*c*f - b*c**2*e + 2*c**3*d))*atan(sqrt(2)*sq
rt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*c**(5/2)*sqrt(b + sqrt(
-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*(b*(-13*a*b*c*g
 + 6*a*c**2*f + 3*b**3*g - b**2*c*f - b*c**2*e + 2*c**3*d) - 2*c*
(-10*a**2*c*g + 3*a*b**2*g - a*b*c*f + 2*a*c**2*e - b*c**2*d) - s
qrt(-4*a*c + b**2)*(-13*a*b*c*g + 6*a*c**2*f + 3*b**3*g - b**2*c*
f - b*c**2*e + 2*c**3*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4
*a*c + b**2)))/(4*c**(5/2)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c
+ b**2)**(3/2))
```

Mathematica [A] time = 4.39267, size = 575, normalized size = 1.22

$$\frac{-\frac{2\sqrt{c}x\left(2c\left(a^2g-ac\left(e+fx^2\right)+c^2dx^2\right)+b^2\left(cfx^2-ag\right)+bc\left(a\left(f+3gx^2\right)+c\left(d-ex^2\right)\right)+b^3(-g)x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2c^2\left(-10a^2g+cd\sqrt{b^2-4ac}+3af\sqrt{b^2-\cdots}\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*g*x - (2*Sqrt[c]*x*(-(b^3*g*x^2) + b^2*(-(a*g) + c*f*x
^2) + 2*c*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2)) + b*c*(c*(d - e*x
^2) + a*(f + 3*g*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (S
qrt[2]*(-3*b^4*g + b^2*c*(c*e - Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2
*c^2*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f -
 10*a^2*g) + b^3*(c*f + 3*Sqrt[b^2 - 4*a*c]*g) - b*c*(4*c^2*d + c
*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f + 13*a*Sqrt[b^2 - 4*a*c]*g))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*g - b^2*c*
(c*e + Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]
*d - 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f + 10*a^2*g) + b^3*(-(c*f)
+ 3*Sqrt[b^2 - 4*a*c]*g) + b*c*(4*c^2*d - c*Sqrt[b^2 - 4*a*c]*e +
 8*a*c*f - 13*a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]))/(4*c^(5/2))
```

Maple [B] time = 0.093, size = 7318, normalized size = 15.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,c^3d-bc^2e+\left(b^2c-2\,ac^2\right)f-\left(b^3-3\,abc\right)g\right)x^3+\left(bc^2d-2\,ac^2e+abcf-\left(ab^2-2\,a^2c\right)g\right)x}{2\,(ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^4+(b^3c^2-4\,abc^3)x^2)}+\frac{gx}{c^2}+\frac{\int\frac{bc^2d-2\,ac^2e+abcf-\left(2\,c^3d-bc^2e-\left(b^2c-6\,ac^2\right)f+\left(3\,b^3-13\,abc\right)g\right)x^2-\left(3\,ab^2-10\,a^2c\right)g}{cx^4+bx^2+a}\,dx}{2\,(b^2c^2-4\,ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^6 + f*x^4 + e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxi
```

```
[Out] -1/2*((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*
g)*x^3 + (b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x)
/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*
a*b*c^3)*x^2) + g*x/c^2 + 1/2*integrate((b*c^2*d - 2*a*c^2*e + a*
b*c*f - (2*c^3*d - b*c^2*e - (b^2*c - 6*a*c^2)*f + (3*b^3 - 13*a*
b*c)*g)*x^2 - (3*a*b^2 - 10*a^2*c)*g)/(c*x^4 + b*x^2 + a), x)/(b^
2*c^2 - 4*a*c^3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^6 + f*x^4 + e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="fric
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^6 + f*x^4 + e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac
```

```
[Out] Exception raised: TypeError
```

3.128 $\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=449

$$\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2g}{c}+\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{ab^2g}{c}-\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(c*(b^2*d - 2*a*(c*d - a*f) - (a*b*(c*e + a*g))/c) + (b*c*(c*d
 + a*f) - a*b^2*g - 2*a*c*(c*e - a*g))*x^2))/(2*a*c*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e
+ 3*a*g) + (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f) - a*b^3*g +
 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*f) + (a*b^2*
g)/c - 2*a*(c*e + 3*a*g) - (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d +
a*f) - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 7.1774, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$

$$\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)-ab(ag+ce)-2ac(cd-af)+b^2cd\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2g}{c}+\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{ab^2g}{c}-\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g) + (b*c*(c*d + a
*f) - a*b^2*g - 2*a*c*(c*e - a*g))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + ((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e + 3*
a*g) + (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f) - a*b^3*g + 4*a
*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*
a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*f) + (a*b^2*g)/c
 - 2*a*(c*e + 3*a*g) - (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f)
 - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*S
qrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 99.3167, size = 508, normalized size = 1.13

$$-\frac{x\left(a^2bg-2a^2cf+abce+2ac^2d-b^2cd+x^2\left(-2a^2cg+ab^2g-abcf+2ac^2e-bc^2d\right)\right)}{2ac\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{2}\left(b\left(-6a^2cg+ab^2g+abcf-2ac^2e+bc^2d\right)+2c\left(-a^2bg+2a^2cf-abce+6ac^2d-b^2cd\right)+\sqrt{-4ac+b^2}\left(-6a^2cg+ab^2g\right.\right.}{4ac^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}$$
$$-\frac{\sqrt{2}\left(b\left(-6a^2cg+ab^2g+abcf-2ac^2e+bc^2d\right)+2c\left(-a^2bg+2a^2cf-abce+6ac^2d-b^2cd\right)-\sqrt{-4ac+b^2}\left(-6a^2cg+ab^2\right.\right.}{4ac^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -x*(a**2*b*g - 2*a**2*c*f + a*b*c*e + 2*a*c**2*d - b**2*c*d + x**
2*(-2*a**2*c*g + a*b**2*g - a*b*c*f + 2*a*c**2*e - b*c**2*d))/(2*
a*c*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)) + sqrt(2)*(b*(-6*a**2*
c*g + a*b**2*g + a*b*c*f - 2*a*c**2*e + b*c**2*d) + 2*c*(-a**2*b*
g + 2*a**2*c*f - a*b*c*e + 6*a*c**2*d - b**2*c*d) + sqrt(-4*a*c +
 b**2)*(-6*a**2*c*g + a*b**2*g + a*b*c*f - 2*a*c**2*e + b*c**2*d)
)*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a*c**(
3/2)*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) - sqrt
(2)*(b*(-6*a**2*c*g + a*b**2*g + a*b*c*f - 2*a*c**2*e + b*c**2*d)
 + 2*c*(-a**2*b*g + 2*a**2*c*f - a*b*c*e + 6*a*c**2*d - b**2*c*d)
 - sqrt(-4*a*c + b**2)*(-6*a**2*c*g + a*b**2*g + a*b*c*f - 2*a*c*
*2*e + b*c**2*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b
**2)))/(4*a*c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2
)**(3/2))
```

Mathematica [A] time = 3.5291, size = 512, normalized size = 1.14

$$\frac{\frac{2\sqrt{c}x\left(b\left(a^2(-g)-ace+acfx^2+c^2dx^2\right)+b^2\left(cd-agx^2\right)+2ac\left(a\left(f+gx^2\right)-c\left(d+ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(bc\left(8a^2g+cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+4ace\right)-a\right.}{\left(b^2-4ac\right.}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b*(-(a*c*e) - a^2*g + c^2*d*x^2 + a*c*f*x^2) + b^2
*(c*d - a*g*x^2) + 2*a*c*(-(c*(d + e*x^2)) + a*(f + g*x^2))))/((b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*g) + b*c*(c*
Sqrt[b^2 - 4*a*c]*d + 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f + 8*a^2*g)
+ b^2*(c^2*d - a*c*f + a*Sqrt[b^2 - 4*a*c]*g) - 2*a*c*(6*c^2*d +
c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f + 3*a*Sqrt[b^2 - 4*a*c]*g))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a*b^3*g + b*c*(c
*Sqrt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f - 8*a^2*g)
 + 2*a*c*(6*c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f - 3*a*Sqrt[b^
2 - 4*a*c]*g) + b^2*(-(c^2*d) + a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a*c^(3/2))
```

Maple [B] time = 0.126, size = 8358, normalized size = 18.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bc^2d-2\,ac^2e+abcf-\left(ab^2-2\,a^2c\right)g\right)x^3-\left(abce-2\,a^2cf+a^2bg-\left(b^2c-2\,ac^2\right)d\right)x}{2\,(a^2b^2c-4\,a^3c^2+(ab^2c^2-4\,a^2c^3)x^4+(ab^3c-4\,a^2bc^2)x^2)}-\frac{-\int\frac{abce-2\,a^2cf+a^2bg+\left(bc^2d-2\,ac^2e+abcf+\left(ab^2-6\,a^2c\right)g\right)x^2+\left(b^2c-6\,ac^2\right)d}{cx^4+bx^2+a}\,dx}{2\,(ab^2c-4\,a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^6 + f*x^4 + e*x^2 + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x^3 -
(a*b*c*e - 2*a^2*c*f + a^2*b*g - (b^2*c - 2*a*c^2)*d)*x)/(a^2*b^2
*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b
*c^2)*x^2) - 1/2*integrate(-(a*b*c*e - 2*a^2*c*f + a^2*b*g + (b*c
^2*d - 2*a*c^2*e + a*b*c*f + (a*b^2 - 6*a^2*c)*g)*x^2 + (b^2*c -
6*a*c^2)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a^2*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^6 + f*x^4 + e*x^2 + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x^6 + f*x^4 + e*x^2 + d)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.129 $\int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=460

$$-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2c(ag+3ce)-ab^2(ce-ag)-4abc(af+4cd)+3b^3cd}{\sqrt{b^2-4ac}}-ab(ag+ce)-2ac(5cd-af)+3b^2cd\right)}{2\sqrt{2}a^2\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4a^2c(ag+3ce)-ab^2(ce-ag)-4abc(af+4cd)+3b^3cd}{\sqrt{b^2-4ac}}-ab(ag+ce)-2ac(5cd-af)+3b^2cd\right)}{2\sqrt{2}a^2\sqrt{c}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{a^2x}$$

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*
f) - 2*a^2*g) + (b^2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g))*x
^2))/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2*c*d - 2*
a*c*(5*c*d - a*f) - a*b*(c*e + a*g) + (3*b^3*c*d - 4*a*b*c*(4*c*d
 + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
2*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
- ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) - (3*b^3*c*
d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e +
a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sq
rt[b^2 - 4*a*c]])
```

Rubi [A] time = 8.15688, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2c(ag+3ce)-ab^2(ce-ag)-4abc(af+4cd)+3b^3cd}{\sqrt{b^2-4ac}}-ab(ag+ce)-2ac(5cd-af)+3b^2cd\right)}{2\sqrt{2}a^2\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4a^2c(ag+3ce)-ab^2(ce-ag)-4abc(af+4cd)+3b^3cd}{\sqrt{b^2-4ac}}-ab(ag+ce)-2ac(5cd-af)+3b^2cd\right)}{2\sqrt{2}a^2\sqrt{c}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{a^2x}-\frac{x\left(-2a^3g+a^2(bf+2ce)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)-ab(be+3cd)+b^3d\right)}{2a^2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(d/(a^2*x)) - (x*(b^3*d - a*b*(3*c*d + b*e) + a^2*(2*c*e + b*f)
- 2*a^3*g + (b^2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g))*x^2))
/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2*c*d - 2*a*c*
(5*c*d - a*f) - a*b*(c*e + a*g) + (3*b^3*c*d - 4*a*b*c*(4*c*d + a
*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((
3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) - (3*b^3*c*d -
4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b
^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((g*x**6+f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.95702, size = 529, normalized size = 1.15

$$-\frac{-\frac{2x\left(2a\left(a^2g-ac\left(e+fx^2\right)+c^2dx^2\right)+b^2\left(ae-cdx^2\right)+ab\left(-af+agx^2+3cd+cex^2\right)+b^3(-d)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2ac\left(2a^2g-5cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+\cdots\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -((4*d)/x - (2*x*(-(b^3*d) + b^2*(a*e - c*d*x^2) + a*b*(3*c*d - a
*f + c*e*x^2 + a*g*x^2) + 2*a*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2
))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(3*b^3*c*d +
b^2*(3*c*Sqrt[b^2 - 4*a*c]*d - a*c*e + a^2*g) + 2*a*c*(-5*c*Sqrt[
b^2 - 4*a*c]*d + 6*a*c*e + a*Sqrt[b^2 - 4*a*c]*f + 2*a^2*g) - a*b
*(16*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f + a*Sqrt[b^2 - 4*a*c
]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2
]*(-3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]*d + a*c*e - a^2*g) - 2
*a*c*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[b^2 - 4*a*c]*f +
 2*a^2*g) + a*b*(16*c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f - a*S
qrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]))/(4*a^2)
```

Maple [B] time = 0.092, size = 6807, normalized size = 14.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(abce-2\,a^2cf+a^2bg-\left(3\,b^2c-10\,ac^2\right)d\right)x^4-\left(a^2bf-2\,a^3g+\left(3\,b^3-11\,abc\right)d-\left(ab^2-2\,a^2c\right)e\right)x^2-2\left(ab^2-4\,a^2c\right)d}{2\left((a^2b^2c-4\,a^3c^2)x^5+(a^2b^3-4\,a^3bc)x^3+(a^3b^2-4\,a^4c)x\right)}+\frac{\int\frac{a^2bf-2\,a^3g+\left(abce-2\,a^2cf+a^2bg-\left(3\,b^2c-10\,ac^2\right)d\right)x^2-\left(3\,b^3-13\,abc\right)d+\left(ab^2-6\,a^2c\right)e}{cx^4+bx^2+a}\,dx}{2\left(a^2b^2-4\,a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="ma
```

```
[Out] 1/2*((a*b*c*e - 2*a^2*c*f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^4
 - (a^2*b*f - 2*a^3*g + (3*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*
e)*x^2 - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a
^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate(
(a^2*b*f - 2*a^3*g + (a*b*c*e - 2*a^2*c*f + a^2*b*g - (3*b^2*c -
10*a*c^2)*d)*x^2 - (3*b^3 - 13*a*b*c)*d + (a*b^2 - 6*a^2*c)*e)/(c
*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="fr
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**6+f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="gi
```

```
[Out] Exception raised: TypeError
```

3.130 $\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=542

$$\frac{2bd-ae}{a^3x}-\frac{d}{3a^2x^3}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2b(ag+4ce)+4a^2c(7cd-3af)-3ab^3e-ab^2(29cd-af)+5b^4d}{\sqrt{b^2-4ac}}+2a^2(5ce-ag)-3ab^2e-ab(19cd-af)+5b^3d\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4a^2b(ag+4ce)+4a^2c(7cd-3af)-3ab^3e-ab^2(29cd-af)+5b^4d}{\sqrt{b^2-4ac}}+2a^2(5ce-ag)-3ab^2e-ab(19cd-af)+5b^3\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2
*c^2*d + 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - a*(2*c*f + b*g
)) + c*(b^3*d - a*b^2*e - a*b*(3*c*d - a*f) + 2*a^2*(c*e - a*g))*
x^2))/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^3
*d - 3*a*b^2*e - a*b*(19*c*d - a*f) + 2*a^2*(5*c*e - a*g) + (5*b^
4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*(29*c*d - a*f)
+ 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*
b*(19*c*d - a*f) + 2*a^2*(5*c*e - a*g) - (5*b^4*d - 3*a*b^3*e + 4
*a^2*c*(7*c*d - 3*a*f) - a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e +
a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 -
 4*a*c]])
```

Rubi [A] time = 19.3327, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$

$$\frac{2bd-ae}{a^3x}-\frac{d}{3a^2x^3}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2b(ag+4ce)+4a^2c(7cd-3af)-3ab^3e-ab^2(29cd-af)+5b^4d}{\sqrt{b^2-4ac}}+2a^2(5ce-ag)-3ab^2e-ab(19cd-af)+5b^3d\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4a^2b(ag+4ce)+4a^2c(7cd-3af)-3ab^3e-ab^2(29cd-af)+5b^4d}{\sqrt{b^2-4ac}}+2a^2(5ce-ag)-3ab^2e-ab(19cd-af)+5b^3\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2
*c^2*d + 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - a*(2*c*f + b*g
)) + c*(b^3*d - a*b^2*e - a*b*(3*c*d - a*f) + 2*a^2*(c*e - a*g))*
x^2))/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^3
*d - 3*a*b^2*e - a*b*(19*c*d - a*f) + 2*a^2*(5*c*e - a*g) + (5*b^
4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*(29*c*d - a*f)
+ 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*
b*(19*c*d - a*f) + 2*a^2*(5*c*e - a*g) - (5*b^4*d - 3*a*b^3*e + 4
```

```
*a^2*c*(7*c*d - 3*a*f) - a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e +
a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 -
 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((g*x**6+f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.88511, size = 612, normalized size = 1.13

$$\frac{\frac{6x\left(ab\left(a^2(-g)+ac\left(3e+fx^2\right)-3c^2dx^2\right)+2a^2c\left(c\left(d+ex^2\right)-a\left(f+gx^2\right)\right)+b^3\left(cdx^2-ae\right)+ab^2\left(af-c\left(4d+ex^2\right)\right)+b^4d\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-2a^2\left(-5ce\sqrt{\cdots}\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) +
 c*d*x^2) + a*b^2*(a*f - c*(4*d + e*x^2)) + a*b*(-(a^2*g) - 3*c^2
*d*x^2 + a*c*(3*e + f*x^2)) + 2*a^2*c*(c*(d + e*x^2) - a*(f + g*x
^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(
5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + a*b^2*(-29*c*d -
3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-19*c*Sqrt[b^2 - 4*a*c]*d + 1
6*a*c*e + a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) - 2*a^2*(-14*c^2*d - 5
*c*Sqrt[b^2 - 4*a*c]*e + 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(5*b^4*d
 - b^3*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(-29*c*d + 3*Sqrt[
b^2 - 4*a*c]*e + a*f) + a*b*(19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e
- a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) + 2*a^2*(14*c^2*d - 5*c*Sqrt[b
^2 - 4*a*c]*e - 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [B] time = 0.095, size = 7512, normalized size = 13.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(a^{2}bcf-2\,a^{3}cg+\left(5\,b^{3}c-19\,abc^{2}\right)d-\left(3\,ab^{2}c-10\,a^{2}c^{2}\right)e\right)x^{6}-\left(3\,a^{3}bg-\left(15\,b^{4}-62\,ab^{2}c+14\,a^{2}c^{2}\right)d+3\left(3\,ab^{3}-11\,a\right.\right.}{6\left(\left(a^{3}b^{2}c-4\,a^{4}c^{2}\right)x^{7}+\left(a^{3}b^{3}-4\,a^{4}bc\right)x^{5}+\left(\right.\right.}$$

$$-\frac{-\int\frac{a^{3}bg+\left(a^{2}bcf-2\,a^{3}cg+\left(5\,b^{3}c-19\,abc^{2}\right)d-\left(3\,ab^{2}c-10\,a^{2}c^{2}\right)e\right)x^{2}+\left(5\,b^{4}-24\,ab^{2}c+14\,a^{2}c^{2}\right)d-\left(3\,ab^{3}-13\,a^{2}bc\right)e+\left(a^{2}b^{2}-6\,a^{3}c\right)f}{cx^{4}+bx^{2}+a}\,dx}{2\left(a^{3}b^{2}-4\,a^{4}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^4),x, algorithm="ma
```

```
[Out] 1/6*(3*(a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b
^2*c - 10*a^2*c^2)*e)*x^6 - (3*a^3*b*g - (15*b^4 - 62*a*b^2*c + 1
4*a^2*c^2)*d + 3*(3*a*b^3 - 11*a^2*b*c)*e - 3*(a^2*b^2 - 2*a^3*c)
*f)*x^4 + 2*(5*(a*b^3 - 4*a^2*b*c)*d - 3*(a^2*b^2 - 4*a^3*c)*e)*x
^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3
*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) - 1/2*integrate(
-(a^3*b*g + (a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*d - (
3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2
)*d - (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a^3*c)*f)/(c*x^4 +
b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^4),x, algorithm="fr
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((g*x**6+f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6 + f*x^4 + e*x^2 + d)/((c*x^4 + b*x^2 + a)^2*x^4),x, algorithm="gi
```

```
[Out] Exception raised: TypeError
```

3.131 $\int x^2 \left(a+bx^2+cx^4\right)^p \left(3a+b(5+2p)x^2+c(7+4p)x^4\right) \, dx$

Optimal. Leaf size=20

$$x^3 \left(a+bx^2+cx^4\right)^{p+1}$$

```
[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Rubi [A] time = 0.0209339, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$

$$x^3 \left(a+bx^2+cx^4\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]
```

```
[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Rubi in Sympy [A] time = 13.6066, size = 17, normalized size = 0.85

$$x^3 \left(a+bx^2+cx^4\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(c*x**4+b*x**2+a)**p*(3*a+b*(5+2*p)*x**2+c*(7+4*p)*x**4),x
```

```
[Out] x**3*(a + b*x**2 + c*x**4)**(p + 1)
```

Mathematica [A] time = 0.0533136, size = 20, normalized size = 1.

$$x^3 \left(a+bx^2+cx^4\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]
```

```
[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Maple [A] time = 0.011, size = 21, normalized size = 1.1

$$x^3 \left(cx^4+bx^2+a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x)
```

```
[Out] x^3*(c*x^4+b*x^2+a)^(1+p)
```

Maxima [A] time = 0.806387, size = 42, normalized size = 2.1

$$\left(cx^{7}+bx^{5}+ax^{3}\right)\left(cx^{4}+bx^{2}+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(4*p + 7)*x^4 + b*(2*p + 5)*x^2 + 3*a)*(c*x^4 + b*x^2 + a)^p*x^2,x, a
```

```
[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

Fricas [A] time = 0.279548, size = 42, normalized size = 2.1

$$\left(cx^{7}+bx^{5}+ax^{3}\right)\left(cx^{4}+bx^{2}+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(4*p + 7)*x^4 + b*(2*p + 5)*x^2 + 3*a)*(c*x^4 + b*x^2 + a)^p*x^2,x, a
```

```
[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**p*(3*a+b*(5+2*p)*x**2+c*(7+4*p)*x**4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.302884, size = 86, normalized size = 4.3

$$cx^{7}e^{\left(p\ln\left(cx^{4}+bx^{2}+a\right)\right)}+bx^{5}e^{\left(p\ln\left(cx^{4}+bx^{2}+a\right)\right)}+ax^{3}e^{\left(p\ln\left(cx^{4}+bx^{2}+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(4*p + 7)*x^4 + b*(2*p + 5)*x^2 + 3*a)*(c*x^4 + b*x^2 + a)^p*x^2,x, a
```

```
[Out] c*x^7*e^(p*ln(c*x^4 + b*x^2 + a)) + b*x^5*e^(p*ln(c*x^4 + b*x^2 +
 a)) + a*x^3*e^(p*ln(c*x^4 + b*x^2 + a))
```

3.132 $\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=210

$$-\frac{(d-ex)^{5/2}(d+ex)^{5/2}\left(ae^4+3bd^2e^2+6cd^4\right)}{5e^{10}}+\frac{d^2(d-ex)^{3/2}(d+ex)^{3/2}\left(2ae^4+3bd^2e^2+4cd^4\right)}{3e^{10}}-\frac{d^4\sqrt{d-ex}\sqrt{d+ex}\left(ae^4+bd^2e^2+cd^4\right)}{e^{10}}+\frac{(d-ex)^{7/2}(d+ex)^{7/2}\left(be^2+4cd^2\right)}{7e^{10}}-\frac{c(d-ex)^{9/2}(d+ex)^{9/2}}{9e^{10}}$$

```
[Out] -((d^4*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d - e*x]*Sqrt[d + e*x])/e
^10) + (d^2*(4*c*d^4 + 3*b*d^2*e^2 + 2*a*e^4)*(d - e*x)^(3/2)*(d
+ e*x)^(3/2))/(3*e^10) - ((6*c*d^4 + 3*b*d^2*e^2 + a*e^4)*(d - e*
x)^(5/2)*(d + e*x)^(5/2))/(5*e^10) + ((4*c*d^2 + b*e^2)*(d - e*x)
^(7/2)*(d + e*x)^(7/2))/(7*e^10) - (c*(d - e*x)^(9/2)*(d + e*x)^(
9/2))/(9*e^10)
```

Rubi [A] time = 0.818034, antiderivative size = 278, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114

$$-\frac{\left(d^2-e^2x^2\right)^3\left(ae^4+3bd^2e^2+6cd^4\right)}{5e^{10}\sqrt{d-ex}\sqrt{d+ex}}+\frac{d^2\left(d^2-e^2x^2\right)^2\left(2ae^4+3bd^2e^2+4cd^4\right)}{3e^{10}\sqrt{d-ex}\sqrt{d+ex}}-\frac{d^4\left(d^2-e^2x^2\right)\left(ae^4+bd^2e^2+cd^4\right)}{e^{10}\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(d^2-e^2x^2\right)^4\left(be^2+4cd^2\right)}{7e^{10}\sqrt{d-ex}\sqrt{d+ex}}-\frac{c\left(d^2-e^2x^2\right)^5}{9e^{10}\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((d^4*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^10*Sqrt[d
- e*x]*Sqrt[d + e*x])) + (d^2*(4*c*d^4 + 3*b*d^2*e^2 + 2*a*e^4)*(
d^2 - e^2*x^2)^2)/(3*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*c*d^
4 + 3*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^3)/(5*e^10*Sqrt[d - e*x]
*Sqrt[d + e*x]) + ((4*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^4)/(7*e^10*S
qrt[d - e*x]*Sqrt[d + e*x]) - (c*(d^2 - e^2*x^2)^5)/(9*e^10*Sqrt[
d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\sqrt{d-ex}\sqrt{d+ex}\left(d^2-e^2x^2\right)^4}{9e^{10}}+\frac{d^2\sqrt{d-ex}\sqrt{d+ex}\left(d^2-e^2x^2\right)\left(2ae^4+3bd^2e^2+4cd^4\right)}{3e^{10}}+\frac{\sqrt{d-ex}\sqrt{d+ex}\left(d^2-e^2x^2\right)^3\left(be^2+4cd^2\right)}{7e^{10}}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(d^2-e^2x^2\right)^2\left(ae^4+3bd^2e^2+6cd^4\right)}{5e^{10}}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(ae^4+bd^2e^2+cd^4\right)\int^{\sqrt{d^2-e^2x^2}}d^4\,dx}{e^{10}\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**5*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -c*sqrt(d - e*x)*sqrt(d + e*x)*(d**2 - e**2*x**2)**4/(9*e**10) +
d**2*sqrt(d - e*x)*sqrt(d + e*x)*(d**2 - e**2*x**2)*(2*a*e**4 + 3
```

```
*b*d**2*e**2 + 4*c*d**4)/(3*e**10) + sqrt(d - e*x)*sqrt(d + e*x)*
(d**2 - e**2*x**2)**3*(b*e**2 + 4*c*d**2)/(7*e**10) - sqrt(d - e*
x)*sqrt(d + e*x)*(d**2 - e**2*x**2)**2*(a*e**4 + 3*b*d**2*e**2 +
6*c*d**4)/(5*e**10) - sqrt(d - e*x)*sqrt(d + e*x)*(a*e**4 + b*d**
2*e**2 + c*d**4)*Integral(d**4, (x, sqrt(d**2 - e**2*x**2)))/(e**
10*sqrt(d**2 - e**2*x**2))
```

Mathematica [A] time = 0.163924, size = 149, normalized size = 0.71

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(21ae^4\left(8d^4+4d^2e^2x^2+3e^4x^4\right)+9b\left(16d^6e^2+8d^4e^4x^2+6d^2e^6x^4+5e^8x^6\right)+c\left(128d^8+64d^6e^2x^2+48d^4e^4x^4+40d^2e^6x^6+35e^8x^8\right)\right)}{315e^{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*a*e^4*(8*d^4 + 4*d^2*e^2*x^2 +
3*e^4*x^4) + 9*b*(16*d^6*e^2 + 8*d^4*e^4*x^2 + 6*d^2*e^6*x^4 + 5*
e^8*x^6) + c*(128*d^8 + 64*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 40*d^2*
e^6*x^6 + 35*e^8*x^8)))/(315*e^10)
```

Maple [A] time = 0.011, size = 145, normalized size = 0.7

$$-\frac{35\,cx^8e^8+45\,be^8x^6+40\,cd^2e^6x^6+63\,ae^8x^4+54\,bd^2e^6x^4+48\,cd^4e^4x^4+84\,ad^2e^6x^2+72\,bd^4e^4x^2+64\,cd^6e^2x^2+168\,ad^4e^4+144\,bd^6e^2+128\,cd^8}{315\,e^{10}}\sqrt{-ex+d}\sqrt{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/315*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(35*c*e^8*x^8+45*b*e^8*x^6+40
*c*d^2*e^6*x^6+63*a*e^8*x^4+54*b*d^2*e^6*x^4+48*c*d^4*e^4*x^4+84*
a*d^2*e^6*x^2+72*b*d^4*e^4*x^2+64*c*d^6*e^2*x^2+168*a*d^4*e^4+144
*b*d^6*e^2+128*c*d^8)/e^10
```

Maxima [A] time = 0.789336, size = 398, normalized size = 1.9

$$-\frac{\sqrt{-e^2x^2+d^2}cx^8}{9\,e^2}-\frac{8\,\sqrt{-e^2x^2+d^2}cd^2x^6}{63\,e^4}-\frac{\sqrt{-e^2x^2+d^2}bx^6}{7\,e^2}-\frac{16\,\sqrt{-e^2x^2+d^2}cd^4x^4}{105\,e^6}-\frac{6\,\sqrt{-e^2x^2+d^2}bd^2x^4}{35\,e^4}-\frac{\sqrt{-e^2x^2+d^2}ax^4}{5\,e^2}-\frac{64\,\sqrt{-e^2x^2+d^2}cd^6x^2}{315\,e^8}-\frac{8\,\sqrt{-e^2x^2+d^2}bd^4x^2}{35\,e^6}-\frac{4\,\sqrt{-e^2x^2+d^2}ad^2x^2}{15\,e^4}-\frac{128\,\sqrt{-e^2x^2+d^2}cd^8}{315\,e^{10}}-\frac{16\,\sqrt{-e^2x^2+d^2}bd^6}{35\,e^8}-\frac{8\,\sqrt{-e^2x^2+d^2}ad^4}{15\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^5/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-e^2*x^2 + d^2)*c*x^8/e^2 - 8/63*sqrt(-e^2*x^2 + d^2)*c
*d^2*x^6/e^4 - 1/7*sqrt(-e^2*x^2 + d^2)*b*x^6/e^2 - 16/105*sqrt(-
e^2*x^2 + d^2)*c*d^4*x^4/e^6 - 6/35*sqrt(-e^2*x^2 + d^2)*b*d^2*x^
4/e^4 - 1/5*sqrt(-e^2*x^2 + d^2)*a*x^4/e^2 - 64/315*sqrt(-e^2*x^2
 + d^2)*c*d^6*x^2/e^8 - 8/35*sqrt(-e^2*x^2 + d^2)*b*d^4*x^2/e^6 -
 4/15*sqrt(-e^2*x^2 + d^2)*a*d^2*x^2/e^4 - 128/315*sqrt(-e^2*x^2
+ d^2)*c*d^8/e^10 - 16/35*sqrt(-e^2*x^2 + d^2)*b*d^6/e^8 - 8/15*s
qrt(-e^2*x^2 + d^2)*a*d^4/e^6
```

Fricas [A] time = 0.283802, size = 554, normalized size = 2.64

$$-\frac{35\, ce^8 x^{18} - 45\left(31\, cd^2 e^6 - be^8\right) x^{16} + 13440\, ad^8 x^6 + 9\left(912\, cd^4 e^4 - 199\, bd^2 e^6 + 7\, ae^8\right) x^{14} - 21\left(704\, cd^6 e^2 - 498\, bd^4 e^4 + 11\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^5/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="fr
```

```
[Out] -1/315*(35*c*e^8*x^18 - 45*(31*c*d^2*e^6 - b*e^8)*x^16 + 13440*a*
d^8*x^6 + 9*(912*c*d^4*e^4 - 199*b*d^2*e^6 + 7*a*e^8)*x^14 - 21*(
704*c*d^6*e^2 - 498*b*d^4*e^4 + 119*a*d^2*e^6)*x^12 + 252*(32*c*d
^8 - 74*b*d^6*e^2 + 57*a*d^4*e^4)*x^10 + 5040*(2*b*d^8 - 5*a*d^6*
e^2)*x^8 + 3*(105*c*d*e^6*x^16 - 5*(256*c*d^3*e^4 - 27*b*d*e^6)*x
^14 - 4480*a*d^7*x^6 + 7*(512*c*d^5*e^2 - 234*b*d^3*e^4 + 27*a*d*
e^6)*x^12 - 84*(32*c*d^7 - 54*b*d^5*e^2 + 27*a*d^3*e^4)*x^10 - 56
0*(6*b*d^7 - 11*a*d^5*e^2)*x^8)*sqrt(e*x + d)*sqrt(-e*x + d))/(9*
d*e^8*x^8 - 120*d^3*e^6*x^6 + 432*d^5*e^4*x^4 - 576*d^7*e^2*x^2 +
 256*d^9 - (e^8*x^8 - 40*d^2*e^6*x^6 + 240*d^4*e^4*x^4 - 448*d^6*
e^2*x^2 + 256*d^8)*sqrt(e*x + d)*sqrt(-e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.300016, size = 328, normalized size = 1.56

$$-\frac{1}{2807562240}\left(315\, cd^8 e^{81} + 315\, bd^6 e^{83} + 315\, ad^4 e^{85} - \left(840\, cd^7 e^{81} + 630\, bd^5 e^{83} + 420\, ad^3 e^{85} - \left(1932\, cd^6 e^{81} + 1071\, bd^4 e^{83} +\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^5/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="gi
```

```
[Out] -1/2807562240*(315*c*d^8*e^81 + 315*b*d^6*e^83 + 315*a*d^4*e^85 -
 (840*c*d^7*e^81 + 630*b*d^5*e^83 + 420*a*d^3*e^85 - (1932*c*d^6*
e^81 + 1071*b*d^4*e^83 + 462*a*d^2*e^85 - (2952*c*d^5*e^81 + 1116
*b*d^3*e^83 + 252*a*d*e^85 - (3098*c*d^4*e^81 + 729*b*d^2*e^83 -
5*(440*c*d^3*e^81 + 54*b*d*e^83 - (204*c*d^2*e^81 + 7*((x*e + d)*
c*e^81 - 8*c*d*e^81)*(x*e + d) + 9*b*e^83)*(x*e + d))*(x*e + d) +
 63*a*e^85)*(x*e + d))*(x*e + d))*(x*e + d))*(x*e + d))*sqrt(x*e
+ d)*sqrt(-x*e + d)*e^(-1)
```

3.133 $\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=159

$$\frac{(d-ex)^{3/2}(d+ex)^{3/2}\left(ae^4+2bd^2e^2+3cd^4\right)}{3e^8}-\frac{d^2\sqrt{d-ex}\sqrt{d+ex}\left(ae^4+bd^2e^2+cd^4\right)}{e^8}-\frac{(d-ex)^{5/2}(d+ex)^{5/2}\left(be^2+3cd^2\right)}{5e^8}+\frac{c(d-ex)^{7/2}(d+ex)^{7/2}}{7e^8}$$

```
[Out] -((d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d - e*x]*Sqrt[d + e*x])/e
^8) + ((3*c*d^4 + 2*b*d^2*e^2 + a*e^4)*(d - e*x)^(3/2)*(d + e*x)^
(3/2))/(3*e^8) - ((3*c*d^2 + b*e^2)*(d - e*x)^(5/2)*(d + e*x)^(5/
2))/(5*e^8) + (c*(d - e*x)^(7/2)*(d + e*x)^(7/2))/(7*e^8)
```

Rubi [A] time = 0.555717, antiderivative size = 213, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$

$$\frac{\left(d^2-e^2x^2\right)^2\left(ae^4+2bd^2e^2+3cd^4\right)}{3e^8\sqrt{d-ex}\sqrt{d+ex}}-\frac{d^2\left(d^2-e^2x^2\right)\left(ae^4+bd^2e^2+cd^4\right)}{e^8\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)^3\left(be^2+3cd^2\right)}{5e^8\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\left(d^2-e^2x^2\right)^4}{7e^8\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^8*Sqrt[d -
 e*x]*Sqrt[d + e*x])) + ((3*c*d^4 + 2*b*d^2*e^2 + a*e^4)*(d^2 - e
^2*x^2)^2)/(3*e^8*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((3*c*d^2 + b*e^
2)*(d^2 - e^2*x^2)^3)/(5*e^8*Sqrt[d - e*x]*Sqrt[d + e*x]) + (c*(d
^2 - e^2*x^2)^4)/(7*e^8*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 29.0047, size = 177, normalized size = 1.11

$$\frac{c\sqrt{d-ex}\sqrt{d+ex}\left(d^2-e^2x^2\right)^3}{7e^8}-\frac{d^2\sqrt{d-ex}\sqrt{d+ex}\left(ae^4+bd^2e^2+cd^4\right)}{e^8}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(d^2-e^2x^2\right)^2\left(be^2+3cd^2\right)}{5e^8}+\frac{\sqrt{d-ex}\sqrt{d+ex}\left(d^2-e^2x^2\right)\left(ae^4+2bd^2e^2+3cd^4\right)}{3e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] c*sqrt(d - e*x)*sqrt(d + e*x)*(d**2 - e**2*x**2)**3/(7*e**8) - d*
*2*sqrt(d - e*x)*sqrt(d + e*x)*(a*e**4 + b*d**2*e**2 + c*d**4)/e*
*8 - sqrt(d - e*x)*sqrt(d + e*x)*(d**2 - e**2*x**2)**2*(b*e**2 +
3*c*d**2)/(5*e**8) + sqrt(d - e*x)*sqrt(d + e*x)*(d**2 - e**2*x**
2)*(a*e**4 + 2*b*d**2*e**2 + 3*c*d**4)/(3*e**8)
```

Mathematica [A] time = 0.128465, size = 116, normalized size = 0.73

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(35ae^4\left(2d^2+e^2x^2\right)+7b\left(8d^4e^2+4d^2e^4x^2+3e^6x^4\right)+3c\left(16d^6+8d^4e^2x^2+6d^2e^4x^4+5e^6x^6\right)\right)}{105e^8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*a*e^4*(2*d^2 + e^2*x^2) + 7*b*(
8*d^4*e^2 + 4*d^2*e^4*x^2 + 3*e^6*x^4) + 3*c*(16*d^6 + 8*d^4*e^2*
x^2 + 6*d^2*e^4*x^4 + 5*e^6*x^6)))/(105*e^8)
```

Maple [A] time = 0.01, size = 109, normalized size = 0.7

$$-\frac{15\,cx^6e^6+21\,be^6x^4+18\,cd^2e^4x^4+35\,ae^6x^2+28\,bd^2e^4x^2+24\,cd^4e^2x^2+70\,ad^2e^4+56\,bd^4e^2+48\,cd^6}{105\,e^8}\sqrt{-ex+d}\sqrt{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/105*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(15*c*e^6*x^6+21*b*e^6*x^4+18
*c*d^2*e^4*x^4+35*a*e^6*x^2+28*b*d^2*e^4*x^2+24*c*d^4*e^2*x^2+70*
a*d^2*e^4+56*b*d^4*e^2+48*c*d^6)/e^8
```

Maxima [A] time = 0.789057, size = 293, normalized size = 1.84

$$-\frac{\sqrt{-e^2x^2+d^2}cx^6}{7\,e^2}-\frac{6\,\sqrt{-e^2x^2+d^2}cd^2x^4}{35\,e^4}-\frac{\sqrt{-e^2x^2+d^2}bx^4}{5\,e^2}-\frac{8\,\sqrt{-e^2x^2+d^2}cd^4x^2}{35\,e^6}-\frac{4\,\sqrt{-e^2x^2+d^2}bd^2x^2}{15\,e^4}-\frac{\sqrt{-e^2x^2+d^2}ax^2}{3\,e^2}-\frac{16\,\sqrt{-e^2x^2+d^2}cd^6}{35\,e^8}-\frac{8\,\sqrt{-e^2x^2+d^2}bd^4}{15\,e^6}-\frac{2\,\sqrt{-e^2x^2+d^2}ad^2}{3\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^3/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="ma
```

```
[Out] -1/7*sqrt(-e^2*x^2 + d^2)*c*x^6/e^2 - 6/35*sqrt(-e^2*x^2 + d^2)*c
*d^2*x^4/e^4 - 1/5*sqrt(-e^2*x^2 + d^2)*b*x^4/e^2 - 8/35*sqrt(-e^
2*x^2 + d^2)*c*d^4*x^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*b*d^2*x^2/
e^4 - 1/3*sqrt(-e^2*x^2 + d^2)*a*x^2/e^2 - 16/35*sqrt(-e^2*x^2 +
d^2)*c*d^6/e^8 - 8/15*sqrt(-e^2*x^2 + d^2)*b*d^4/e^6 - 2/3*sqrt(-
e^2*x^2 + d^2)*a*d^2/e^4
```

Fricas [A] time = 0.274299, size = 435, normalized size = 2.74

$$-\frac{15\,ce^6x^{14}-21\left(17\,cd^2e^4-be^6\right)x^{12}-1680\,ad^6x^4+7\left(162\,cd^4e^2-71\,bd^2e^4+5\,ae^6\right)x^{10}-35\left(24\,cd^6-44\,bd^4e^2+23\,ad^2e^4\right.}{105\left(7\,de^6x^6-56\,d^3e^4x^4+11\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^3/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="fr
```

```
[Out] -1/105*(15*c*e^6*x^14 - 21*(17*c*d^2*e^4 - b*e^6)*x^12 - 1680*a*d
^6*x^4 + 7*(162*c*d^4*e^2 - 71*b*d^2*e^4 + 5*a*e^6)*x^10 - 35*(24
*c*d^6 - 44*b*d^4*e^2 + 23*a*d^2*e^4)*x^8 - 140*(8*b*d^6 - 17*a*d
^4*e^2)*x^6 + 7*(15*c*d*e^4*x^12 - 3*(34*c*d^3*e^2 - 7*b*d*e^4)*x
^10 + 240*a*d^5*x^4 + 5*(24*c*d^5 - 28*b*d^3*e^2 + 7*a*d*e^4)*x^8
```

```
 + 20*(8*b*d^5 - 11*a*d^3*e^2)*x^6)*sqrt(e*x + d)*sqrt(-e*x + d))
/(7*d*e^6*x^6 - 56*d^3*e^4*x^4 + 112*d^5*e^2*x^2 - 64*d^7 - (e^6*
x^6 - 24*d^2*e^4*x^4 + 80*d^4*e^2*x^2 - 64*d^6)*sqrt(e*x + d)*sqr
t(-e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.295257, size = 239, normalized size = 1.5

$$-\frac{1}{44728320}\left(105\,cd^6e^{49}+105\,bd^4e^{51}+105\,ad^2e^{53}-\left(210\,cd^5e^{49}+140\,bd^3e^{51}+70\,ade^{53}-\left(357\,cd^4e^{49}+154\,bd^2e^{51}-3\,(12\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^3/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="gi
```

```
[Out] -1/44728320*(105*c*d^6*e^49 + 105*b*d^4*e^51 + 105*a*d^2*e^53 - (
210*c*d^5*e^49 + 140*b*d^3*e^51 + 70*a*d*e^53 - (357*c*d^4*e^49 +
 154*b*d^2*e^51 - 3*(124*c*d^3*e^49 + 28*b*d*e^51 - (81*c*d^2*e^4
9 + 5*((x*e + d)*c*e^49 - 6*c*d*e^49)*(x*e + d) + 7*b*e^51)*(x*e
+ d))*(x*e + d) + 35*a*e^53)*(x*e + d))*(x*e + d))*sqrt(x*e + d)*
sqrt(-x*e + d)*e^(-1)
```

3.134 $\int \frac{x\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(ae^4+bd^2e^2+cd^4\right)}{e^6}+\frac{(d-ex)^{3/2}(d+ex)^{3/2}\left(be^2+2cd^2\right)}{3e^6}-\frac{c(d-ex)^{5/2}(d+ex)^{5/2}}{5e^6}$$

```
[Out] -(((c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d - e*x]*Sqrt[d + e*x])/e^6)
+ ((2*c*d^2 + b*e^2)*(d - e*x)^(3/2)*(d + e*x)^(3/2))/(3*e^6) - (
c*(d - e*x)^(5/2)*(d + e*x)^(5/2))/(5*e^6)
```

Rubi [A] time = 0.35672, antiderivative size = 149, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{\left(d^2-e^2x^2\right)\left(ae^4+bd^2e^2+cd^4\right)}{e^6\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(d^2-e^2x^2\right)^2\left(be^2+2cd^2\right)}{3e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{c\left(d^2-e^2x^2\right)^3}{5e^6\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(((c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^6*Sqrt[d - e*x
]*Sqrt[d + e*x])) + ((2*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^2)/(3*e^6*
Sqrt[d - e*x]*Sqrt[d + e*x]) - (c*(d^2 - e^2*x^2)^3)/(5*e^6*Sqrt[
d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 19.777, size = 117, normalized size = 1.07

$$-\frac{c\sqrt{d-ex}\sqrt{d+ex}\left(d^2-e^2x^2\right)^2}{5e^6}+\frac{\sqrt{d-ex}\sqrt{d+ex}\left(d^2-e^2x^2\right)\left(be^2+2cd^2\right)}{3e^6}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(ae^4+bd^2e^2+cd^4\right)}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -c*sqrt(d - e*x)*sqrt(d + e*x)*(d**2 - e**2*x**2)**2/(5*e**6) + s
qrt(d - e*x)*sqrt(d + e*x)*(d**2 - e**2*x**2)*(b*e**2 + 2*c*d**2)
/(3*e**6) - sqrt(d - e*x)*sqrt(d + e*x)*(a*e**4 + b*d**2*e**2 + c
*d**4)/e**6
```

Mathematica [A] time = 0.0930085, size = 80, normalized size = 0.73

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(5\left(3ae^4+2bd^2e^2+be^4x^2\right)+c\left(8d^4+4d^2e^2x^2+3e^4x^4\right)\right)}{15e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(5*(2*b*d^2*e^2 + 3*a*e^4 + b*e^4*x
^2) + c*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^4*x^4)))/(15*e^6)
```

Maple [A] time = 0.007, size = 73, normalized size = 0.7

$$-\frac{3\,cx^{4}e^{4}+5\,be^{4}x^{2}+4\,cd^{2}e^{2}x^{2}+15\,ae^{4}+10\,bd^{2}e^{2}+8\,cd^{4}}{15\,e^{6}}\sqrt{-ex+d}\sqrt{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(3*c*e^4*x^4+5*b*e^4*x^2+4*c*d
^2*e^2*x^2+15*a*e^4+10*b*d^2*e^2+8*c*d^4)/e^6
```

Maxima [A] time = 0.797571, size = 188, normalized size = 1.72

$$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}cx^{4}}{5\,e^{2}}-\frac{4\,\sqrt{-e^{2}x^{2}+d^{2}}cd^{2}x^{2}}{15\,e^{4}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}bx^{2}}{3\,e^{2}}-\frac{8\,\sqrt{-e^{2}x^{2}+d^{2}}cd^{4}}{15\,e^{6}}-\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}bd^{2}}{3\,e^{4}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}a}{e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="maxi
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*c*x^4/e^2 - 4/15*sqrt(-e^2*x^2 + d^2)*c
*d^2*x^2/e^4 - 1/3*sqrt(-e^2*x^2 + d^2)*b*x^2/e^2 - 8/15*sqrt(-e^
2*x^2 + d^2)*c*d^4/e^6 - 2/3*sqrt(-e^2*x^2 + d^2)*b*d^2/e^4 - sqr
t(-e^2*x^2 + d^2)*a/e^2
```

Fricas [A] time = 0.271901, size = 313, normalized size = 2.87

$$-\frac{3\,ce^{4}x^{10}-5\left(7\,cd^{2}e^{2}-be^{4}\right)x^{8}+120\,ad^{4}x^{2}+5\left(8\,cd^{4}-11\,bd^{2}e^{2}+3\,ae^{4}\right)x^{6}+60\left(bd^{4}-2\,ad^{2}e^{2}\right)x^{4}+5\left(3\,cde^{2}x^{8}-\left(8\,cd^{3}\right.\right.}{15\left(5\,de^{4}x^{4}-20\,d^{3}e^{2}x^{2}+16\,d^{5}-(e^{4}x^{4}-12\,d^{2}e^{2}x^{2}+16\,d^{4})\sqrt{ex+d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="fric
```

```
[Out] -1/15*(3*c*e^4*x^10 - 5*(7*c*d^2*e^2 - b*e^4)*x^8 + 120*a*d^4*x^2
 + 5*(8*c*d^4 - 11*b*d^2*e^2 + 3*a*e^4)*x^6 + 60*(b*d^4 - 2*a*d^2
*e^2)*x^4 + 5*(3*c*d*e^2*x^8 - (8*c*d^3 - 5*b*d*e^2)*x^6 - 24*a*d
^3*x^2 - 12*(b*d^3 - a*d*e^2)*x^4)*sqrt(e*x + d)*sqrt(-e*x + d))/
(5*d*e^4*x^4 - 20*d^3*e^2*x^2 + 16*d^5 - (e^4*x^4 - 12*d^2*e^2*x^
2 + 16*d^4)*sqrt(e*x + d)*sqrt(-e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.286287, size = 153, normalized size = 1.4

$$-\frac{1}{276480}\left(15\,cd^4e^{25} + 15\,bd^2e^{27} - \left(20\,cd^3e^{25} + 10\,bde^{27} - \left(22\,cd^2e^{25} + 3\left((xe+d)ce^{25} - 4\,cde^{25}\right)(xe+d) + 5\,be^{27}\right)(xe+d)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="giac
```

```
[Out] -1/276480*(15*c*d^4*e^25 + 15*b*d^2*e^27 - (20*c*d^3*e^25 + 10*b*
d*e^27 - (22*c*d^2*e^25 + 3*((x*e + d)*c*e^25 - 4*c*d*e^25)*(x*e
+ d) + 5*b*e^27)*(x*e + d))*(x*e + d) + 15*a*e^29)*sqrt(x*e + d)*
sqrt(-x*e + d)*e^(-1)
```

3.135 $\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=93

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{d}-\frac{\sqrt{d-ex}\sqrt{d+ex}\,(be^2+cd^2)}{e^4}+\frac{c(d-ex)^{3/2}(d+ex)^{3/2}}{3e^4}$$

```
[Out] -(((c*d^2 + b*e^2)*Sqrt[d - e*x]*Sqrt[d + e*x])/e^4) + (c*(d - e*
x)^(3/2)*(d + e*x)^(3/2))/(3*e^4) - (a*ArcTanh[(Sqrt[d - e*x]*Sqr
t[d + e*x])/d])/d
```

Rubi [A] time = 0.482115, antiderivative size = 151, normalized size of antiderivative = 1.62, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{a\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)\,(be^2+cd^2)}{e^4\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\,(d^2-e^2x^2)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(((c*d^2 + b*e^2)*(d^2 - e^2*x^2))/(e^4*Sqrt[d - e*x]*Sqrt[d + e
*x])) + (c*(d^2 - e^2*x^2)^2)/(3*e^4*Sqrt[d - e*x]*Sqrt[d + e*x])
 - (a*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(d*Sqrt
[d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 33.8527, size = 114, normalized size = 1.23

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}\,\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d^2-e^2x^2}}+\frac{c\sqrt{d-ex}\sqrt{d+ex}\,(d^2-e^2x^2)}{3e^4}-\frac{\sqrt{d-ex}\sqrt{d+ex}\,(be^2+cd^2)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -a*sqrt(d - e*x)*sqrt(d + e*x)*atanh(sqrt(d**2 - e**2*x**2)/d)/(d
*sqrt(d**2 - e**2*x**2)) + c*sqrt(d - e*x)*sqrt(d + e*x)*(d**2 -
e**2*x**2)/(3*e**4) - sqrt(d - e*x)*sqrt(d + e*x)*(b*e**2 + c*d**
2)/e**4
```

Mathematica [A] time = 0.209577, size = 84, normalized size = 0.9

$$-\frac{a\log\left(\sqrt{d-ex}\sqrt{d+ex}+d\right)}{d}+\frac{a\log(x)}{d}-\frac{\sqrt{d-ex}\sqrt{d+ex}\,(3be^2+2cd^2+ce^2x^2)}{3e^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(2*c*d^2 + 3*b*e^2 + c*e^2*x^2))/(3
*e^4) + (a*Log[x])/d - (a*Log[d + Sqrt[d - e*x]*Sqrt[d + e*x]])/d
```

Maple [C] time = 0.048, size = 143, normalized size = 1.5

$$-\frac{csgn(d)}{3\,de^4}\sqrt{-ex+d}\sqrt{ex+d}\left(csgn(d)\,x^2cde^2\sqrt{-e^2x^2+d^2}+3\,\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2+d^2}csgn(d)+d\right)}{x}\right)ae^4+3\,csgn(d)\,\sqrt{-e^2x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d*(csgn(d)*x^2*c*d*e^2*(-e^2*x^
2+d^2)^(1/2)+3*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*a*e^4+3
*csgn(d)*(-e^2*x^2+d^2)^(1/2)*b*d*e^2+2*csgn(d)*(-e^2*x^2+d^2)^(1
/2)*c*d^3)*csgn(d)/(-e^2*x^2+d^2)^(1/2)/e^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x),x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279206, size = 270, normalized size = 2.9

$$-\frac{cde^2x^6-6\,bd^3x^2-3\left(cd^3-bde^2\right)x^4+3\left(cd^2x^4+2\,bd^2x^2\right)\sqrt{ex+d}\sqrt{-ex+d}-3\left(3\,ade^2x^2-4\,ad^3-\left(ae^2x^2-4\,ad^2\right)\sqrt{e}\right.}{3\left(3\,d^2e^2x^2-4\,d^4-(de^2x^2-4\,d^3)\sqrt{ex+d}\sqrt{-ex+d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x),x, algorithm="fric
```

```
[Out] -1/3*(c*d*e^2*x^6 - 6*b*d^3*x^2 - 3*(c*d^3 - b*d*e^2)*x^4 + 3*(c*
d^2*x^4 + 2*b*d^2*x^2)*sqrt(e*x + d)*sqrt(-e*x + d) - 3*(3*a*d*e^
2*x^2 - 4*a*d^3 - (a*e^2*x^2 - 4*a*d^2)*sqrt(e*x + d)*sqrt(-e*x +
 d))*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x))/(3*d^2*e^2*x^2 -
4*d^4 - (d*e^2*x^2 - 4*d^3)*sqrt(e*x + d)*sqrt(-e*x + d))
```

Sympy [A] time = 101.102, size = 304, normalized size = 3.27

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibdG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2}-\frac{bdG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2}-\frac{icd^3G_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}-\frac{cd^3G_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2
), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d) - a*meijerg(((0, 1/4,
 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_p
olar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d) - I*b*d*meijerg(((-1/4
, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), d**2
/(e**2*x**2))/(4*pi**(3/2)*e**2) - b*d*meijerg(((-1, -3/4, -1/2,
-1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_p
olar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*c*d**3*meijerg(
((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2,
0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**4) - c*d**3*meijerg(((
-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2
, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**4)
```

GIAC/XCAS [A] time = 0.670355, size = 4, normalized size = 0.04

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x),x, algorithm="giac
```

```
[Out] sage0*x
```

3.136 $\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=99

$$-\frac{\left(ae^2+2bd^2\right)\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{2d^3}-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{2d^2x^2}-\frac{c\sqrt{d-ex}\sqrt{d+ex}}{e^2}$$

```
[Out] -((c*Sqrt[d - e*x]*Sqrt[d + e*x])/e^2) - (a*Sqrt[d - e*x]*Sqrt[d
+ e*x])/(2*d^2*x^2) - ((2*b*d^2 + a*e^2)*ArcTanh[(Sqrt[d - e*x]*S
qrt[d + e*x])/d])/(2*d^3)
```

Rubi [A] time = 0.55525, antiderivative size = 155, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$

$$-\frac{\sqrt{d^2-e^2x^2}\left(ae^2+2bd^2\right)\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{c\left(d^2-e^2x^2\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((c*(d^2 - e^2*x^2))/(e^2*Sqrt[d - e*x]*Sqrt[d + e*x])) - (a*(d^
2 - e^2*x^2))/(2*d^2*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((2*b*d^2
 + a*e^2)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*
d^3*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 36.2349, size = 114, normalized size = 1.15

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{2d^2x^2}-\frac{c\sqrt{d-ex}\sqrt{d+ex}}{e^2}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(ae^2+2bd^2\right)\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -a*sqrt(d - e*x)*sqrt(d + e*x)/(2*d**2*x**2) - c*sqrt(d - e*x)*sq
rt(d + e*x)/e**2 - sqrt(d - e*x)*sqrt(d + e*x)*(a*e**2 + 2*b*d**2
)*atanh(sqrt(d**2 - e**2*x**2)/d)/(2*d**3*sqrt(d**2 - e**2*x**2))
```

Mathematica [A] time = 0.272164, size = 102, normalized size = 1.03

$$\frac{\log(x)\left(ae^2+2bd^2\right)}{2d^3}-\frac{\left(ae^2+2bd^2\right)\log\left(\sqrt{d-ex}\sqrt{d+ex}+d\right)}{2d^3}+\sqrt{d-ex}\sqrt{d+ex}\left(-\frac{a}{2d^2x^2}-\frac{c}{e^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(c/e^2) - a/(2*d^2*x^2))*Sqrt[d - e*x]*Sqrt[d + e*x] + ((2*b*d^
2 + a*e^2)*Log[x])/(2*d^3) - ((2*b*d^2 + a*e^2)*Log[d + Sqrt[d -
e*x]*Sqrt[d + e*x]])/(2*d^3)
```

Maple [C] time = 0.032, size = 163, normalized size = 1.7

$$-\frac{csgn(d)}{2\,d^3x^2e^2}\sqrt{-ex+d}\sqrt{ex+d}\left(2\,csgn(d)\,x^2cd^3\sqrt{-e^2x^2+d^2}+\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2+d^2}csgn(d)+d\right)}{x}\right)x^2ae^4+2\,\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/2*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^3*(2*csgn(d)*x^2*c*d^3*(-e^2*
x^2+d^2)^(1/2)+ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*x^2*a*e
^4+2*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*x^2*b*d^2*e^2+csg
n(d)*a*d*e^2*(-e^2*x^2+d^2)^(1/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/x
^2/e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^3),x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265945, size = 378, normalized size = 3.82

$$-\frac{2\,cd^3e^2x^6-5\,ad^3e^2x^2+4\,ad^5-\left(4\,cd^5-ade^4\right)x^4+\left(4\,cd^4x^4+3\,ad^2e^2x^2-4\,ad^4\right)\sqrt{ex+d}\sqrt{-ex+d}-\left(3\left(2\,bd^3e^2+ade^4\right)\right.}{2\left(3\,d^4e^2x^4-4\,d^6x^2-(d^3e^2x^4-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^3),x, algorithm="fr
```

```
[Out] -1/2*(2*c*d^3*e^2*x^6 - 5*a*d^3*e^2*x^2 + 4*a*d^5 - (4*c*d^5 - a*
d*e^4)*x^4 + (4*c*d^4*x^4 + 3*a*d^2*e^2*x^2 - 4*a*d^4)*sqrt(e*x +
 d)*sqrt(-e*x + d) - (3*(2*b*d^3*e^2 + a*d*e^4)*x^4 - 4*(2*b*d^5
+ a*d^3*e^2)*x^2 - ((2*b*d^2*e^2 + a*e^4)*x^4 - 4*(2*b*d^4 + a*d^
2*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))*log((sqrt(e*x + d)*sqrt
(-e*x + d) - d)/x))/(3*d^4*e^2*x^4 - 4*d^6*x^2 - (d^3*e^2*x^4 - 4
*d^5*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))
```

Sympy [A] time = 144.224, size = 270, normalized size = 2.73

$$\frac{iae^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{ae^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{ibG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{bG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{icdG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2}-\frac{cdG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4
, 5/2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**3) - a*e**2*meij
erg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)
), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**3) + I*b*
meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (
0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d) - b*meijerg(((0, 1/4, 1/2
, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_polar
(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d) - I*c*d*meijerg(((-1/4, 1/
4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), d**2/(e*
*2*x**2))/(4*pi**(3/2)*e**2) - c*d*meijerg(((-1, -3/4, -1/2, -1/4
, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_polar
(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2)
```

GIAC/XCAS [A] time = 0.684055, size = 4, normalized size = 0.04

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^3),x, algorithm="gi
```

```
[Out] sage0*x
```

3.137 $\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=126

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)\left(3ae^4+4bd^2e^2+8cd^4\right)}{8d^5}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(3ae^2+4bd^2\right)}{8d^4x^2}-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{4d^2x^4}$$

```
[Out] -(a*Sqrt[d - e*x]*Sqrt[d + e*x])/(4*d^2*x^4) - ((4*b*d^2 + 3*a*e^
2)*Sqrt[d - e*x]*Sqrt[d + e*x])/(8*d^4*x^2) - ((8*c*d^4 + 4*b*d^2
*e^2 + 3*a*e^4)*ArcTanh[(Sqrt[d - e*x]*Sqrt[d + e*x])/d])/(8*d^5)
```

Rubi [A] time = 0.62503, antiderivative size = 182, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$-\frac{\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(3ae^4+4bd^2e^2+8cd^4\right)}{8d^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(3ae^2+4bd^2\right)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(4*d^2*x^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((
4*b*d^2 + 3*a*e^2)*(d^2 - e^2*x^2))/(8*d^4*x^2*Sqrt[d - e*x]*Sqrt
[d + e*x]) - ((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*Sqrt[d^2 - e^2*x^
2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^5*Sqrt[d - e*x]*Sqrt[d +
e*x])
```

Rubi in Sympy [A] time = 38.3906, size = 143, normalized size = 1.13

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{4d^2x^4}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(3ae^2+4bd^2\right)}{8d^4x^2}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(3ae^4+4bd^2e^2+8cd^4\right)\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^5\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)/x**5/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -a*sqrt(d - e*x)*sqrt(d + e*x)/(4*d**2*x**4) - sqrt(d - e*x)*sqrt
(d + e*x)*(3*a*e**2 + 4*b*d**2)/(8*d**4*x**2) - sqrt(d - e*x)*sqr
t(d + e*x)*(3*a*e**4 + 4*b*d**2*e**2 + 8*c*d**4)*atanh(sqrt(d**2
- e**2*x**2)/d)/(8*d**5*sqrt(d**2 - e**2*x**2))
```

Mathematica [A] time = 0.247853, size = 135, normalized size = 1.07

$$-\frac{x^4\log(x)\left(-\left(3ae^4+4bd^2e^2+8cd^4\right)\right)+x^4\log\left(\sqrt{d-ex}\sqrt{d+ex}+d\right)\left(3ae^4+4bd^2e^2+8cd^4\right)+d\sqrt{d-ex}\sqrt{d+ex}\left(2ad^2+\right.}{8d^5x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(d*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*d^2 + 4*b*d^2*x^2 + 3*a*e^2*
x^2) - (8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*x^4*Log[x] + (8*c*d^4 +
4*b*d^2*e^2 + 3*a*e^4)*x^4*Log[d + Sqrt[d - e*x]*Sqrt[d + e*x]])/
(8*d^5*x^4)
```

Maple [C] time = 0.033, size = 222, normalized size = 1.8

$$-\frac{csgn(d)}{8\,d^5x^4}\sqrt{-ex+d}\sqrt{ex+d}\left(3\,\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2+d^2}\,csgn(d)+d\right)}{x}\right)x^4ae^4+4\,\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2+d^2}\,csgn(d)+d\right)}{x}\right)x^4bd^2e^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/8*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^5*(3*ln(2*d*((-e^2*x^2+d^2)^(
1/2)*csgn(d)+d)/x)*x^4*a*e^4+4*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(
d)+d)/x)*x^4*b*d^2*e^2+8*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/
x)*x^4*c*d^4+3*csgn(d)*x^2*a*d*e^2*(-e^2*x^2+d^2)^(1/2)+4*csgn(d)
*x^2*b*d^3*(-e^2*x^2+d^2)^(1/2)+2*csgn(d)*a*d^3*(-e^2*x^2+d^2)^(1
/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^5),x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.303873, size = 567, normalized size = 4.5

$$\frac{32\,bd^8x^2+16\,ad^8+4\left(4\,bd^4e^4+3\,ad^2e^6\right)x^6-4\left(12\,bd^6e^2+7\,ad^4e^4\right)x^4-\left(16\,ad^7+\left(4\,bd^3e^4+3\,ade^6\right)x^6-2\left(16\,bd^5e^2+11\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^5),x, algorithm="fr
```

```
[Out] 1/8*(32*b*d^8*x^2 + 16*a*d^8 + 4*(4*b*d^4*e^4 + 3*a*d^2*e^6)*x^6
- 4*(12*b*d^6*e^2 + 7*a*d^4*e^4)*x^4 - (16*a*d^7 + (4*b*d^3*e^4 +
 3*a*d*e^6)*x^6 - 2*(16*b*d^5*e^2 + 11*a*d^3*e^4)*x^4 + 8*(4*b*d^
7 + a*d^5*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d) + ((8*c*d^4*e^4
+ 4*b*d^2*e^6 + 3*a*e^8)*x^8 - 8*(8*c*d^6*e^2 + 4*b*d^4*e^4 + 3*a
*d^2*e^6)*x^6 + 8*(8*c*d^8 + 4*b*d^6*e^2 + 3*a*d^4*e^4)*x^4 + 4*(
(8*c*d^5*e^2 + 4*b*d^3*e^4 + 3*a*d*e^6)*x^6 - 2*(8*c*d^7 + 4*b*d^
5*e^2 + 3*a*d^3*e^4)*x^4)*sqrt(e*x + d)*sqrt(-e*x + d))*log((sqrt
(e*x + d)*sqrt(-e*x + d) - d)/x))/(d^5*e^4*x^8 - 8*d^7*e^2*x^6 +
8*d^9*x^4 + 4*(d^6*e^2*x^6 - 2*d^8*x^4)*sqrt(e*x + d)*sqrt(-e*x +
 d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**5/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.705993, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^5),x, algorithm="gi
```

```
[Out] sage0*x
```

3.138 $\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=212

$$-\frac{e^2\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^6x^2}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(5ae^2+6bd^2\right)}{24d^4x^4}-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{6d^2x^6}$$

```
[Out] -(a*Sqrt[d - e*x]*Sqrt[d + e*x])/(6*d^2*x^6) - ((6*b*d^2 + 5*a*e^
2)*Sqrt[d - e*x]*Sqrt[d + e*x])/(24*d^4*x^4) - ((8*c*d^4 + 6*b*d^
2*e^2 + 5*a*e^4)*Sqrt[d - e*x]*Sqrt[d + e*x])/(16*d^6*x^2) - (e^2
*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d])/(16*d^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi [A] time = 0.758547, antiderivative size = 248, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^6x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(5ae^2+6bd^2\right)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(6*d^2*x^6*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((
6*b*d^2 + 5*a*e^2)*(d^2 - e^2*x^2))/(24*d^4*x^4*Sqrt[d - e*x]*Sqr
t[d + e*x]) - ((8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*(d^2 - e^2*x^2))
/(16*d^6*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - (e^2*(8*c*d^4 + 6*b*d
^2*e^2 + 5*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d])/(16*d^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 42.9818, size = 196, normalized size = 0.92

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{6d^2x^6}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(5ae^2+6bd^2\right)}{24d^4x^4}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^6x^2}-\frac{e^2\sqrt{d-ex}\sqrt{d+ex}\left(5ae^4+6bd^2e^2+8cd^4\right)\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^7\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)/x**7/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -a*sqrt(d - e*x)*sqrt(d + e*x)/(6*d**2*x**6) - sqrt(d - e*x)*sqrt
(d + e*x)*(5*a*e**2 + 6*b*d**2)/(24*d**4*x**4) - sqrt(d - e*x)*sq
rt(d + e*x)*(5*a*e**4 + 6*b*d**2*e**2 + 8*c*d**4)/(16*d**6*x**2)
- e**2*sqrt(d - e*x)*sqrt(d + e*x)*(5*a*e**4 + 6*b*d**2*e**2 + 8*
c*d**4)*atanh(sqrt(d**2 - e**2*x**2)/d)/(16*d**7*sqrt(d**2 - e**2
*x**2))
```

Mathematica [A] time = 0.409857, size = 177, normalized size = 0.83

$$\frac{e^2\log(x)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^7}-\frac{e^2\log\left(\sqrt{d-ex}\sqrt{d+ex}+d\right)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^7}+\sqrt{d-ex}\sqrt{d+ex}\left(\frac{-5ae^4-6bd^2e^2-8cd^4}{16d^6x^2}+\frac{-5ae^2-6bd^2}{24d^4x^4}-\frac{a}{6d^2x^6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-a/(6*d^2*x^6) + (-6*b*d^2 - 5*a*e^2)/(24*d^4*x^4) + (-8*c*d^4 -
 6*b*d^2*e^2 - 5*a*e^4)/(16*d^6*x^2))*Sqrt[d - e*x]*Sqrt[d + e*x]
 + (e^2*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*Log[x])/(16*d^7) - (e^2
*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*Log[d + Sqrt[d - e*x]*Sqrt[d +
 e*x]])/(16*d^7)
```

Maple [C] time = 0.052, size = 306, normalized size = 1.4

$$-\frac{csgn(d)}{48\,d^7x^6}\sqrt{-ex+d}\sqrt{ex+d}\left(15\,\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2+d^2}csgn(d)+d\right)}{x}\right)x^6ae^6+18\,\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2+d^2}csgn(d)+d\right)}{x}\right)x^6bd^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/48*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^7*(15*ln(2*d*((-e^2*x^2+d^2)
^(1/2)*csgn(d)+d)/x)*x^6*a*e^6+18*ln(2*d*((-e^2*x^2+d^2)^(1/2)*cs
gn(d)+d)/x)*x^6*b*d^2*e^4+24*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)
+d)/x)*x^6*c*d^4*e^2+15*csgn(d)*x^4*a*d*e^4*(-e^2*x^2+d^2)^(1/2)+
18*csgn(d)*x^4*b*d^3*e^2*(-e^2*x^2+d^2)^(1/2)+24*csgn(d)*x^4*c*d^
5*(-e^2*x^2+d^2)^(1/2)+10*csgn(d)*x^2*a*d^3*e^2*(-e^2*x^2+d^2)^(1
/2)+12*csgn(d)*x^2*b*d^5*(-e^2*x^2+d^2)^(1/2)+8*csgn(d)*a*d^5*(-e
^2*x^2+d^2)^(1/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/x^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^7),x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.483568, size = 923, normalized size = 4.35

$$-\frac{256\,ad^{12}-18\left(8\,cd^6e^6+6\,bd^4e^8+5\,ad^2e^{10}\right)x^{10}+6\left(152\,cd^8e^4+102\,bd^6e^6+85\,ad^4e^8\right)x^8-4\left(384\,cd^{10}e^2+174\,bd^8e^4+15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^7),x, algorithm="fr
```

```
[Out] -1/48*(256*a*d^12 - 18*(8*c*d^6*e^6 + 6*b*d^4*e^8 + 5*a*d^2*e^10)
*x^10 + 6*(152*c*d^8*e^4 + 102*b*d^6*e^6 + 85*a*d^4*e^8)*x^8 - 4*
(384*c*d^10*e^2 + 174*b*d^8*e^4 + 157*a*d^6*e^6)*x^6 + 48*(16*c*d
^12 - 4*b*d^10*e^2 + 3*a*d^8*e^4)*x^4 + 192*(2*b*d^12 - a*d^10*e^
2)*x^2 - (256*a*d^11 - 3*(8*c*d^5*e^6 + 6*b*d^3*e^8 + 5*a*d*e^10)
*x^10 + 4*(108*c*d^7*e^4 + 78*b*d^5*e^6 + 65*a*d^3*e^8)*x^8 - 4*(
288*c*d^9*e^2 + 162*b*d^7*e^4 + 137*a*d^5*e^6)*x^6 + 48*(16*c*d^1
1 + 3*a*d^7*e^4)*x^4 + 64*(6*b*d^11 - a*d^9*e^2)*x^2)*sqrt(e*x +
d)*sqrt(-e*x + d) - 3*((8*c*d^4*e^8 + 6*b*d^2*e^10 + 5*a*e^12)*x^
12 - 18*(8*c*d^6*e^6 + 6*b*d^4*e^8 + 5*a*d^2*e^10)*x^10 + 48*(8*c
*d^8*e^4 + 6*b*d^6*e^6 + 5*a*d^4*e^8)*x^8 - 32*(8*c*d^10*e^2 + 6*
b*d^8*e^4 + 5*a*d^6*e^6)*x^6 + 2*(3*(8*c*d^5*e^6 + 6*b*d^3*e^8 +
5*a*d*e^10)*x^10 - 16*(8*c*d^7*e^4 + 6*b*d^5*e^6 + 5*a*d^3*e^8)*x
^8 + 16*(8*c*d^9*e^2 + 6*b*d^7*e^4 + 5*a*d^5*e^6)*x^6)*sqrt(e*x +
 d)*sqrt(-e*x + d))*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x))/(d
^7*e^6*x^12 - 18*d^9*e^4*x^10 + 48*d^11*e^2*x^8 - 32*d^13*x^6 + 2
*(3*d^8*e^4*x^10 - 16*d^10*e^2*x^8 + 16*d^12*x^6)*sqrt(e*x + d)*s
qrt(-e*x + d))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: MellinTransformStripError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**7/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Exception raised: MellinTransformStripError
```

GIAC/XCAS [A] time = 0.743374, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^7),x, algorithm="gi
```

```
[Out] sage0*x
```

3.139 $\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=216

$$\frac{d^2\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(8ae^4+6bd^2e^2+5cd^4)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{x\sqrt{d-ex}\sqrt{d+ex}\,(8ae^4+6bd^2e^2+5cd^4)}{16e^6}-\frac{x^3\sqrt{d-ex}\sqrt{d+ex}\,(6be^2+5cd^2)}{24e^4}+\frac{cx^5(ex-d)\sqrt{d+ex}}{6e^2\sqrt{d-ex}}$$

```
[Out] -((5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*x*Sqrt[d - e*x]*Sqrt[d + e*x]
)/(16*e^6) - ((5*c*d^2 + 6*b*e^2)*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]
)/(24*e^4) + (c*x^5*(-d + e*x)*Sqrt[d + e*x])/(6*e^2*Sqrt[d - e*x
]) + (d^2*(5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^7*Sqrt[d - e*x]*Sqrt[d +
e*x])
```

Rubi [A] time = 0.601384, antiderivative size = 245, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$

$$\frac{d^2\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(8ae^4+6bd^2e^2+5cd^4)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{x\,(d^2-e^2x^2)\,(8ae^4+6bd^2e^2+5cd^4)}{16e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{x^3\,(d^2-e^2x^2)\,(6be^2+5cd^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx^5\,(d^2-e^2x^2)}{6e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*x*(d^2 - e^2*x^2))/(16*e^6*Sq
rt[d - e*x]*Sqrt[d + e*x]) - ((5*c*d^2 + 6*b*e^2)*x^3*(d^2 - e^2*
x^2))/(24*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - (c*x^5*(d^2 - e^2*x^
2))/(6*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + (d^2*(5*c*d^4 + 6*b*d^2
*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x
^2]])/(16*e^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 27.8546, size = 194, normalized size = 0.9

$$-\frac{cx^5\sqrt{d-ex}\sqrt{d+ex}}{6e^2}+\frac{d^2\sqrt{d-ex}\sqrt{d+ex}\,(8ae^4+6bd^2e^2+5cd^4)\operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^7\sqrt{d^2-e^2x^2}}-\frac{x^3\sqrt{d-ex}\sqrt{d+ex}\,(6be^2+5cd^2)}{24e^4}-\frac{x\sqrt{d-ex}\sqrt{d+ex}\,(8ae^4+6bd^2e^2+5cd^4)}{16e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -c*x**5*sqrt(d - e*x)*sqrt(d + e*x)/(6*e**2) + d**2*sqrt(d - e*x)
*sqrt(d + e*x)*(8*a*e**4 + 6*b*d**2*e**2 + 5*c*d**4)*atan(e*x/sqr
t(d**2 - e**2*x**2))/(16*e**7*sqrt(d**2 - e**2*x**2)) - x**3*sqrt
(d - e*x)*sqrt(d + e*x)*(6*b*e**2 + 5*c*d**2)/(24*e**4) - x*sqrt(
d - e*x)*sqrt(d + e*x)*(8*a*e**4 + 6*b*d**2*e**2 + 5*c*d**4)/(16*
e**6)
```

Mathematica [A] time = 0.229209, size = 136, normalized size = 0.63

$$\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d-ex}\sqrt{d+ex}}\right)\left(8ae^4+6bd^2e^2+5cd^4\right)-ex\sqrt{d-ex}\sqrt{d+ex}\left(6\left(4ae^4+3bd^2e^2+2be^4x^2\right)+c\left(15d^4+10d^2e^2x^2+8\right.\right.}{48e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(3*b*d^2*e^2 + 4*a*e^4 + 2*
b*e^4*x^2) + c*(15*d^4 + 10*d^2*e^2*x^2 + 8*e^4*x^4))) + 3*d^2*(5
*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*ArcTan[(e*x)/(Sqrt[d - e*x]*Sqrt[
d + e*x])])/(48*e^7)
```

Maple [C] time = 0.046, size = 273, normalized size = 1.3

$$-\frac{csgn(e)}{48\,e^7}\sqrt{-ex+d}\sqrt{ex+d}\left(8\,csgn(e)\,x^5ce^5\sqrt{-e^2x^2+d^2}+12\,csgn(e)\,x^3be^5\sqrt{-e^2x^2+d^2}+10\,csgn(e)\,x^3cd^2e^3\sqrt{-e^2x^2+d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/48*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(8*csgn(e)*x^5*c*e^5*(-e^2*x^2
+d^2)^(1/2)+12*csgn(e)*x^3*b*e^5*(-e^2*x^2+d^2)^(1/2)+10*csgn(e)*
x^3*c*d^2*e^3*(-e^2*x^2+d^2)^(1/2)+24*a*x*(-e^2*x^2+d^2)^(1/2)*e^
5*csgn(e)+18*d^2*x*(-e^2*x^2+d^2)^(1/2)*b*e^3*csgn(e)+15*c*d^4*x*
(-e^2*x^2+d^2)^(1/2)*csgn(e)*e-24*a*d^2*arctan(csgn(e)*e*x/(-e^2*
x^2+d^2)^(1/2))*e^4-18*d^4*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2
))*b*e^2-15*c*d^6*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2)))*csgn(
e)/(-e^2*x^2+d^2)^(1/2)/e^7
```

Maxima [A] time = 0.787224, size = 309, normalized size = 1.43

$$-\frac{\sqrt{-e^2x^2+d^2}cx^5}{6\,e^2}-\frac{5\,\sqrt{-e^2x^2+d^2}cd^2x^3}{24\,e^4}-\frac{\sqrt{-e^2x^2+d^2}bx^3}{4\,e^2}+\frac{5\,cd^6\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{16\,\sqrt{e^2}e^6}+\frac{3\,bd^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\,\sqrt{e^2}e^4}+\frac{ad^2\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\,\sqrt{e^2}e^2}-\frac{5\,\sqrt{-e^2x^2+d^2}cd^4x}{16\,e^6}-\frac{3\,\sqrt{-e^2x^2+d^2}bd^2x}{8\,e^4}-\frac{\sqrt{-e^2x^2+d^2}ax}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^2/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="ma
```

```
[Out] -1/6*sqrt(-e^2*x^2 + d^2)*c*x^5/e^2 - 5/24*sqrt(-e^2*x^2 + d^2)*c
*d^2*x^3/e^4 - 1/4*sqrt(-e^2*x^2 + d^2)*b*x^3/e^2 + 5/16*c*d^6*ar
csin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^6) + 3/8*b*d^4*arcsin(e^2*
x/sqrt(d^2*e^2))/(sqrt(e^2)*e^4) + 1/2*a*d^2*arcsin(e^2*x/sqrt(d^
2*e^2))/(sqrt(e^2)*e^2) - 5/16*sqrt(-e^2*x^2 + d^2)*c*d^4*x/e^6 -
 3/8*sqrt(-e^2*x^2 + d^2)*b*d^2*x/e^4 - 1/2*sqrt(-e^2*x^2 + d^2)*
a*x/e^2
```

Fricas [A] time = 0.386573, size = 921, normalized size = 4.26

$$48\,cde^{11}x^{11} - 4\left(61\,cd^3e^9 - 18\,bde^{11}\right)x^9 + 6\left(37\,cd^5e^7 - 58\,bd^3e^9 + 24\,ade^{11}\right)x^7 - 6\left(31\,cd^7e^5 - 14\,bd^5e^7 + 152\,ad^3e^9\right)x^5 + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^2/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="fr
```

```
[Out] 1/48*(48*c*d*e^11*x^11 - 4*(61*c*d^3*e^9 - 18*b*d*e^11)*x^9 + 6*(
37*c*d^5*e^7 - 58*b*d^3*e^9 + 24*a*d*e^11)*x^7 - 6*(31*c*d^7*e^5
- 14*b*d^5*e^7 + 152*a*d^3*e^9)*x^5 + 128*(5*c*d^9*e^3 + 6*b*d^7*
e^5 + 12*a*d^5*e^7)*x^3 - (8*c*e^11*x^11 - 2*(67*c*d^2*e^9 - 6*b*
e^11)*x^9 + 3*(73*c*d^4*e^7 - 66*b*d^2*e^9 + 8*a*e^11)*x^7 - 2*(2
3*c*d^6*e^5 - 126*b*d^4*e^7 + 216*a*d^2*e^9)*x^5 + 16*(25*c*d^8*e
^3 + 30*b*d^6*e^5 + 72*a*d^4*e^7)*x^3 - 96*(5*c*d^10*e + 6*b*d^8*
e^3 + 8*a*d^6*e^5)*x)*sqrt(e*x + d)*sqrt(-e*x + d) - 96*(5*c*d^11
*e + 6*b*d^9*e^3 + 8*a*d^7*e^5)*x + 6*(160*c*d^12 + 192*b*d^10*e^
2 + 256*a*d^8*e^4 - (5*c*d^6*e^6 + 6*b*d^4*e^8 + 8*a*d^2*e^10)*x^
6 + 18*(5*c*d^8*e^4 + 6*b*d^6*e^6 + 8*a*d^4*e^8)*x^4 - 48*(5*c*d^
10*e^2 + 6*b*d^8*e^4 + 8*a*d^6*e^6)*x^2 - 2*(80*c*d^11 + 96*b*d^9
*e^2 + 128*a*d^7*e^4 + 3*(5*c*d^7*e^4 + 6*b*d^5*e^6 + 8*a*d^3*e^8
)*x^4 - 16*(5*c*d^9*e^2 + 6*b*d^7*e^4 + 8*a*d^5*e^6)*x^2)*sqrt(e*
x + d)*sqrt(-e*x + d))*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/
(e*x)))/(e^13*x^6 - 18*d^2*e^11*x^4 + 48*d^4*e^9*x^2 - 32*d^6*e^7
 + 2*(3*d*e^11*x^4 - 16*d^3*e^9*x^2 + 16*d^5*e^7)*sqrt(e*x + d)*s
qrt(-e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.315731, size = 257, normalized size = 1.19

$$\frac{1}{34603008}\left(\left(33\,cd^5e^{36} + 30\,bd^3e^{38} + 24\,ade^{40} - \left(85\,cd^4e^{36} + 54\,bd^2e^{38} - 2\left(55\,cd^3e^{36} + 18\,bde^{38} - \left(45\,cd^2e^{36} + 4\left((xe+d)ce\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^2/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="gi
```

```
[Out] 1/34603008*((33*c*d^5*e^36 + 30*b*d^3*e^38 + 24*a*d*e^40 - (85*c*
d^4*e^36 + 54*b*d^2*e^38 - 2*(55*c*d^3*e^36 + 18*b*d*e^38 - (45*c
*d^2*e^36 + 4*((x*e + d)*c*e^36 - 5*c*d*e^36)*(x*e + d) + 6*b*e^3
8)*(x*e + d))*(x*e + d) + 24*a*e^40)*(x*e + d))*sqrt(x*e + d)*sqr
t(-x*e + d) + 6*(5*c*d^6*e^36 + 6*b*d^4*e^38 + 8*a*d^2*e^40)*arcs
in(1/2*sqrt(2)*sqrt(x*e + d)/sqrt(d)))*e^(-1)
```

3.140 $\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}} dx$

Optimal. Leaf size=128

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)\left(8ae^4+4bd^2e^2+3cd^4\right)}{4e^5}-\frac{x\sqrt{d-ex}\sqrt{d+ex}\left(4be^2+3cd^2\right)}{8e^4}+\frac{cx^3(ex-d)\sqrt{d+ex}}{4e^2\sqrt{d-ex}}$$

```
[Out] -((3*c*d^2 + 4*b*e^2)*x*Sqrt[d - e*x]*Sqrt[d + e*x])/(8*e^4) + (c
*x^3*(-d + e*x)*Sqrt[d + e*x])/(4*e^2*Sqrt[d - e*x]) - ((3*c*d^4
+ 4*b*d^2*e^2 + 8*a*e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/(4*
e^5)
```

Rubi [A] time = 0.259615, antiderivative size = 179, normalized size of antiderivative = 1.4, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$

$$\frac{\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(8ae^4+4bd^2e^2+3cd^4\right)}{8e^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{x\left(d^2-e^2x^2\right)\left(4be^2+3cd^2\right)}{8e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((3*c*d^2 + 4*b*e^2)*x*(d^2 - e^2*x^2))/(8*e^4*Sqrt[d - e*x]*Sqr
t[d + e*x]) - (c*x^3*(d^2 - e^2*x^2))/(4*e^2*Sqrt[d - e*x]*Sqrt[d
 + e*x]) + ((3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^5*Sqrt[d - e*x]*Sqrt[d +
 e*x])
```

Rubi in Sympy [A] time = 19.0009, size = 141, normalized size = 1.1

$$-\frac{cx^3\sqrt{d-ex}\sqrt{d+ex}}{4e^2}-\frac{x\sqrt{d-ex}\sqrt{d+ex}\left(4be^2+3cd^2\right)}{8e^4}+\frac{\sqrt{d-ex}\sqrt{d+ex}\left(8ae^4+4bd^2e^2+3cd^4\right)\operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -c*x**3*sqrt(d - e*x)*sqrt(d + e*x)/(4*e**2) - x*sqrt(d - e*x)*sq
rt(d + e*x)*(4*b*e**2 + 3*c*d**2)/(8*e**4) + sqrt(d - e*x)*sqrt(d
 + e*x)*(8*a*e**4 + 4*b*d**2*e**2 + 3*c*d**4)*atan(e*x/sqrt(d**2
- e**2*x**2))/(8*e**5*sqrt(d**2 - e**2*x**2))
```

Mathematica [A] time = 0.14877, size = 99, normalized size = 0.77

$$\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d-ex}\sqrt{d+ex}}\right)\left(8ae^4+4bd^2e^2+3cd^4\right)-ex\sqrt{d-ex}\sqrt{d+ex}\left(4be^2+3cd^2+2ce^2x^2\right)}{8e^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(3*c*d^2 + 4*b*e^2 + 2*c*e^2*x
^2)) + (3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*ArcTan[(e*x)/(Sqrt[d - e
*x]*Sqrt[d + e*x])])/(8*e^5)
```

Maple [C] time = 0.029, size = 191, normalized size = 1.5

$$-\frac{csgn(e)}{8e^5}\sqrt{-ex+d}\sqrt{ex+d}\left(2\,csgn(e)\,x^3ce^3\sqrt{-e^2x^2+d^2}+4\,bx\sqrt{-e^2x^2+d^2}e^3\,csgn(e)+3\,cd^2x\sqrt{-e^2x^2+d^2}csgn(e)\,e-8\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/8*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(2*csgn(e)*x^3*c*e^3*(-e^2*x^2+
d^2)^(1/2)+4*b*x*(-e^2*x^2+d^2)^(1/2)*e^3*csgn(e)+3*c*d^2*x*(-e^2
*x^2+d^2)^(1/2)*csgn(e)*e-8*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/
2))*a*e^4-4*b*d^2*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*e^2-3*
c*d^4*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2)))*csgn(e)/(-e^2*x^2
+d^2)^(1/2)/e^5
```

Maxima [A] time = 0.795528, size = 201, normalized size = 1.57

$$-\frac{\sqrt{-e^2x^2+d^2}cx^3}{4\,e^2}+\frac{a\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}}+\frac{3\,cd^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\sqrt{e^2}e^4}+\frac{bd^2\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\sqrt{e^2}e^2}-\frac{3\sqrt{-e^2x^2+d^2}cd^2x}{8\,e^4}-\frac{\sqrt{-e^2x^2+d^2}bx}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="maxima
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*c*x^3/e^2 + a*arcsin(e^2*x/sqrt(d^2*e^2
))/sqrt(e^2) + 3/8*c*d^4*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e
^4) + 1/2*b*d^2*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) - 3/8
*sqrt(-e^2*x^2 + d^2)*c*d^2*x/e^4 - 1/2*sqrt(-e^2*x^2 + d^2)*b*x/
e^2
```

Fricas [A] time = 0.282719, size = 571, normalized size = 4.46

$$\frac{8\,cde^7x^7-4\left(3\,cd^3e^5-4\,bde^7\right)x^5-4\left(5\,cd^5e^3+12\,bd^3e^5\right)x^3-\left(2\,ce^7x^7-\left(13\,cd^2e^5-4\,be^7\right)x^5-8\left(cd^4e^3+4\,bd^2e^5\right)x^3+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="fricas
```

```
[Out] 1/8*(8*c*d*e^7*x^7 - 4*(3*c*d^3*e^5 - 4*b*d*e^7)*x^5 - 4*(5*c*d^5
*e^3 + 12*b*d^3*e^5)*x^3 - (2*c*e^7*x^7 - (13*c*d^2*e^5 - 4*b*e^7
)*x^5 - 8*(c*d^4*e^3 + 4*b*d^2*e^5)*x^3 + 8*(3*c*d^6*e + 4*b*d^4*
e^3)*x)*sqrt(e*x + d)*sqrt(-e*x + d) + 8*(3*c*d^7*e + 4*b*d^5*e^3
)*x - 2*(24*c*d^8 + 32*b*d^6*e^2 + 64*a*d^4*e^4 + (3*c*d^4*e^4 +
4*b*d^2*e^6 + 8*a*e^8)*x^4 - 8*(3*c*d^6*e^2 + 4*b*d^4*e^4 + 8*a*d
^2*e^6)*x^2 - 4*(6*c*d^7 + 8*b*d^5*e^2 + 16*a*d^3*e^4 - (3*c*d^5*
e^2 + 4*b*d^3*e^4 + 8*a*d*e^6)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))
```

```
*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)))/(e^9*x^4 - 8*d
^2*e^7*x^2 + 8*d^4*e^5 + 4*(d*e^7*x^2 - 2*d^3*e^5)*sqrt(e*x + d)*
sqrt(-e*x + d))
```

Sympy [A] time = 104.666, size = 325, normalized size = 2.54

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}+\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}-\frac{ibd^2G_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^3}+\frac{bd^2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^3}-\frac{icd^4G_{6,6}^{6,2}\left(\begin{matrix}-\frac{7}{4},-\frac{5}{4} & -\frac{3}{2},-\frac{3}{2},-1,1\\ -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^5}+\frac{cd^4G_{6,6}^{2,6}\left(\begin{matrix}-\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},1 & \\ -\frac{9}{4},-\frac{7}{4} & -\frac{5}{2},-2,-2,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4,
1, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e) + a*meijerg(((-1/2,
 -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), d**2
*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e) - I*b*d**2*meije
rg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0,
 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**3) + b*d**2*meijerg((
(-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -
1, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**3) -
 I*c*d**4*meijerg(((-7/4, -5/4), (-3/2, -3/2, -1, 1)), ((-2, -7/4
, -3/2, -5/4, -1, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**5) +
 c*d**4*meijerg(((-5/2, -9/4, -2, -7/4, -3/2, 1), ()), ((-9/4, -7
/4), (-5/2, -2, -2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*
pi**(3/2)*e**5)
```

GIAC/XCAS [A] time = 0.308596, size = 170, normalized size = 1.33

$$\frac{1}{114688}\left(\left(5\,cd^3e^{16}+4\,bde^{18}-\left(9\,cd^2e^{16}+2\left((xe+d)ce^{16}-3\,cde^{16}\right)(xe+d)+4\,be^{18}\right)(xe+d)\right)\sqrt{xe+d}\sqrt{-xe+d}+2\left(3\,cd^4e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)),x, algorithm="giac")
```

```
[Out] 1/114688*((5*c*d^3*e^16 + 4*b*d*e^18 - (9*c*d^2*e^16 + 2*((x*e +
d)*c*e^16 - 3*c*d*e^16)*(x*e + d) + 4*b*e^18)*(x*e + d))*sqrt(x*e
 + d)*sqrt(-x*e + d) + 2*(3*c*d^4*e^16 + 4*b*d^2*e^18 + 8*a*e^20)
*arcsin(1/2*sqrt(2)*sqrt(x*e + d)/sqrt(d)))*e^(-1)
```

3.141 $\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=102

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{d^2x}-\frac{\left(2be^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e^3}+\frac{cx(ex-d)\sqrt{d+ex}}{2e^2\sqrt{d-ex}}$$

```
[Out] -((a*Sqrt[d - e*x]*Sqrt[d + e*x])/(d^2*x)) + (c*x*(-d + e*x)*Sqrt
[d + e*x])/(2*e^2*Sqrt[d - e*x]) - ((c*d^2 + 2*b*e^2)*ArcTan[Sqrt
[d - e*x]/Sqrt[d + e*x]])/e^3
```

Rubi [A] time = 0.392118, antiderivative size = 155, normalized size of antiderivative = 1.52, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$-\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}}+\frac{\sqrt{d^2-e^2x^2}\left(2be^2+cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx\left(d^2-e^2x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((a*(d^2 - e^2*x^2))/(d^2*x*Sqrt[d - e*x]*Sqrt[d + e*x])) - (c*x
*(d^2 - e^2*x^2))/(2*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((c*d^2 +
 2*b*e^2)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/
(2*e^3*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 20.1334, size = 114, normalized size = 1.12

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{d^2x}-\frac{cx\sqrt{d-ex}\sqrt{d+ex}}{2e^2}+\frac{\sqrt{d-ex}\sqrt{d+ex}\left(2be^2+cd^2\right)\operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -a*sqrt(d - e*x)*sqrt(d + e*x)/(d**2*x) - c*x*sqrt(d - e*x)*sqrt(
d + e*x)/(2*e**2) + sqrt(d - e*x)*sqrt(d + e*x)*(2*b*e**2 + c*d**
2)*atan(e*x/sqrt(d**2 - e**2*x**2))/(2*e**3*sqrt(d**2 - e**2*x**2
))
```

Mathematica [A] time = 0.166743, size = 82, normalized size = 0.8

$$\sqrt{d-ex}\sqrt{d+ex}\left(-\frac{a}{d^2x}-\frac{cx}{2e^2}\right)+\frac{\left(2be^2+cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d-ex}\sqrt{d+ex}}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(a/(d^2*x)) - (c*x)/(2*e^2))*Sqrt[d - e*x]*Sqrt[d + e*x] + ((c*
d^2 + 2*b*e^2)*ArcTan[(e*x)/(Sqrt[d - e*x]*Sqrt[d + e*x])])/(2*e^
```

3)

Maple [C] time = 0.031, size = 148, normalized size = 1.5

$$-\frac{csgn(e)}{2d^2e^3x}\sqrt{-ex+d}\sqrt{ex+d}\left(csgn(e)x^2cd^2e\sqrt{-e^2x^2+d^2}-2\arctan\left(\frac{csgn(e)ex}{\sqrt{-e^2x^2+d^2}}\right)xbd^2e^2-\arctan\left(csgn(e)ex\frac{1}{\sqrt{-e^2x^2+\ldots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/2*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2*(csgn(e)*x^2*c*d^2*e*(-e^2*
x^2+d^2)^(1/2)-2*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*x*b*d^2
*e^2-arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*x*c*d^4+2*a*(-e^2*x
^2+d^2)^(1/2)*e^3*csgn(e))*csgn(e)/(-e^2*x^2+d^2)^(1/2)/e^3/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^2),x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277357, size = 424, normalized size = 4.16

$$-\frac{cd^2e^5x^6+8\,ad^4e^3-\left(5\,cd^4e^3-2\,ae^7\right)x^4+2\left(2\,cd^6e-5\,ad^2e^5\right)x^2+\left(3\,cd^3e^3x^4-8\,ad^3e^3-2\left(2\,cd^5e-3\,ade^5\right)x^2\right)\sqrt{ex+d}\ldots}{2\left(3\,d^3e^5x^3-4\,d^5\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^2),x, algorithm="fr
```

```
[Out] -1/2*(c*d^2*e^5*x^6 + 8*a*d^4*e^3 - (5*c*d^4*e^3 - 2*a*e^7)*x^4 +
 2*(2*c*d^6*e - 5*a*d^2*e^5)*x^2 + (3*c*d^3*e^3*x^4 - 8*a*d^3*e^3
 - 2*(2*c*d^5*e - 3*a*d*e^5)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d) +
2*(3*(c*d^5*e^2 + 2*b*d^3*e^4)*x^3 - ((c*d^4*e^2 + 2*b*d^2*e^4)*x
^3 - 4*(c*d^6 + 2*b*d^4*e^2)*x)*sqrt(e*x + d)*sqrt(-e*x + d) - 4*
(c*d^7 + 2*b*d^5*e^2)*x)*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d
)/(e*x)))/(3*d^3*e^5*x^3 - 4*d^5*e^3*x - (d^2*e^5*x^3 - 4*d^4*e^3
*x)*sqrt(e*x + d)*sqrt(-e*x + d))
```

Sympy [A] time = 118.302, size = 287, normalized size = 2.81

$$\frac{iaeG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{aeG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}+\frac{bG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}-\frac{icd^2G_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^3}+\frac{cd^2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4,
 2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**2) + a*e*meijerg(((
1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), d**
2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**2) - I*b*meijer
g(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
 d**2/(e**2*x**2))/(4*pi**(3/2)*e) + b*meijerg(((-1/2, -1/4, 0, 1
/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), d**2*exp_polar(
-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e) - I*c*d**2*meijerg(((-3/4,
-1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), d
**2/(e**2*x**2))/(4*pi**(3/2)*e**3) + c*d**2*meijerg(((-3/2, -5/4
, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), d**
2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**3)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^2),x, algorithm="gi
```

```
[Out] Exception raised: NotImplementedError
```

3.142 $\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=157

$$-\frac{(d^2-e^2x^2)(2ae^2+3bd^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -(a*(d^2 - e^2*x^2))/(3*d^2*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((
3*b*d^2 + 2*a*e^2)*(d^2 - e^2*x^2))/(3*d^4*x*Sqrt[d - e*x]*Sqrt[d
 + e*x]) + (c*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/(e*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi [A] time = 0.417943, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$

$$-\frac{(d^2-e^2x^2)(2ae^2+3bd^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(3*d^2*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((
3*b*d^2 + 2*a*e^2)*(d^2 - e^2*x^2))/(3*d^4*x*Sqrt[d - e*x]*Sqrt[d
 + e*x]) + (c*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2
]])/(e*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 19.8509, size = 133, normalized size = 0.85

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{3d^2x^3}-\frac{2ae^2\sqrt{d-ex}\sqrt{d+ex}}{3d^4x}-\frac{b\sqrt{d-ex}\sqrt{d+ex}}{d^2x}+\frac{c\sqrt{d-ex}\sqrt{d+ex}\,\mathrm{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**4/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -a*sqrt(d - e*x)*sqrt(d + e*x)/(3*d**2*x**3) - 2*a*e**2*sqrt(d -
e*x)*sqrt(d + e*x)/(3*d**4*x) - b*sqrt(d - e*x)*sqrt(d + e*x)/(d*
*2*x) + c*sqrt(d - e*x)*sqrt(d + e*x)*atan(e*x/sqrt(d**2 - e**2*x
**2))/(e*sqrt(d**2 - e**2*x**2))
```

Mathematica [A] time = 0.181367, size = 84, normalized size = 0.54

$$\sqrt{d-ex}\sqrt{d+ex}\left(\frac{-2ae^2-3bd^2}{3d^4x}-\frac{a}{3d^2x^3}\right)+\frac{c\tan^{-1}\left(\frac{ex}{\sqrt{d-ex}\sqrt{d+ex}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-a/(3*d^2*x^3) + (-3*b*d^2 - 2*a*e^2)/(3*d^4*x))*Sqrt[d - e*x]*S
qrt[d + e*x] + (c*ArcTan[(e*x)/(Sqrt[d - e*x]*Sqrt[d + e*x])])/e
```

Maple [C] time = 0.031, size = 146, normalized size = 0.9

$$-\frac{csgn\left(e\right)}{3\,d^{4}x^{3}e}\sqrt{-ex+d}\sqrt{ex+d}\left(-3\,\arctan\left(\frac{csgn\left(e\right)ex}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)x^{3}cd^{4}+2\,\sqrt{-e^{2}x^{2}+d^{2}}ae^{3}x^{2}csgn\left(e\right)+3\,\sqrt{-e^{2}x^{2}+d^{2}}bd^{2}x^{2}csgn\left(e\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^4*(-3*arctan(csgn(e)*e*x/(-e^
2*x^2+d^2)^(1/2))*x^3*c*d^4+2*(-e^2*x^2+d^2)^(1/2)*a*e^3*x^2*csgn
(e)+3*(-e^2*x^2+d^2)^(1/2)*b*d^2*x^2*csgn(e)*e+a*(-e^2*x^2+d^2)^(
1/2)*d^2*csgn(e)*e)*csgn(e)/(-e^2*x^2+d^2)^(1/2)/x^3/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^4),x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288593, size = 396, normalized size = 2.52

$$-\frac{4\,ad^{6}e+\left(3\,bd^{2}e^{5}+2\,ae^{7}\right)x^{6}-3\,\left(5\,bd^{4}e^{3}+3\,ad^{2}e^{5}\right)x^{4}+3\,\left(4\,bd^{6}e+ad^{4}e^{3}\right)x^{2}-\left(4\,ad^{5}e-3\,\left(3\,bd^{3}e^{3}+2\,ade^{5}\right)x^{4}+\left(12\,bd\right.\right.}{3\,\left(3\,d^{5}e^{3}x^{5}-4\,d^{7}ex^{3}-\left(d^{4}e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^4),x, algorithm="fr
```

```
[Out] -1/3*(4*a*d^6*e + (3*b*d^2*e^5 + 2*a*e^7)*x^6 - 3*(5*b*d^4*e^3 +
3*a*d^2*e^5)*x^4 + 3*(4*b*d^6*e + a*d^4*e^3)*x^2 - (4*a*d^5*e - 3
*(3*b*d^3*e^3 + 2*a*d*e^5)*x^4 + (12*b*d^5*e + 5*a*d^3*e^3)*x^2)*
sqrt(e*x + d)*sqrt(-e*x + d) + 6*(3*c*d^5*e^2*x^5 - 4*c*d^7*x^3 -
 (c*d^4*e^2*x^5 - 4*c*d^6*x^3)*sqrt(e*x + d)*sqrt(-e*x + d))*arct
an((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)))/(3*d^5*e^3*x^5 - 4*
d^7*e*x^3 - (d^4*e^3*x^5 - 4*d^6*e*x^3)*sqrt(e*x + d)*sqrt(-e*x +
 d))
```

Sympy [A] time = 167.912, size = 257, normalized size = 1.64

$$\frac{iae^{3}G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{d^{2}}{e^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{4}}+\frac{ae^{3}G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\\frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{d^{2}e^{-2i\pi}}{e^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{4}}+\frac{ibeG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{d^{2}}{e^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{2}}+\frac{beG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\\frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{d^{2}e^{-2i\pi}}{e^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{2}}-\frac{icG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{d^{2}}{e^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}e}+\frac{cG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{d^{2}e^{-2i\pi}}{e^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**4/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2,
11/4, 3), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**4) + a*e**3*me
ijerg(((3/2, 7/4, 2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2,
0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**4) + I*
b*e*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2
), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**2) + b*e*meijerg(((1/
2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), d**2*
exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(
((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), d
**2/(e**2*x**2))/(4*pi**(3/2)*e) + c*meijerg(((-1/2, -1/4, 0, 1/4
, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), d**2*exp_polar(-2
*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^4),x, algorithm="gi
```

```
[Out] Exception raised: NotImplementedError
```

3.143 $\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=160

$$-\frac{(d^2-e^2x^2)\,(8ae^4+10bd^2e^2+15cd^4)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)\,(4ae^2+5bd^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\,(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -(a*(d^2 - e^2*x^2))/(5*d^2*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((
5*b*d^2 + 4*a*e^2)*(d^2 - e^2*x^2))/(15*d^4*x^3*Sqrt[d - e*x]*Sqr
t[d + e*x]) - ((15*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*(d^2 - e^2*x^2
))/(15*d^6*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi [A] time = 0.44472, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$

$$-\frac{(d^2-e^2x^2)\,(8ae^4+10bd^2e^2+15cd^4)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)\,(4ae^2+5bd^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\,(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(5*d^2*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((
5*b*d^2 + 4*a*e^2)*(d^2 - e^2*x^2))/(15*d^4*x^3*Sqrt[d - e*x]*Sqr
t[d + e*x]) - ((15*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*(d^2 - e^2*x^2
))/(15*d^6*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 21.9054, size = 153, normalized size = 0.96

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{5d^2x^5}+\frac{c\sqrt{d-ex}\sqrt{d+ex}}{2e^2x^3}-\frac{\sqrt{d-ex}\sqrt{d+ex}\,(8ae^4+10bd^2e^2+15cd^4)}{30d^4e^2x^3}-\frac{\sqrt{d-ex}\sqrt{d+ex}\,(8ae^4+10bd^2e^2+15cd^4)}{15d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**6/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -a*sqrt(d - e*x)*sqrt(d + e*x)/(5*d**2*x**5) + c*sqrt(d - e*x)*sq
rt(d + e*x)/(2*e**2*x**3) - sqrt(d - e*x)*sqrt(d + e*x)*(8*a*e**4
 + 10*b*d**2*e**2 + 15*c*d**4)/(30*d**4*e**2*x**3) - sqrt(d - e*x
)*sqrt(d + e*x)*(8*a*e**4 + 10*b*d**2*e**2 + 15*c*d**4)/(15*d**6*
x)
```

Mathematica [A] time = 0.118727, size = 87, normalized size = 0.54

$$\sqrt{d-ex}\sqrt{d+ex}\left(\frac{-8ae^4-10bd^2e^2-15cd^4}{15d^6x}+\frac{-4ae^2-5bd^2}{15d^4x^3}-\frac{a}{5d^2x^5}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-a/(5*d^2*x^5) + (-5*b*d^2 - 4*a*e^2)/(15*d^4*x^3) + (-15*c*d^4
- 10*b*d^2*e^2 - 8*a*e^4)/(15*d^6*x))*Sqrt[d - e*x]*Sqrt[d + e*x]
```

Maple [A] time = 0.009, size = 82, normalized size = 0.5

$$-\frac{8\,a e^{4} x^{4}+10\,b d^{2} e^{2} x^{4}+15\,c d^{4} x^{4}+4\,a d^{2} e^{2} x^{2}+5\,b d^{4} x^{2}+3\,a d^{4}}{15\,x^{5} d^{6}}\sqrt{ex+d}\sqrt{-ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(8*a*e^4*x^4+10*b*d^2*e^2*x^4+
15*c*d^4*x^4+4*a*d^2*e^2*x^2+5*b*d^4*x^2+3*a*d^4)/x^5/d^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^6),x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.334463, size = 505, normalized size = 3.16

$$\frac{48\,ad^{10}-\left(15\,cd^{4}e^{6}+10\,bd^{2}e^{8}+8\,ae^{10}\right)x^{10}+5\left(39\,cd^{6}e^{4}+25\,bd^{4}e^{6}+20\,ad^{2}e^{8}\right)x^{8}-5\left(84\,cd^{8}e^{2}+43\,bd^{6}e^{4}+35\,ad^{4}e^{6}\right)x^{6}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^6),x, algorithm="fr
```

```
[Out] 1/15*(48*a*d^10 - (15*c*d^4*e^6 + 10*b*d^2*e^8 + 8*a*e^10)*x^10 +
 5*(39*c*d^6*e^4 + 25*b*d^4*e^6 + 20*a*d^2*e^8)*x^8 - 5*(84*c*d^8
*e^2 + 43*b*d^6*e^4 + 35*a*d^4*e^6)*x^6 + 5*(48*c*d^10 + 4*b*d^8*
e^2 + 11*a*d^6*e^4)*x^4 + 20*(4*b*d^10 - a*d^8*e^2)*x^2 - (48*a*d
^9 + 5*(15*c*d^5*e^4 + 10*b*d^3*e^6 + 8*a*d*e^8)*x^8 - 5*(60*c*d^
7*e^2 + 35*b*d^5*e^4 + 28*a*d^3*e^6)*x^6 + 3*(80*c*d^9 + 20*b*d^7
*e^2 + 21*a*d^5*e^4)*x^4 + 4*(20*b*d^9 + a*d^7*e^2)*x^2)*sqrt(e*x
 + d)*sqrt(-e*x + d))/(5*d^7*e^4*x^9 - 20*d^9*e^2*x^7 + 16*d^11*x
^5 - (d^6*e^4*x^9 - 12*d^8*e^2*x^7 + 16*d^10*x^5)*sqrt(e*x + d)*s
qrt(-e*x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**6/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.535734, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^6),x, algorithm="gi
```

```
[Out] Done
```

3.144 $\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=226

$$-\frac{2e^2\left(d^2-e^2x^2\right)\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(6ae^2+7bd^2\right)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -(a*(d^2 - e^2*x^2))/(7*d^2*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((
7*b*d^2 + 6*a*e^2)*(d^2 - e^2*x^2))/(35*d^4*x^5*Sqrt[d - e*x]*Sqr
t[d + e*x]) - ((35*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^
2))/(105*d^6*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - (2*e^2*(35*c*d^4
+ 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^2))/(105*d^8*x*Sqrt[d - e
*x]*Sqrt[d + e*x])
```

Rubi [A] time = 0.54504, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{2e^2\left(d^2-e^2x^2\right)\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(6ae^2+7bd^2\right)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(7*d^2*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((
7*b*d^2 + 6*a*e^2)*(d^2 - e^2*x^2))/(35*d^4*x^5*Sqrt[d - e*x]*Sqr
t[d + e*x]) - ((35*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^
2))/(105*d^6*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - (2*e^2*(35*c*d^4
+ 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^2))/(105*d^8*x*Sqrt[d - e
*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 26.3829, size = 207, normalized size = 0.92

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{7d^2x^7}+\frac{c\sqrt{d-ex}\sqrt{d+ex}}{4e^2x^5}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(24ae^4+28bd^2e^2+35cd^4\right)}{140d^4e^2x^5}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^6x^3}-\frac{2e^2\sqrt{d-ex}\sqrt{d+ex}\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**8/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -a*sqrt(d - e*x)*sqrt(d + e*x)/(7*d**2*x**7) + c*sqrt(d - e*x)*sq
rt(d + e*x)/(4*e**2*x**5) - sqrt(d - e*x)*sqrt(d + e*x)*(24*a*e**
4 + 28*b*d**2*e**2 + 35*c*d**4)/(140*d**4*e**2*x**5) - sqrt(d - e
*x)*sqrt(d + e*x)*(24*a*e**4 + 28*b*d**2*e**2 + 35*c*d**4)/(105*d
**6*x**3) - 2*e**2*sqrt(d - e*x)*sqrt(d + e*x)*(24*a*e**4 + 28*b*
d**2*e**2 + 35*c*d**4)/(105*d**8*x)
```

Mathematica [A] time = 0.150053, size = 122, normalized size = 0.54

$$\sqrt{d-ex}\sqrt{d+ex}\left(-\frac{2e^2\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^8x}+\frac{-24ae^4-28bd^2e^2-35cd^4}{105d^6x^3}+\frac{-6ae^2-7bd^2}{35d^4x^5}-\frac{a}{7d^2x^7}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-a/(7*d^2*x^7) + (-7*b*d^2 - 6*a*e^2)/(35*d^4*x^5) + (-35*c*d^4
- 28*b*d^2*e^2 - 24*a*e^4)/(105*d^6*x^3) - (2*e^2*(35*c*d^4 + 28*
b*d^2*e^2 + 24*a*e^4))/(105*d^8*x))*Sqrt[d - e*x]*Sqrt[d + e*x]
```

Maple [A] time = 0.009, size = 118, normalized size = 0.5

$$-\frac{48\,ae^{6}x^{6}+56\,bd^{2}e^{4}x^{6}+70\,cd^{4}e^{2}x^{6}+24\,ad^{2}e^{4}x^{4}+28\,bd^{4}e^{2}x^{4}+35\,cd^{6}x^{4}+18\,ad^{4}e^{2}x^{2}+21\,bd^{6}x^{2}+15\,ad^{6}}{105\,x^{7}d^{8}}\sqrt{ex+d}\sqrt{-ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(48*a*e^6*x^6+56*b*d^2*e^4*x^
6+70*c*d^4*e^2*x^6+24*a*d^2*e^4*x^4+28*b*d^4*e^2*x^4+35*c*d^6*x^4
+18*a*d^4*e^2*x^2+21*b*d^6*x^2+15*a*d^6)/x^7/d^8
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^8),x, algorithm="ma
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.556946, size = 713, normalized size = 3.15

$$-\frac{960\,ad^{14}+2\left(35\,cd^{4}e^{10}+28\,bd^{2}e^{12}+24\,ae^{14}\right)x^{14}-49\left(35\,cd^{6}e^{8}+28\,bd^{4}e^{10}+24\,ad^{2}e^{12}\right)x^{12}+105\left(61\,cd^{8}e^{6}+49\,bd^{6}e^{8}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^8),x, algorithm="fr
```

```
[Out] -1/105*(960*a*d^14 + 2*(35*c*d^4*e^10 + 28*b*d^2*e^12 + 24*a*e^14
)*x^14 - 49*(35*c*d^6*e^8 + 28*b*d^4*e^10 + 24*a*d^2*e^12)*x^12 +
 105*(61*c*d^8*e^6 + 49*b*d^6*e^8 + 42*a*d^4*e^10)*x^10 - 7*(920*
c*d^10*e^4 + 811*b*d^8*e^6 + 693*a*d^6*e^8)*x^8 - 7*(80*c*d^12*e^
2 - 248*b*d^10*e^4 - 159*a*d^8*e^6)*x^6 + 56*(40*c*d^14 - 22*b*d^
12*e^2 + 9*a*d^10*e^4)*x^4 + 336*(4*b*d^14 - 3*a*d^12*e^2)*x^2 -
(960*a*d^13 - 14*(35*c*d^5*e^8 + 28*b*d^3*e^10 + 24*a*d*e^12)*x^1
2 + 105*(35*c*d^7*e^6 + 28*b*d^5*e^8 + 24*a*d^3*e^10)*x^10 - 21*(
280*c*d^9*e^4 + 231*b*d^7*e^6 + 198*a*d^5*e^8)*x^8 + (560*c*d^11*
e^2 + 1624*b*d^9*e^4 + 1287*a*d^7*e^6)*x^6 + 40*(56*c*d^13 - 14*b
*d^11*e^2 + 9*a*d^9*e^4)*x^4 + 48*(28*b*d^13 - 11*a*d^11*e^2)*x^2
)*sqrt(e*x + d)*sqrt(-e*x + d))/(7*d^9*e^6*x^13 - 56*d^11*e^4*x^1
1 + 112*d^13*e^2*x^9 - 64*d^15*x^7 - (d^8*e^6*x^13 - 24*d^10*e^4*
x^11 + 80*d^12*e^2*x^9 - 64*d^14*x^7)*sqrt(e*x + d)*sqrt(-e*x + d
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**8/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.864267, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^8),x, algorithm="gi
```

```
[Out] Done
```

3.145 $\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=292

$$-\frac{8e^4\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}}-\frac{4e^2\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(8ae^2+9bd^2\right)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -(a*(d^2 - e^2*x^2))/(9*d^2*x^9*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((
9*b*d^2 + 8*a*e^2)*(d^2 - e^2*x^2))/(63*d^4*x^7*Sqrt[d - e*x]*Sqr
t[d + e*x]) - ((21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^
2))/(105*d^6*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - (4*e^2*(21*c*d^4
+ 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(315*d^8*x^3*Sqrt[d -
 e*x]*Sqrt[d + e*x]) - (8*e^4*(21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4
)*(d^2 - e^2*x^2))/(315*d^10*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi [A] time = 0.684247, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{8e^4\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}}-\frac{4e^2\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(8ae^2+9bd^2\right)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(9*d^2*x^9*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((
9*b*d^2 + 8*a*e^2)*(d^2 - e^2*x^2))/(63*d^4*x^7*Sqrt[d - e*x]*Sqr
t[d + e*x]) - ((21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^
2))/(105*d^6*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - (4*e^2*(21*c*d^4
+ 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(315*d^8*x^3*Sqrt[d -
 e*x]*Sqrt[d + e*x]) - (8*e^4*(21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4
)*(d^2 - e^2*x^2))/(315*d^10*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi in Sympy [A] time = 31.3793, size = 262, normalized size = 0.9

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{9d^2x^9}+\frac{c\sqrt{d-ex}\sqrt{d+ex}}{6e^2x^7}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(16ae^4+18bd^2e^2+21cd^4\right)}{126d^4e^2x^7}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(16ae^4+18bd^2e^2+21cd^4\right)}{105d^6x^5}-\frac{4e^2\sqrt{d-ex}\sqrt{d+ex}\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^8x^3}-\frac{8e^4\sqrt{d-ex}\sqrt{d+ex}\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^{10}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)/x**10/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -a*sqrt(d - e*x)*sqrt(d + e*x)/(9*d**2*x**9) + c*sqrt(d - e*x)*sq
rt(d + e*x)/(6*e**2*x**7) - sqrt(d - e*x)*sqrt(d + e*x)*(16*a*e**
4 + 18*b*d**2*e**2 + 21*c*d**4)/(126*d**4*e**2*x**7) - sqrt(d - e
*x)*sqrt(d + e*x)*(16*a*e**4 + 18*b*d**2*e**2 + 21*c*d**4)/(105*d
**6*x**5) - 4*e**2*sqrt(d - e*x)*sqrt(d + e*x)*(16*a*e**4 + 18*b*
d**2*e**2 + 21*c*d**4)/(315*d**8*x**3) - 8*e**4*sqrt(d - e*x)*sqr
```

```
t(d + e*x)*(16*a*e**4 + 18*b*d**2*e**2 + 21*c*d**4)/(315*d**10*x)
```

Mathematica [A] time = 0.186516, size = 157, normalized size = 0.54

$$\sqrt{d-ex}\sqrt{d+ex}\left(-\frac{8e^4\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^{10}x}-\frac{4e^2\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^8x^3}+\frac{-16ae^4-18bd^2e^2-21cd^4}{105d^6x^5}+\frac{-8ae^2-9bd^2}{63d^4x^7}-\frac{a}{9d^2x^9}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-a/(9*d^2*x^9) + (-9*b*d^2 - 8*a*e^2)/(63*d^4*x^7) + (-21*c*d^4
- 18*b*d^2*e^2 - 16*a*e^4)/(105*d^6*x^5) - (4*e^2*(21*c*d^4 + 18*
b*d^2*e^2 + 16*a*e^4))/(315*d^8*x^3) - (8*e^4*(21*c*d^4 + 18*b*d^
2*e^2 + 16*a*e^4))/(315*d^10*x))*Sqrt[d - e*x]*Sqrt[d + e*x]
```

Maple [A] time = 0.014, size = 154, normalized size = 0.5

$$-\frac{128\,ae^8x^8+144\,bd^2e^6x^8+168\,cd^4e^4x^8+64\,ad^2e^6x^6+72\,bd^4e^4x^6+84\,cd^6e^2x^6+48\,ad^4e^4x^4+54\,bd^6e^2x^4+63\,cd^8x^4+40}{315\,x^9d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(128*a*e^8*x^8+144*b*d^2*e^6*
x^8+168*c*d^4*e^4*x^8+64*a*d^2*e^6*x^6+72*b*d^4*e^4*x^6+84*c*d^6*
e^2*x^6+48*a*d^4*e^4*x^4+54*b*d^6*e^2*x^4+63*c*d^8*x^4+40*a*d^6*e
^2*x^2+45*b*d^8*x^2+35*a*d^8)/x^9/d^10
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^10),x, algorithm="m
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.14287, size = 922, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^10),x, algorithm="f
```

```
[Out] 1/315*(8960*a*d^18 - 8*(21*c*d^4*e^14 + 18*b*d^2*e^16 + 16*a*e^18
)*x^18 + 324*(21*c*d^6*e^12 + 18*b*d^4*e^14 + 16*a*d^2*e^16)*x^16
```

```
 - 2079*(21*c*d^8*e^10 + 18*b*d^6*e^12 + 16*a*d^4*e^14)*x^14 + 21
*(4507*c*d^10*e^8 + 3861*b*d^8*e^10 + 3432*a*d^6*e^12)*x^12 - 45*
(1736*c*d^12*e^6 + 1447*b*d^10*e^8 + 1287*a*d^8*e^10)*x^10 + 9*(3
024*c*d^14*e^4 + 1192*b*d^12*e^6 + 1219*a*d^10*e^8)*x^8 - 24*(952
*c*d^16*e^2 - 474*b*d^14*e^4 - 13*a*d^12*e^6)*x^6 + 144*(112*c*d^
18 - 124*b*d^16*e^2 + 57*a*d^14*e^4)*x^4 + 2880*(4*b*d^18 - 5*a*d
^16*e^2)*x^2 - (8960*a*d^17 + 72*(21*c*d^5*e^12 + 18*b*d^3*e^14 +
 16*a*d*e^16)*x^16 - 924*(21*c*d^7*e^10 + 18*b*d^5*e^12 + 16*a*d^
3*e^14)*x^14 + 3003*(21*c*d^9*e^8 + 18*b*d^7*e^10 + 16*a*d^5*e^12
)*x^12 - 45*(1512*c*d^11*e^6 + 1287*b*d^9*e^8 + 1144*a*d^7*e^10)*
x^10 + 5*(4368*c*d^13*e^4 + 2664*b*d^11*e^6 + 2431*a*d^9*e^8)*x^8
 - 8*(1848*c*d^15*e^2 - 846*b*d^13*e^4 - 227*a*d^11*e^6)*x^6 + 33
6*(48*c*d^17 - 36*b*d^15*e^2 + 13*a*d^13*e^4)*x^4 + 320*(36*b*d^1
7 - 31*a*d^15*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(9*d^11*e^8
*x^17 - 120*d^13*e^6*x^15 + 432*d^15*e^4*x^13 - 576*d^17*e^2*x^11
 + 256*d^19*x^9 - (d^10*e^8*x^17 - 40*d^12*e^6*x^15 + 240*d^14*e^
4*x^13 - 448*d^16*e^2*x^11 + 256*d^18*x^9)*sqrt(e*x + d)*sqrt(-e*
x + d))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**10/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 1.16691, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/(sqrt(e*x + d)*sqrt(-e*x + d)*x^10),x, algorithm="g

[Out] Done
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```